Resistive Wall Modes in Quasi-Stationary Collisionless Plasmas

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Motivations/Summary

- High fusion power density requires high-β operation
- Rotational stabilization of RWM may not be effective in ITER
- Active feedback control may not completely suppress RWM due to wall-shielding

$$\beta_{max}^{optimum\ feedback} \approx \frac{\beta_{\infty} + \beta_{b}}{2} \quad *$$

- Fluid theory does not give accurate predictions
- Consider mode-particle interaction. Trapped particles are stabilizing
- The RWM can be suppressed without rotation

* Liu, Bondeson, Gribov, Polevoi, Nuc. Fus.44, 232 (2004)

Previous RWM theories predict stabilization for large plasma flows

- Rotation and dissipations are the essential ingredients for stabilization
- The main dissipative effects in high-temperature plasmas require large flow velocities: the continuum damping

$$\omega_{doppler} \equiv \cancel{\omega} - \Omega_{rot} \cong -\Omega_{rot}$$

≈ 0 for RWM

$$\boxed{\Omega_{rot} = k_{\parallel} v_{sound}}$$

$$\boxed{\Omega_{rot} = k_{\parallel} v_{Alfven}}$$

the ion Landau damping $\boxed{\Omega_{rot} = k_{\parallel} v_{\parallel i}}$ Bondeson and Chu ('98) Liu et al, 2004

Bondeson and Ward ('94), Betti and Freidberg ('95), Finn ('95), Fitzpatrick and Aydemir ('96)

RWM growth rate from the Energy Principle

$$\gamma\tau_w \simeq -\frac{\delta W_{tot}^{\infty}}{\delta W_{tot}^{b}}$$

Haney & Freidberg ('89)

$$\delta W_{tot}^{b,\infty} = \boxed{\underbrace{\delta W_F}_{\text{Plasma}} + \underbrace{\delta W_V^{b,\infty}}_{\text{Vacuum}}} + \underbrace{\delta W_K}_{\text{Kinetic}}$$

$$\boxed{\delta W_{MHD}^{b,\infty}} \qquad \boxed{\mathrm{Re}(\delta W_K) + i\mathrm{Im}(\delta W_K)}$$

Stability condition:

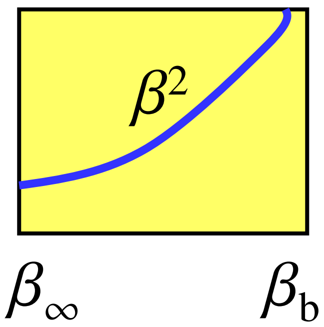
$$\boxed{|\delta W_K|^2 + \mathrm{Re}(\delta W_K)(\delta W_{MHD}^{b} + \delta W_{MHD}^{\infty}) > -\delta W_{MHD}^{\infty}\delta W_{MHD}^{b}}$$

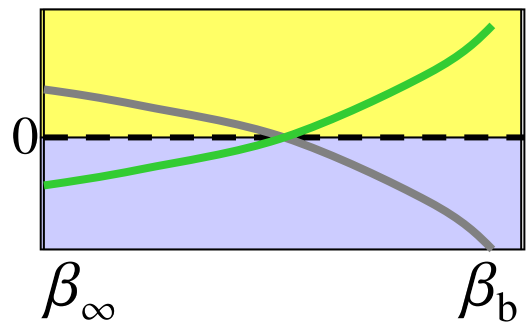
Qualitative analysis of the stability condition

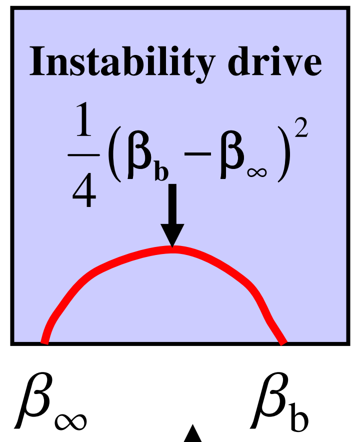
$$
\begin{aligned}
\delta W_{MHD}^{\infty} &\sim \beta_\infty - \beta \\
\delta W_{MHD}^{b} &\sim \beta_b - \beta \\
\delta W_K &\propto \beta
\end{aligned}
$$

