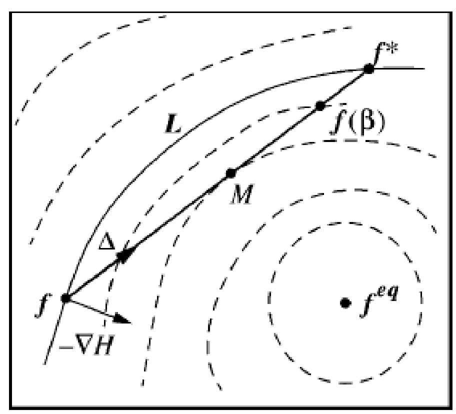
Entropic Lattice Boltzmann Schemes

G. Vahala[1], L. Vahala[2]
[1]College of William & Mary, Williamsburg, VA 23185
[2]Old Dominion University, Norfolk, VA 23529

Abstract

The straightforward Lattice Boltzmann scheme to solve nonlinear macroscopic equations like MHD is a very powerful, highly parallelizeable simple algorithm that has attained 3.6 Tflops/s on the *Earth Simulator*[1]. However, more refined schemes must be developed to avoid nonlinear numerical instabilities – especially as one pushes to regimes of smaller transport coefficients. The instabilities typically manifest themselves by the distribution function going negative in some regions of phase space. Recently[2], an elegant formulation of LBM has been introduced which ensures nonlinear numerical stability and the realizability constraint of non-negative distribution functions at every time step through the introduction of a discrete Liapunov functional. There are 4 basic steps : (1) determination of a discrete convex H-function subject to the appropriate moment constraints, (2) the determination of a bare collision operator Δ (not necessarily BGK); (3) the existence of a functional $\alpha[f]$ so that $H[f] = H[f + \alpha\Delta]$; (4) the evolution of the discrete kinetic equation is through the dressed collision operator $\Delta^* = \beta\alpha[f]\Delta$ where β is a parameter that controls the transport coefficient.

This entropic scheme is unconditionally stable. A plot of surfaces of $H = const.$ has an initial f on the surface L, with $-\nabla H$ having a component towards f^{eq}. Bare collision operator Δ drives the system to lower H, with $\alpha[f]$ so that $H[f] = H[f+\alpha\Delta] = H[f^*]$. Thus α is the maximal that will still permit a local H-theorem and realizability. The parameter $0 \le \beta \le 1$ yields the dressed collision operator $f \to (1-\beta) f + \beta f^*$, with $\beta \to 1$ yielding transport coefficients $\to 0$.



We shall employ this entropic lattice Boltzmann approach to the KdV and 1D MHD models.

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