

# Hessian matrix approach for determining error field sensitivity to coil deviations.

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## Abstract

The presence of error fields has been shown to degrade plasma confinement and drive instabilities [1]. Error fields can arise from many sources, but are predominantly attributed to deviations in the coil geometry. Controlling the error field is critical for stellarators where most the confining magnetic field is produced by carefully optimized coils. The accuracy requirements were the largest cost growth of the NCSX stellarator, which was cancelled because of cost overruns [4].

Conventional approaches to studying the error field caused by coil deviations are to apply possible displacements and calculate the resulting error fields [2]. Heavy computations are required and only certain limited deviations could be explored. Here, we introduce a Hessian matrix approach for determining error field sensitivity to coil deviations. The FOCUS code [5], provides fast and accurate calculations of an error field evaluation function ( $f_B$ ) and its second derivatives (i.e. Hessian). Near a local minimum,  $f_B$  is linearly proportional to the eigenvalues of the Hessian matrix, when decomposing the coil perturbation in the basis of eigenvectors. The sensitivities of error fields to coil displacements are then determined by the eigenvalues.

A proof-of-principle example is given on the CNT configuration [3]. The results show that misalignments at the inner parts of the interlinked coils will cause significant error fields while others are relatively less important. We anticipate that this new method could provide information to avoid dominant coil misalignments, simplify coil designs and ultimately reduce the cost of stellarator coils.

## References

- [1] R.J Buttery, M. De'Benedetti, D.A Gates, Yu Gribov, T.C Hender, R.J. La Haye, P Leahy, J.A Leuer, A.W Morris, A Santagiustina, J.T Scoville, B.J.D Tubbing, JET Team, COMPASS-D Research Team, and DIII-D Team. Error field mode studies on JET, COMPASS-d and DIII-d, and implications for ITER. *Nuclear Fusion*, 39(11Y):1827–1835, nov 1999.
- [2] K C Hammond, A Anichowski, P W Brenner, T S Pedersen, S Raftopoulos, P Traverso, and F A Volpe. Experimental and numerical study of error fields in the CNT stellarator. *Plasma Physics and Controlled Fusion*, 58(7):074002, may 2016.
- [3] Thomas Sunn Pedersen, Allen H. Boozer, Jason Paul Kremer, Remi G. Lefrancois, Wayne T. Reiersen, Fred Dahlgren, and Neil Pomphrey. The columbia nonneutral torus: A new experiment to confine nonneutral and positron-electron plasmas in a stellarator. *Fusion Science and Technology*, 46(1):200–208, jul 2004.
- [4] RL Strykowski, T Brown, J Chrzanowski, M Cole, P Heitzenroeder, GH Neilson, Donald Rej, and M Viol. Engineering cost & schedule lessons learned on ncsx. In *Fusion Engineering, 2009. SOFE 2009. 23rd IEEE/NPSS Symposium on*, pages 1–4. IEEE, 2009.
- [5] Caixiang Zhu, Stuart R. Hudson, Yuntao Song, and Yuanxi Wan. New method to design stellarator coils without the winding surface. *Nuclear Fusion*, 58(1):016008, 2018.