Nonlinear Mode Penetration Caused by Transient Magnetic Perturbations

Sherwood Fusion Theory Conference
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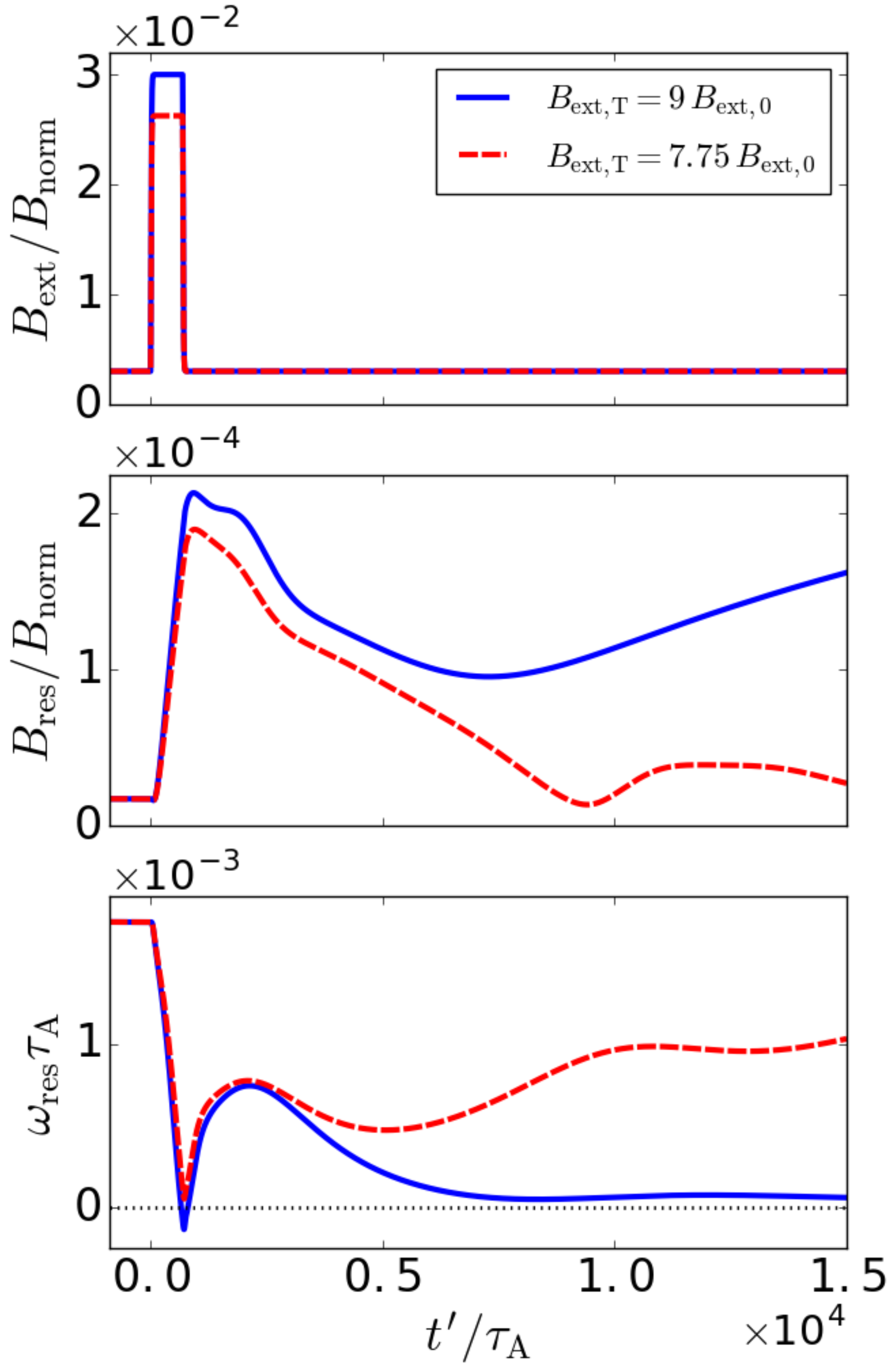
