Nonlinear Evolution of Magnetic Shear-Localized Interchange Instabilities

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The Suydam's/Mercier criterion ($D_I < 1/4$) defines the stability boundary of a toroidally confined plasma for a magnetic shear localized ideal MHD pressure driven interchange instability. The linear stability analysis of these instabilities indicates a very weakly growing mode well above the marginal stability condition[1]. The nonlinear regime of these weakly growing instabilities will be presented in this work. For studying the nonlinear dynamics of these modes, the nonlinear vorticity equation, Ohm's law and pressure evolution equations are solved numerically in two spatial dimension. The perturbed quantities are expanded in Fourier modes in y direction. Then, each Fourier mode is evolved in both space x ("radial", pressure gradient direction) and time. Numerical results indicates that the instability saturate at low amplitude with the formation of current sheet at $ky = \pi/2$. Here, the magnetics plays a dominant role in the saturation of instability in contrast to the usual pressure flattening mechanism in the magnetic shear free interchange instability. In the nonlinear phase, the vorticity equation evolves as $d\nabla_\perp^2 \tilde{\phi}/dt \simeq 0$ which implies that the pressure gradient drive is balanced by magnetic field line bending. In this limit, pressure profile can be solved exactly and given by $\tilde{p} = X \pm \sqrt{X^2 - 2\tilde{\psi}}$ where $\tilde{\psi}$ is the perturbed magnetic flux function. Thus, from the vorticity equation, we get a grad shafronov type "equilibrium" equation in the presence of pressure gradient drive as

$$\nabla^2 \tilde{\psi}_N \mp \frac{D_I X}{\sqrt{2\tilde{\psi}_N}} = f(\tilde{\psi}_N)$$

where $\tilde{\psi}_N = X^2/2 - \tilde{\psi}$. The possible solution of this equation will be presented.

[1]Sangeeta Gupta, J. D. Callen and C. C. Hegna, "Violating Suydam criterion produces feeble instabilities," Phys. Plasmas **9**, 3395 (2002).