Extended MHD calculations with high order-of-continuity finite elements

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Abstract

The majority of recent magnetic fusion MHD applications using high-order finite elements have utilized elements of the C^0 class in which continuity of the function across element boundaries is imposed, but not continuity of derivatives. [1,2] Here, we describe some features and advantages of using high-order C^1 elements that are constructed so as to have the first derivatives continuous across element boundaries. The high order-of-continuity leads to smaller rank matrices, with similar sparseness patterns, compared to C^0 elements of the same order-of-accuracy, and also allows it to be used on differential operators up to fourth order. These occur, for example, in the viscosity operator in the vorticity equation, or in the hyperresistivity operator in the magnetic flux equation. The fact that it can be directly applied to high-order operators eliminates the need for auxiliary variables in many cases, which in turn leads to even smaller rank matrices. These features make high-accuracy fully implicit solutions of the multi-dimensional extended MHD equations feasible. Examples are presented with a particularly attractive triangular C^1 element with h^5 accuracy. [3] The integrations to form the matrix elements are all done in closed form, even for the nonlinear terms. Applications are illustrated for elliptic problems, anisotropic diffusion, the Grad-Shafranov-Schlüter equation, and the two-dimensional time-dependent MHD and extended MHD equations.

Request oral

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[2] A. H. Glasser and X. Tang, “The SEL macroscopic modeling code”, Comput. Phys. Comm., to appear 2004
[3] S. C. Jardin, “A triangular Finite Element with first-derivative continuity applied to Fusion MHD Applications”, PPPL-2931 Princeton Plasma Physics Laboratory (February 2004)