Spectral methods for multi-scale plasma physics simulations

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Abstract

The Vlasov-Maxwell (VM) equations are extremely challenging numerically because of their high dimensionality, nonlinearities and the wide disparity of spatial and temporal scales.

In this work, we present a spectral method for the VM equations based on a decomposition of the plasma phase-space density in Hermite or Legendre modes. It leads to a truncated system for the expansion coefficients (i.e. moments). Its most important feature is that, with a suitable spectral basis, the low-order moments are akin to the typical moments (mass, momentum, energy) of a fluid/macroscopic description of the plasma, while the kinetic/microscopic physics can be retained by adding more moments. The accuracy of the scheme for a given number of modes can be further improved by using generalized spectral bases with carefully chosen scaling and shifting parameters. In addition, spectral convergence, stability and exact conservation laws in the limit of finite time step can be proven [1].

Selected results illustrating the properties and the potential of the method will be presented. A comparison between PIC and the spectral method on standard electrostatic test problems shows that the spectral method can be orders of magnitude faster/more accurate than PIC [2]. Some attempts to optimize the spectral decomposition in velocity space [3] and multi-dimensional fully electromagnetic tests with efficient preconditioning techniques [1, 4] will also be presented.

With the 'built-in' fluid/kinetic coupling and favorable numerical properties, spectral methods might offer an optimal way to perform accurate large-scale simulations including microscopic physics.

References

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