→ **stabilizing**

→ **destabilizing**

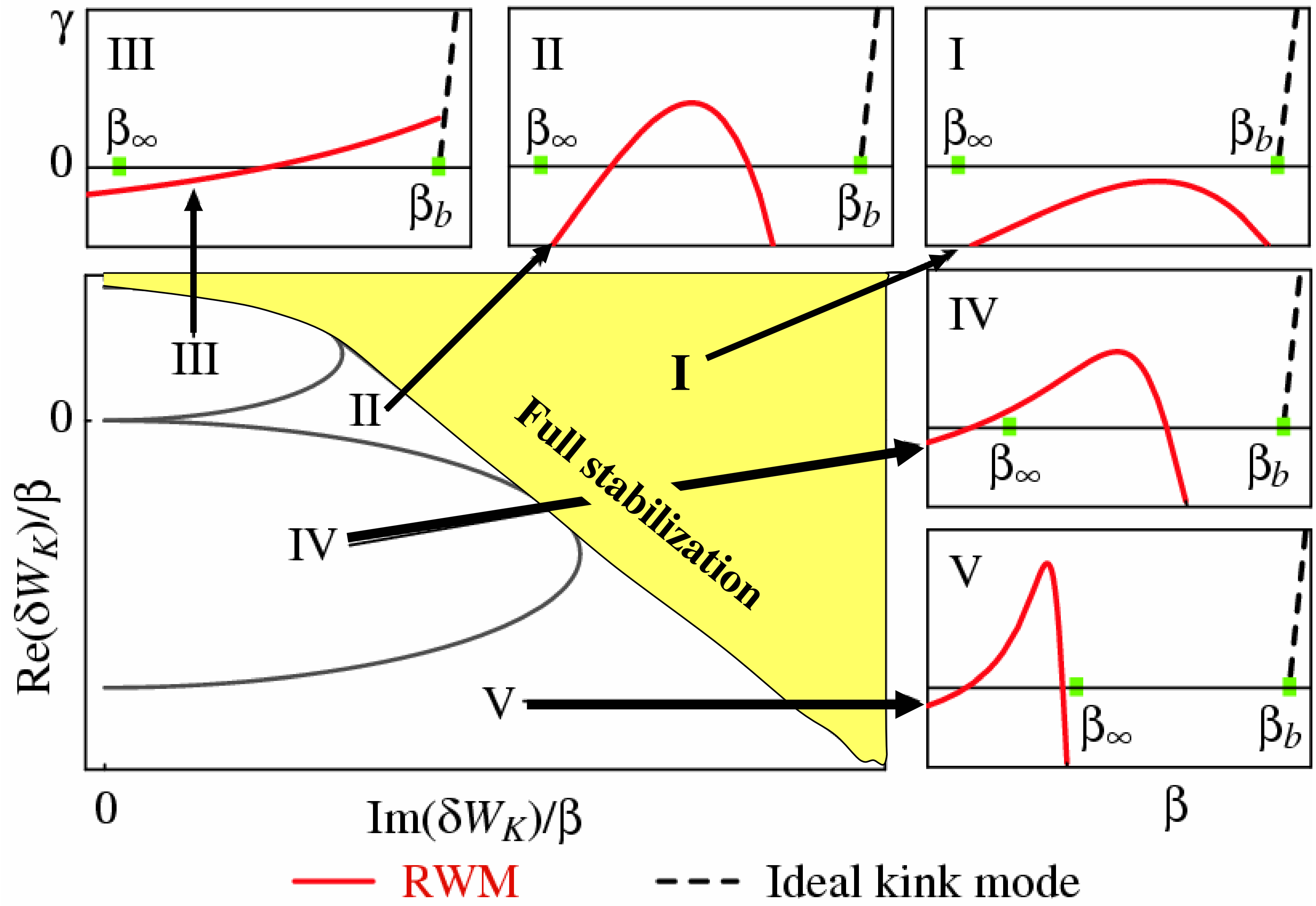






$$
|\delta W_K|^2 + \mathrm{Re}(\delta W_K)(\delta W_{MHD}^{b} + \delta W_{MHD}^{\infty}) > -\delta W_{MHD}^{\infty}\delta W_{MHD}^{b}
$$

Five regimes of RWM stability/instability



Kinetic theory of the RWM: approximations

- RWM frequency: $\omega \sim 1/\tau_w$ - $100/\tau_w$

- $\omega \ll \omega_{Di} \Leftarrow$ zero mode frequency
  (ω_{Di} magnetic drift frequency)

- $\nu_{eff} \ll \omega_{Di} \Leftarrow$ collisionless ions

- $\Omega_{rot} \lesssim \omega_{*i} \Leftarrow$ quasi-stationary plasma

- Retain finite equilibrium electric field E

Kinetic trapped-ion effects enter through the perturbed perpendicular pressure

UR LLE

$$\tilde{p}_\perp^K \approx m_i \int d\boldsymbol{v} \frac{v_\perp^2}{2} \tilde{f}_i^K$$

Kinetic pressure

Perturbed distribution function

$$\tilde{f}_i \approx -\tilde{\boldsymbol{\xi}}_\perp \cdot \nabla f_i \overbrace{-i \frac{\partial f_i}{\partial \Psi} \int_{-\infty}^{t} dt' \left(\frac{\varepsilon}{e} \tilde{\boldsymbol{\xi}}_\perp \cdot \boldsymbol{\kappa} + \tilde{\mathcal{Z}} \right) + e \frac{\partial f_i}{\partial \varepsilon} \tilde{\mathcal{Z}}}^{\tilde{f}_i^K}$$

Electrostatic term $\tilde{\mathcal{Z}} \equiv \tilde{\Phi} + \tilde{\boldsymbol{\xi}}_\perp \cdot \nabla \Phi$ includes the equilibrium electric field Φ and depends on ξ through quasi-neutrality

Large aspect ratio approximation for $\tilde{p}^K$ with nonzero equilibrium E

m-th poloidal harmonic

$$\tilde{p}_m^K = \frac{2^{3/2}\epsilon^{1/2}}{\pi^{3/2}} \int_0^\infty \frac{d\varepsilon\, \varepsilon^{3/2}}{T^{5/2}} e^{-\varepsilon/T} \int_0^1 du K(u) p_i \boxed{\lambda} \sigma_m \sum_{\ell=-\infty}^{+\infty} \sigma_\ell \Upsilon_\ell$$