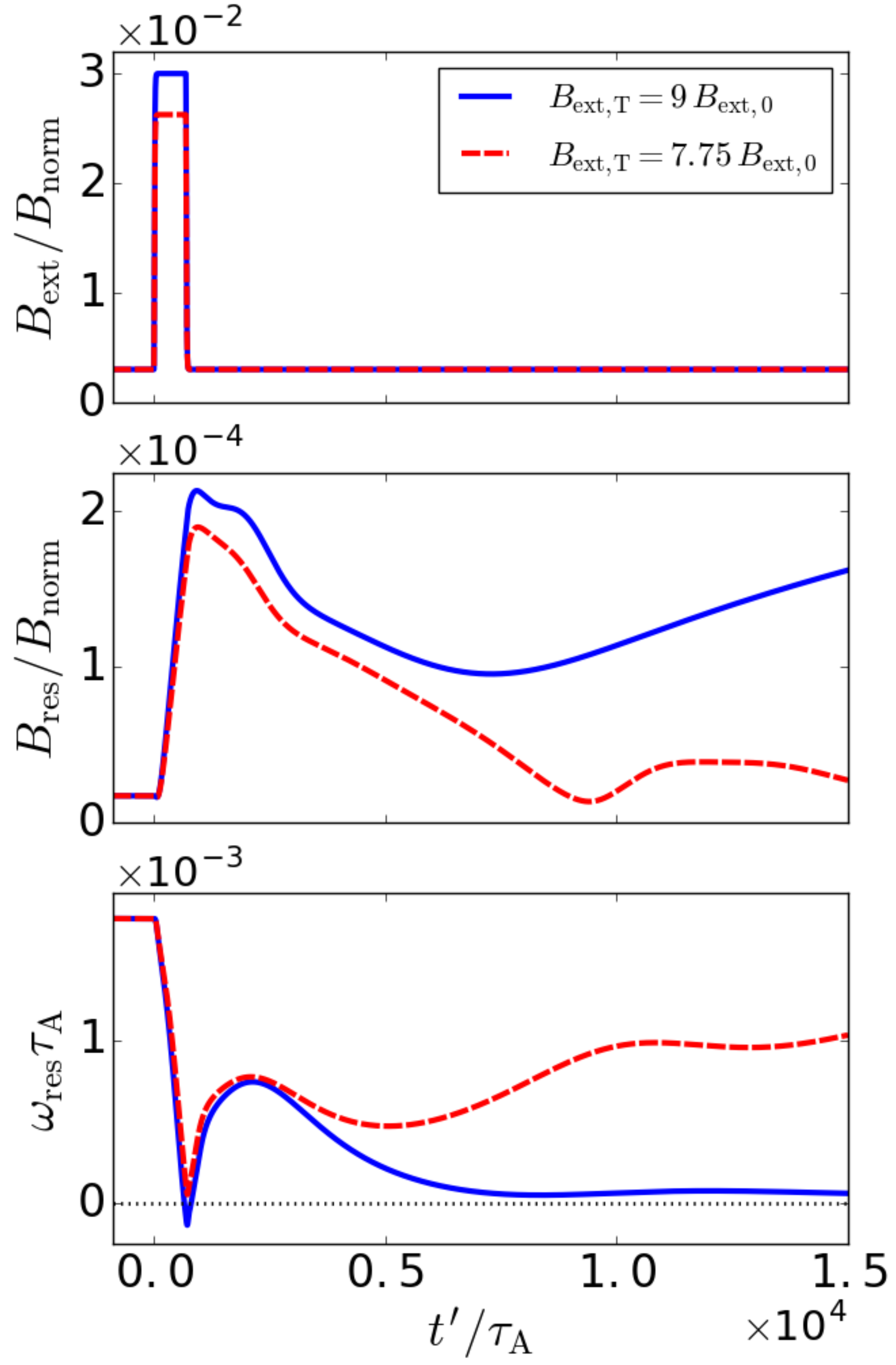
M. T. Beidler, J. D. Callen,
C. C. Hegna, and C. R. Sovinec

Department of Engineering Physics,
University of Wisconsin - Madison



