A fully implicit 3D MHD Newton-Krylov algorithm: a numerical proof-of-principle

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Abstract

Extended magnetohydrodynamics (XMHD) includes nonideal effects such as nonlinear, anisotropic transport and two-fluid (Hall) effects. XMHD supports multiple, separate time scales that make explicit time differencing approaches extremely inefficient. While a fully implicit implementation promises efficiency without sacrificing numerical accuracy,[1] the nonlinear nature of the XMHD system and the numerical stiffness associated with the fast waves make this endeavor difficult.

Newton-Krylov methods are, however, ideally suited for such a task. These synergistically combine Newton's method for nonlinear convergence, and Krylov techniques to solve the associated Jacobian (linear) systems. Krylov methods can be implemented Jacobian-free and can be preconditioned for efficiency. Successful preconditioning strategies have been developed for 2D incompressible resistive[2] and Hall[3] MHD models. These are based on "physics-based" ideas, in which knowledge of the physics is exploited to derive well-conditioned (diagonally-dominant) approximations to the original system that are amenable to optimal solver technologies (multigrid).

Recently,[4] a novel finite-volume discretization for implicit 3D MHD applications has been developed for general, non-orthogonal geometries that is: 1) conservative, 2) solenoidal in $\vec{B}$ and $\vec{J}$, 3) numerically non-dissipative, and 4) linearly and nonlinearly stable in the absence of physical dissipation. A fully implicit Newton-Krylov solver has been developed for 3D single-fluid MHD using this discretization concept. A physics-based preconditioner, developed by extending the ideas put forth in [2,3], has been implemented. Multigrid methods are employed to solve the required systems in the preconditioning stage. We will present preliminary numerical results of the performance of the implicit solver in simple geometries.

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[3] L. Chacón and D. A. Knoll, *J. Comput. Phys.*, **188** (2), 573-592 (2003)

[4] L. Chacón, *J. Comput. Phys.*, submitted (2004); L. Chacón, D. A. Knoll, *Bull. Am. Phys. Soc.*, FP1 108, 116 (2003)