Diamagnetic drift frequencies

Doppler shifted frequency in the E×B frame

$$\boxed{\lambda = \frac{\omega_{*N} + (\varepsilon/T - 3/2)\omega_{*T} - \omega_{doppler}^{E\times B}}{\omega_{Di} - \omega_{doppler}^{E\times B}}}$$

$$\omega_{\text{doppler}}^{\text{E×B}} = \omega - \omega_{\text{E×B}}$$

$$\sigma_m = \int_0^{\pi/2} d\chi \frac{\cos[2(m-q)\arcsin(\sqrt{u}\sin\chi)]}{K(u)\sqrt{1-u\sin^2\chi}} \qquad \Upsilon_\ell = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\ell\theta + i\phi + i\omega t}\left(\frac{\varepsilon}{T_i}\tilde{\boldsymbol{\xi}}_\perp \cdot \boldsymbol{\kappa} + \frac{Z_i e}{T_i}\tilde{\mathcal{Z}}\right)$$

Resonance depends on toroidal rotation

Mode-particle resonance:
Doppler shifted frequency = magnetic drift frequency

$$\omega_{doppler}^{E\times B} \equiv \cancel{\omega} - \omega_{E\times B}$$

zero frequency approx.

$$\omega_{doppler}^{E\times B} = \omega_{Di}$$

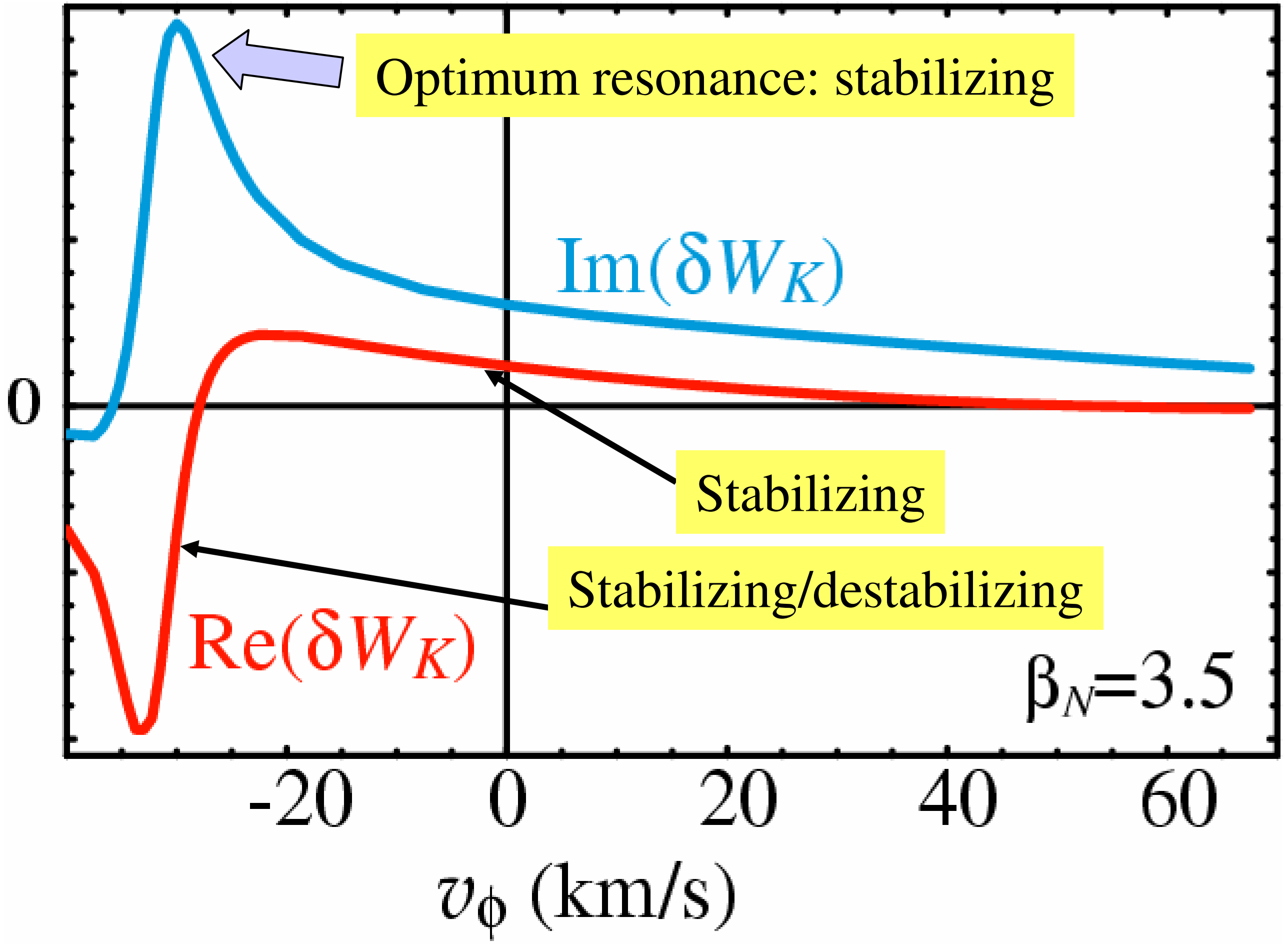