Frequently Used Quantities Need to Be Introduced



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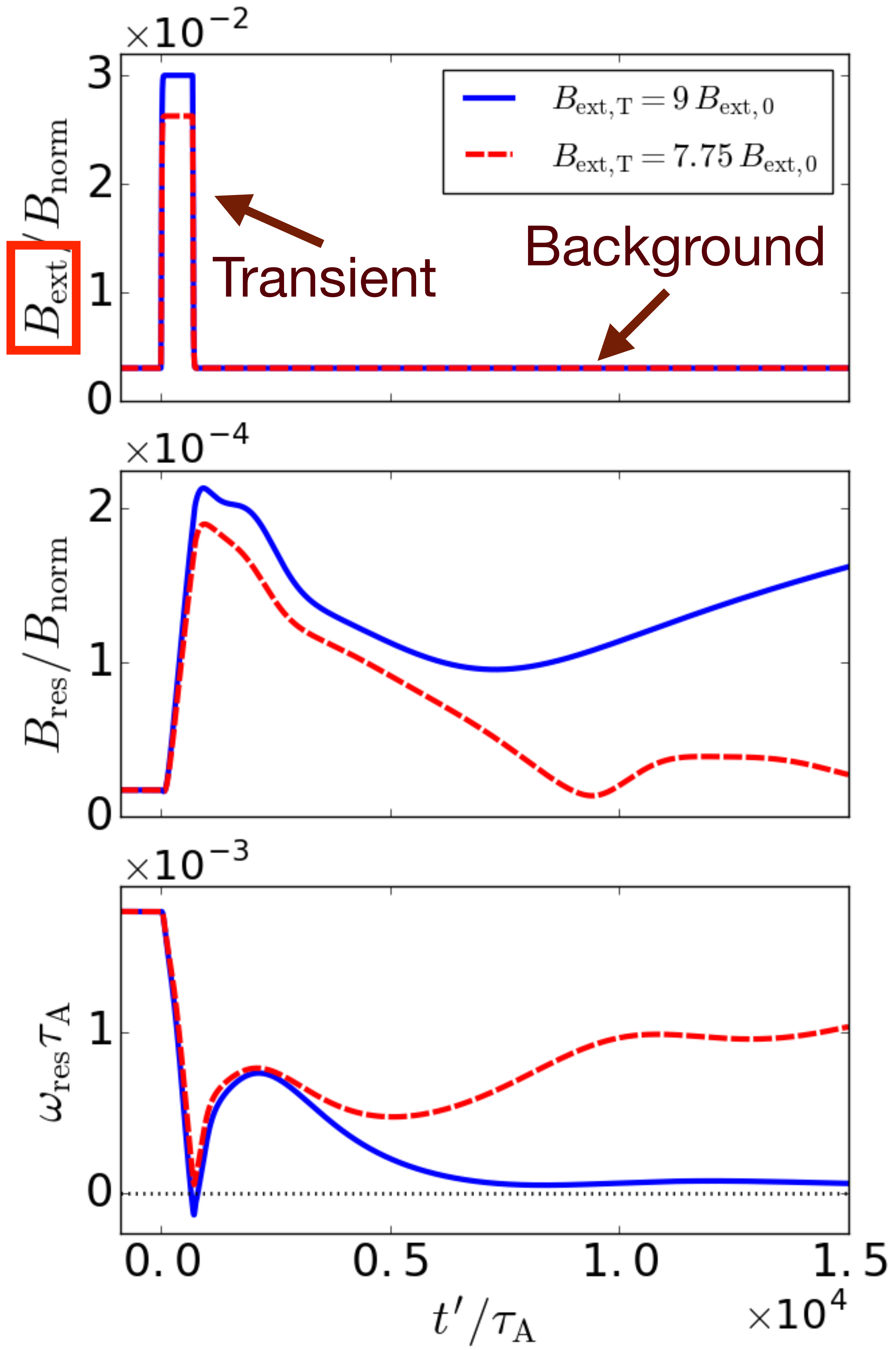
- B_{ext} **: Magnitude of externally-applied field**
