The Case-van Kampen Theorem and the Morrison-Pfirsch Free Energy for Generalized Vlasov Systems*

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In this work, the Case-van Kampen spectral theorem [1-2] is generalized to the case of the Vlasov-Maxwell system with dynamic fields and multiple charged particle species. The proof applies to almost any collisionless kinetic system using a straightforward Laplace transform approach and represents a generalization of the pioneering work of Morrison, Pfirsch, and Shadwick [3-4]. Just as in the Vlasov-Poisson case, there is both a discrete point spectrum, which satisfies the principle value "Vlasov dispersion relation," and a continuous spectrum of van Kampen modes. For Hamiltonian dynamics, these modes must arise in pairs of opposite signs and pairs of complex conjugates. In a multi-species plasma, a charge-neutral continuous spectrum is needed for completeness. Neutrally stable discrete modes can also arise. The spectrum defines the wave action density and diagonalizes the Morrison-Pfirsch free energy, including the dynamic free-field dielectric energy.

Phase mixing of the continuous spectrum is responsible for Landau damping and leads to the appearance of the arrow of time. The linear response of the fields has an equivalent representation in terms of both branches of the analytic "Landau dispersion relation." One branch corresponds to the retarded propagator, while the other corresponds to the advanced propagator; both have discrete spectra that include the discrete Vlasov modes. Time-reversal symmetry is restored by the fact that Landau "damping" is observed in either direction of time.

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[1] N. G. van Kampen, Physica 21, 949 (1955).
[2] K. M. Case, Ann. Phys. 7, 349 (1959); K. M. Case, Phys. Fluids 21, 249 (1978).
[3] P. Morrison and D. Pfirsch, Phys. Fluids B 4, 3038 (1992).
[4] B. A. Shadwick, *On the Hamiltonian Structure of the Linearized Maxwell-Vlasov System*, Ph.D. Thesis, U. Texas, Austin (1995).