$$\omega_{E\times B} = -\omega_{Di}$$

$$\omega_{E\times B} = \Omega_{rot} - \omega_{*p}^{i}$$

Ion force balance equation

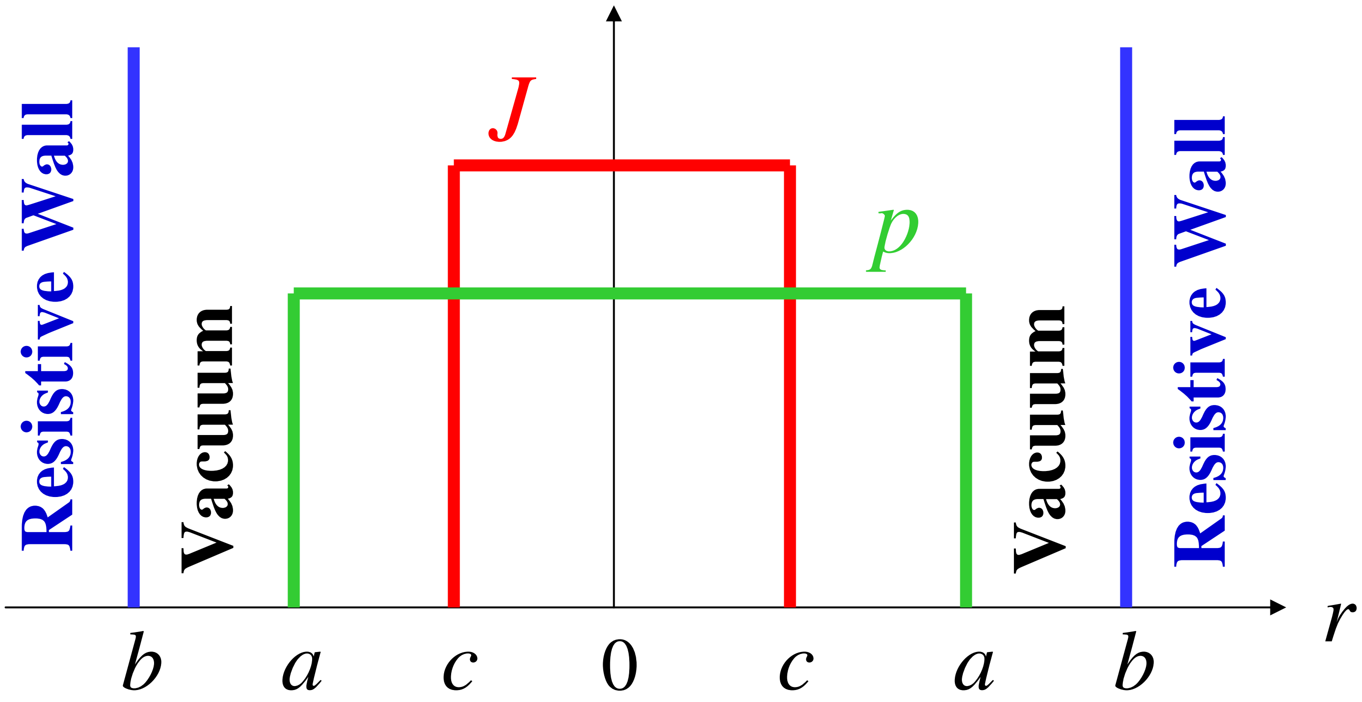
No resonance if $\Omega_{rot} \geq \omega_{*i}$
No resonance if $|\Omega_{rot}| >> \omega_{*i}$

Kinetic-δW versus rotation velocity for ITER-like plasmas



A simplified sharp boundary equilibrium* is used to solve the eingenvalue problem for the RWM

- **Flat current density and pressure profiles**



*Betti, Phys. Plasmas 5, 3615 (1998)

Stability problem is reduced to simple fluid theory in plasma core

- **Kinetic pressure enters in momentum equation**

$$\nabla(p^F + \tilde{p}^K + B^2/2) - \boldsymbol{\kappa}\tilde{p}^K - \boldsymbol{B}\cdot\nabla\boldsymbol{B} \quad = \quad 0$$

- **Kinetic contribution vanishes in plasma core because equilibrium pressure is flat**

$$\tilde{p}^K \sim \frac{\partial f}{\partial r} = 0 \quad \text{inside plasma } (r < a)$$

