Influence of pressure-gradient and shear on ballooning stability in stellarators

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A simple, semi-analytic method for expressing the ballooning growth rate as functions of the pressure-gradient and the averaged magnetic shear is introduced. For a given configuration, the the ballooning equation is written $[\partial_\eta P \partial_\eta + Q - \lambda \sqrt{g}^2 P]\xi = 0$, where the ballooning coefficients $P = B^2/g^{\psi\psi} + g^{\psi\psi}L^2$ and $Q = 2p'\sqrt{g}(G + \iota I)(\kappa_n + \kappa_g L)$. Here L is the integrated local shear, $L = \int_{\eta_k}^{\eta} s(\eta')d\eta'$, where $s = \iota' + \tilde{s}$ is the local shear, and κ_n, κ_g represent the normal and geodesic curvatures. This is an eigenvalue equation and for realistic geometry must be solved numerically.

An analytic variation in the pressure-gradient, $p' \rightarrow p' + \delta p'$, and average shear, $\iota' \rightarrow \iota' + \delta\iota'$, is imposed at an arbitrary flux surface of an MHD equilibrium. The relevant equilibrium quantities are then adjusted to preserve force balance. The variations $(\delta p', \delta\iota')$ alter the ballooning coefficients, $P \rightarrow P + \delta P$, $Q \rightarrow Q + \delta Q$, and the impact of the variations on the ballooning growth rate may be determined using eigenvalue perturbation theory. An expression for the change in the ballooning eigenvalue can be derived

$$\lambda(\delta p', \delta\iota') = \lambda_o + \frac{\partial\lambda}{\partial p'}\delta p' + \frac{\partial\lambda}{\partial\iota'}\delta\iota' + \frac{\partial^2\lambda}{\partial p'^2}\delta p'^2 + \frac{\partial^2\lambda}{\partial p'\partial\iota'}\delta p'\delta\iota' + \frac{\partial^2\lambda}{\partial\iota'^2}\delta\iota'^2 + \ldots, \qquad (1)$$

where λ_0 is the eigenvalue of the original equilibrium and explicit expressions for the derivatives, up to arbitrary order, are determined from a *single eigenfunction calculation*. Using only this information, whether increased pressure-gradient is stabilizing or de-stabilizing, and the existence of a second stable region, can be determined.

Marginal stability diagrams for an LHD-like configuration (left), a quasi-poloidal configuration (center) and an NCSX-like quasi-symmetric configuration(right) are presented in the figure below. The solid line shows the exact calculation: that is, the eigenvalue equation is resolved numerically for each variation $(\delta p', \delta\iota')$ on a grid of 200×200 points; and the dotted line shows the approximation provided by Eqn.(1). Also shown is the location in (p', ι') space of the original equilibrium, indicated with a ' + ' if that equilibrium is unstable or a ' □ ' if it is stable.

The analysis presented here provides both a numerically efficient and physically insightful approach to determining second stability in stellarators.