Helicity in two-dimensional cold-plasma linear mode conversion

Allan N Kaufman, UC Berkeley & LBNL
Alain J Brizard, St Michael's College
Eugene R Tracy, College of William & Mary

When plasma parameters, such as density $n(\mathbf{x})$ and magnetic-field strength $B(\mathbf{x})$, have non-parallel gradients in a mode-conversion region, the analysis of conversion cannot be reduced to the standard local one-dimensional slab model. For 2 spatial dimensions (such as a poloidal cross section of a tokamak), the ray phase space $(\mathbf{x,k})$ is 4-dimensional, and the rays lie in the 3-dimensional surface $D=0$, where D, the determinant of the WKB dispersion matrix $\mathbf{D}(\mathbf{k,x})$, is the ray Hamiltonian. In previous work [*Tracy & Kaufman*, PRL **91** (2003) 130402], we have shown that generically the rays are helical: in one 2-dimensional subspace of the 4-dimensional phase space, a ray orbit is hyperbolic, while in the other sub-space it is elliptic.

We apply these ideas to the simplest non-trivial model, representing a unidirectional magnetic field (in the $\hat{z}$-direction) $B(x,y)$ and plasma density $n(x,y)$, with their gradients non-parallel. (In particular, we consider the extreme case $B(x)$, $n(y)$.) We adopt the cold-plasma model in the ion-gyrofreqency range, with the wave electric field in the (x,y)-plane. For this model, we obtain an explicit expression for ray helicity. We calculate and display ray orbits, and exhibit and calculate their helicity.

The wave field $\mathbf{E}(x,y)$, associated with a set of ray orbits, is obtained by first performing a linear canonical transformation from $(\mathbf{x,k})$ to a new set of coordinates (q_1, q_2, p_1, p_2). In these coordinates, the ray Hamiltonian is separable: $D(\mathbf{q,p}) = D_1(q_1,p_1) + D_2(q_2,p_2)$. The related wave equation is separable, and can be solved in the conversion region in terms of standard functions. The wave field is then expressed in terms of these local solutions by a generalized Fourier transform. In this way, incoming eikonal waves can be split into transmitted and converted outgoing eikonal waves, even though the ray helicity in the conversion region complicates the analysis.

Sherwood Fusion Theory Conference, Missoula, MT, April 26-28, 2004
Research supported by US DOE/OFES.