Only fluid terms in the plasma core. Solve using the small *a/R* expansion

- The perturbed magnetic flux follows simple power laws of r

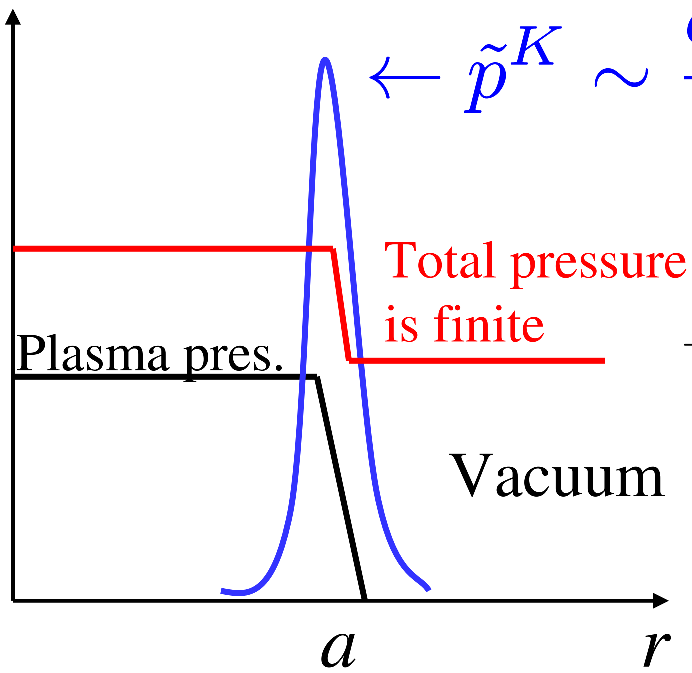
$$\tilde{\psi}_m(r \le c) = \psi_m^0 \left(\frac{r}{c}\right)^{|m|} \qquad \tilde{\psi}_m(c \le r \le a) = \psi_{1m}\left(\frac{r}{c}\right)^{|m|} + \psi_{2m}\left(\frac{c}{r}\right)^{|m|}$$

- The constants are determined through the matching conditions at the current tube boundary $r=c$

$\tilde{\psi}_m$ and $d\tilde{\xi}_m / dr$ are continuous at $r = c$

$$\tilde{\psi}_m = rBh_m\tilde{\xi}_m / m \qquad h_m = n - m/q$$

Delta-function-like kinetic pressure at plasma edge



$\leftarrow \tilde{p}^K \sim \dfrac{\partial p^F}{\partial r}$ is singular at edge

$$\tilde{p}^K + \widetilde{B^2}/2 \text{ is finite}$$

$$\widetilde{B^2}(r \simeq a) = -2\tilde{p}^K(r \simeq a)$$

Kinetic pressure enters through the boundary conditions at the plasma edge

- Boundary condition at the plasma edge

$$\widetilde{B^2}(r \simeq a) = -2\tilde{p}^K(r \simeq a)$$

$$\begin{aligned} \nabla(p^F + \tilde{p}^K + B^2/2) &= \boldsymbol{\kappa}\left(\widetilde{B^2} + \tilde{p}^K\right) \\ &= -\boldsymbol{\kappa}\tilde{p}^K \end{aligned}$$

- Boundary condition includes kinetic effects

$$\left[\!\left[p^F + B^2/2 \right]\!\right]_a = -\boldsymbol{\kappa}\cdot\hat{n}\int_{a-}^{a+} dr\,\tilde{p}^K$$

A linear system is derived by matching the solutions at the plasma boundary

- **Dispersion relation is derived by matching the vacuum to the plasma solution**

$$-h_m a\tilde{\psi}'_m/m$$

Plasma column magnetic term

$$\underbrace{+\frac{3}{2}\frac{\beta}{\epsilon}\left(\frac{m+1}{h_{m+1}}\tilde{\psi}_{m+1}+\frac{m-1}{h_{m-1}}\tilde{\psi}_{m-1}\right)}_{\text{Fluid instability drive}}$$

Kinetic term

$$+\text{K.T.} = \sum_j \delta_{mj}(\gamma\tau_w)\tilde{\psi}_j(a)$$

Vacuum term

Resistive wall term

Kinetic terms are frequency dependent and can be of the same size as the fluid terms

$$\text{Kinetic Terms} = -\frac{1}{2\sqrt{2}\pi}\frac{\beta}{\sqrt{\epsilon}}\sum_k \left[\mathcal{K}^-_{mk} r\partial_r + k\mathcal{K}^+_{mk} + \sum_{l,p}\Delta_{ml}\left(\mathcal{A}^{-1}\right)_{lp}\left(\mathcal{B}^-_{pk} r\partial_r + k\mathcal{B}^+_{pk}\right)\right]\frac{\tilde{\psi}_k}{h_k}$$

