

MHD equilibria with arbitrary flow

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Abstract

In the last few years, an increasing interest has been devoted to plasma equilibria with macroscopic flow. As toroidal and poloidal rotation velocities have been measured in modern tokamak plasmas, their effects on the equilibrium need to be determined. As a basis for our analysis, we use Maxwell equations and Ohm's law for the fields, and continuity and momentum equations for the macroscopic fluid quantities. Plasma resistivity is neglected. Under the assumption of axisymmetry the previous set of equations can be reduced to 2 (3 for the anisotropic case) equations, depending on a certain number of free functions of the magnetic flux Ψ (5 for the isotropic, and 6 for the anisotropic case) [1] [2] [3]. More precisely, those are an equation for the magnetic flux, analogous to the classical Grad-Shafranov (GS) equation, one for the density, analogous to the Bernoulli equation, and in the anisotropic case an equation for the toroidal component of the magnetic field B_ϕ . The models of [1], [2] and [3] can be shown to be equivalent, the free functions of each of them being reducible to the functions of the others by the means of simple algebraic relations. The natural choice of free functions for the numerical solution of the equilibrium problem is then clearly to use the most easily implemented ones in the numerical solver, and the ones most easily related to meaningful physical quantities, such as density, pressures and sonic Mach numbers as input for the code. The code FLOW –developed at Rochester- is capable to compute anisotropic equilibria with arbitrary flow. FLOW has been applied to compute toroidally rotating anisotropic NSTX equilibria. Here the centrifugal force can produce a remarkable outward shift of the density profile if the rotation is sufficiently fast ($V_\phi \sim C_s$). The density profile is also influenced by the anisotropy. Namely, the outward shift of the density is increased if $\beta_\parallel < \beta_\perp$, decreased if $\beta_\parallel > \beta_\perp$. FLOW has also been used to determine equilibria with poloidal flow. When the poloidal flow is of the order of the poloidal sound speed [$C_{s\theta} = C_s B_\theta / B$], the pressure, density and velocity profiles develop radial discontinuities at the transonic surface. When the poloidal flow is even faster, exceeding the poloidal Alfvén velocity, a different kind of equilibrium can arise, with the magnetic axis moving to the inboard side, i.e. on the opposite side with respect to the traditional Shafranov shift. Examples of all these equilibria computed with FLOW are shown and discussed.

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