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Use of Nonsymmetric Algebraic Operators in a Semi-Implicit MHD Advance*

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Abstract

Semi-implicit numerical methods for hyperbolic PDEs are typically formulated with implicit coupling that is represented by symmetric- or Hermitian-positive-definite matrices in the time-advance equations. Having positive and wavenumber-dependent eigenvalues, the operators provide numerical stability for oscillatory behavior at arbitrarily large time-step by introducing unphysical dispersion.[1] This is useful for solving stiff systems, like resistive MHD in application to macroscopic behavior in fusion devices, where magnetic tearing behavior is orders of magnitude slower than the propagation of compressional waves. Here, the numerical dispersion allows computations with time-steps that are large in comparison to wave propagation times, and the accuracy of the computed slow behavior depends on how well the semi-implicit operator represents the modes of the MHD system.[1,2]

Effects that are not described by symmetric operators fall outside the strength of traditional semi-implicit methods. Advection due to fluid flow is the most important example for single-fluid MHD (see Ref. 3), but the Hall term in an extended-MHD description is also more easily described through nonsymmetric operators in a numerical algorithm. In addition, formulating the MHD advance in terms of a single nonsymmetric system for flow velocity, magnetic field, and pressure may provide better accuracy with finite element spatial representation than the traditional symmetric system for flow alone, because a 'mixed' discretization can separate shear and compressional behavior to a greater extent.[4]

Here, the use of nonsymmetric operators for semi-implicit MHD algorithms is explored through numerical analysis and implementation in the NIMROD code[5] (http://nimrodteam.org). A recently updated coupling to the AZTEC parallel linear system library (www.cs.sandia.gov/CRF/aztec1.html) has made it possible to use nonsymmetric systems in NIMROD, and their application is tested for advection, the Hall term, and the MHD advance. We also report on the performance of symmetric system solution methods in the AZTEC library, when applied to NIMROD computations, and compare with the native solver performance.

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