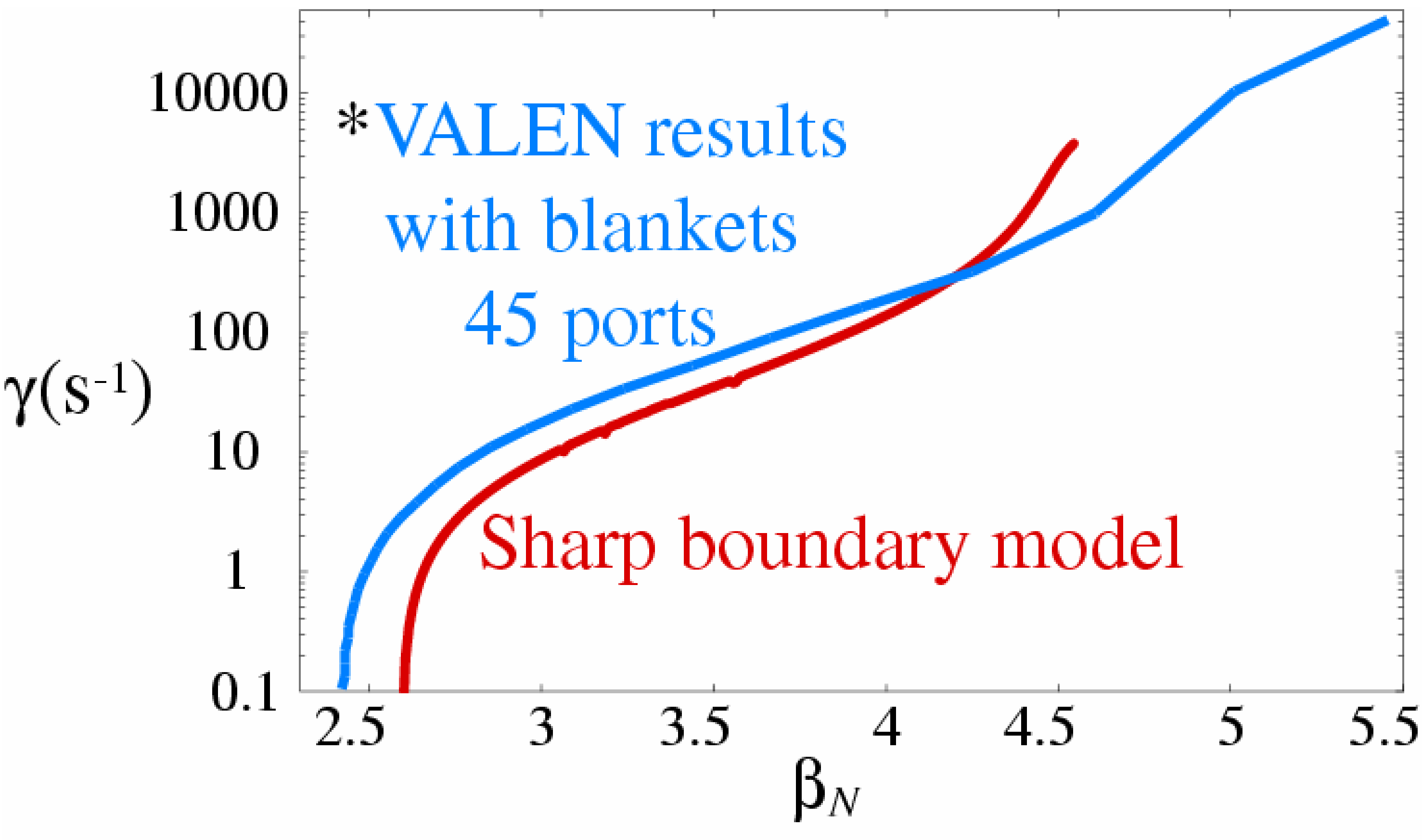
$$\text{Fluid Terms} = \frac{3}{2}\frac{\beta}{\epsilon}\left(\frac{m+1}{h_{m+1}}\tilde{\psi}_{m+1} + \frac{m-1}{h_{m-1}}\tilde{\psi}_{m-1}\right)$$

RWM growth rate is found by setting to zero the determinant of the linear system

Use ITER-like parameters for advanced tokamak mode

- Set $2<q<2.5$ for $r<a$ and $q \rightarrow \infty$ for $r=a$,
- $\epsilon = a\sqrt{\kappa}/R \approx 0.4$
- Wall radius/plasma radius = $b/a \approx 1.2$
- RWM growth rate is calculated for varying β

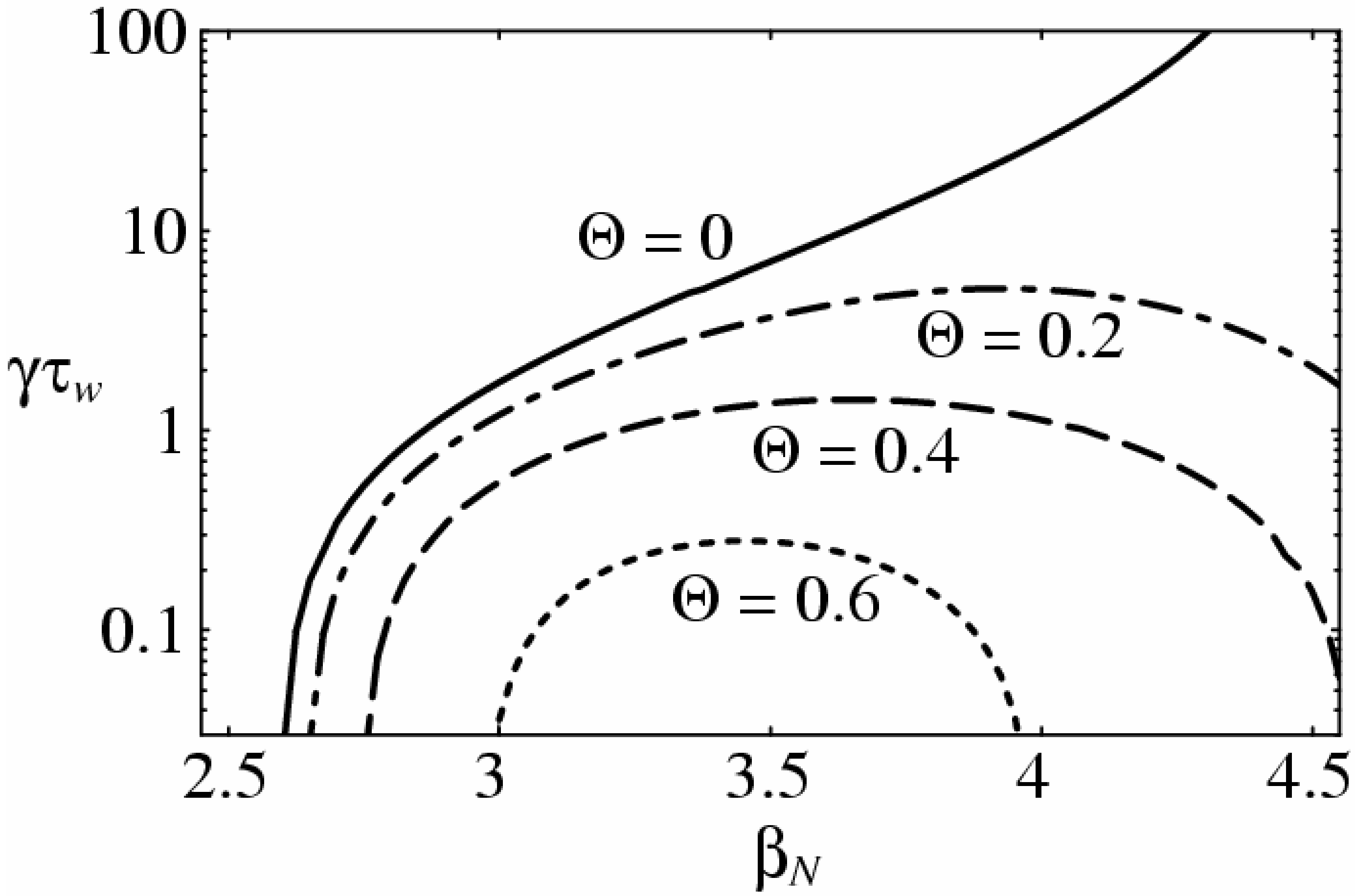
Without kinetic effects the sharp boundary model approximately reproduces VALEN* results for ITER



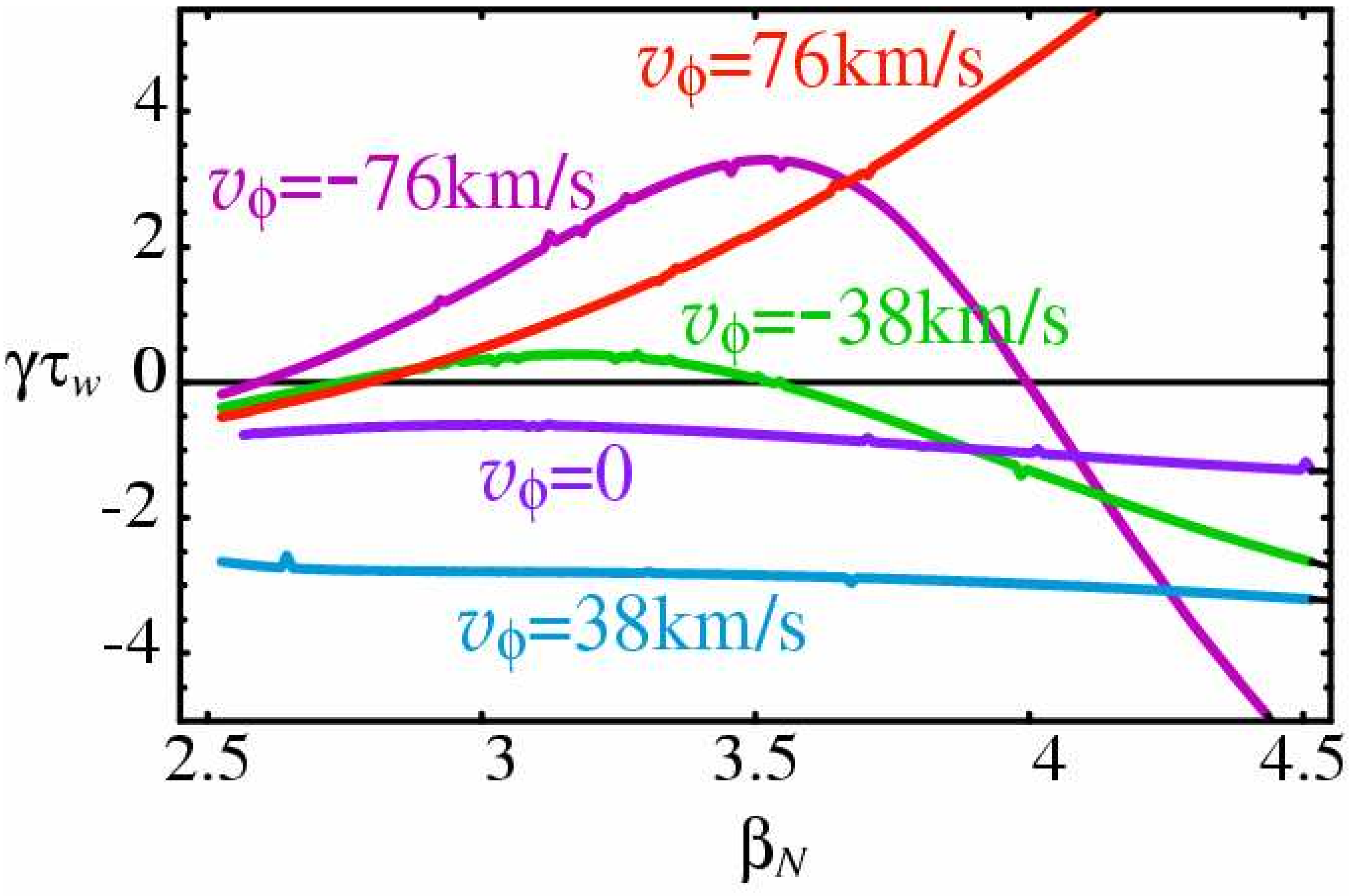
*Navratil, Bialek, Boozer & Katsuro-Hopkins, MHD Workshop, Nov 3-5, 2003, Austin, TX

Trapped-ion kinetic effects suppress the RWM for stationary ITER-like plasmas

Kinetic term is multiplied by Θ



Trapped-ion stabilization is ineffective for large plasma flows

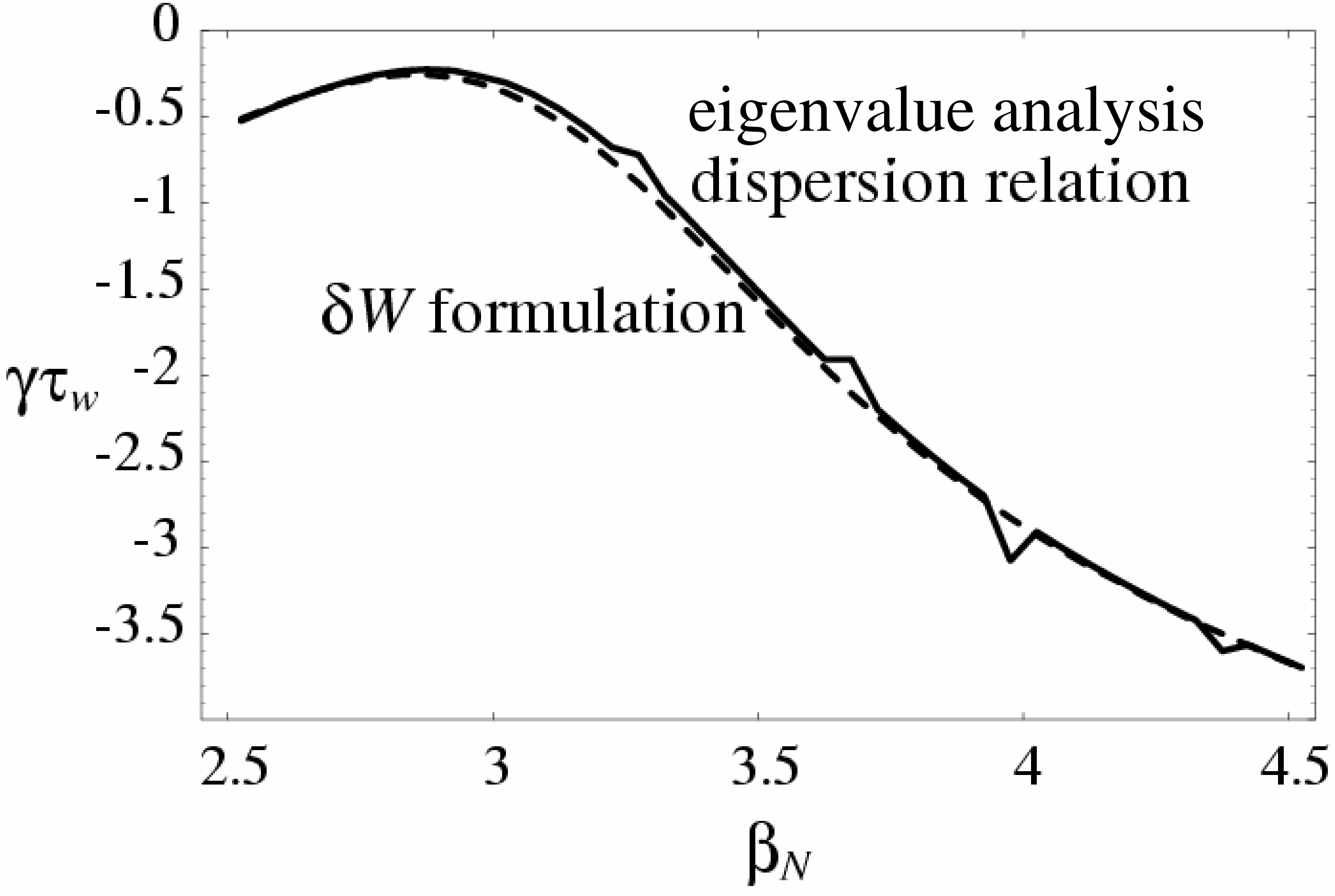


Calculate RWM growth rate with a MHD stability code

- Kinetic effects does not significantly change mode eigenfunction
- Obtain RWM eigenfunction from ideal MHD code
- Calculate δW's including δW_K
- Calculate growth rate from energy principle

$$\boxed{\gamma\tau_w = -\frac{\delta W^{\infty}_{MHD} + \delta W_K}{\delta W^{b}_{MHD} + \delta W_K}}$$

Energy principle vs eigenvalue analysis: comparison of numerical results



Conclusions

- Magnetic drift resonance is stabilizing for the RWM
- Non-resonant part of kinetic effect counteracts fluid instability drive
- **Theory from a simple model of ITER-like plasma indicates that RWM can be suppressed without plasma rotation**
- More realistic predictions can be obtained with minor changes to ideal MHD stability codes