  - Background (0) and transient (T) contributions



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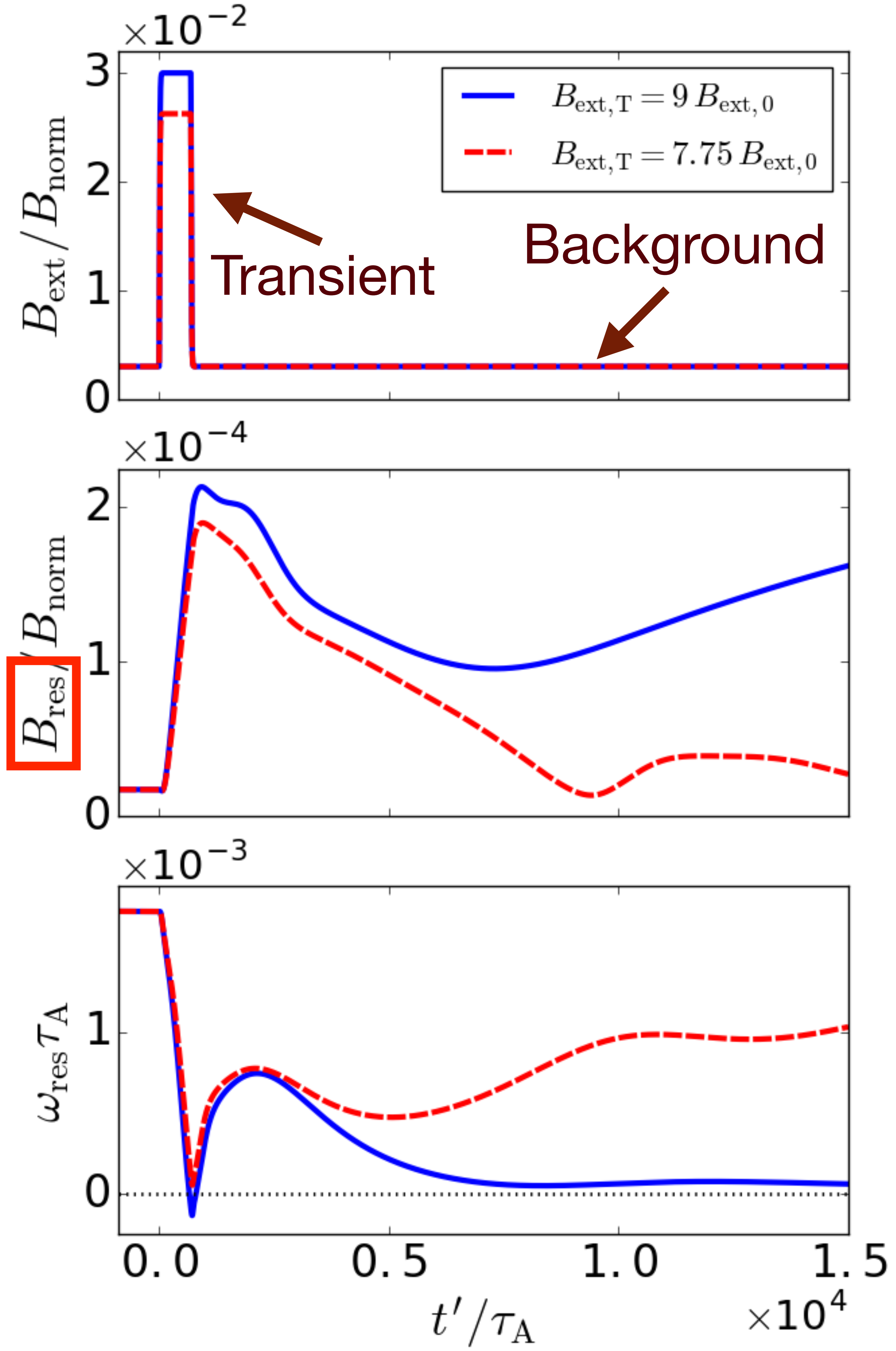

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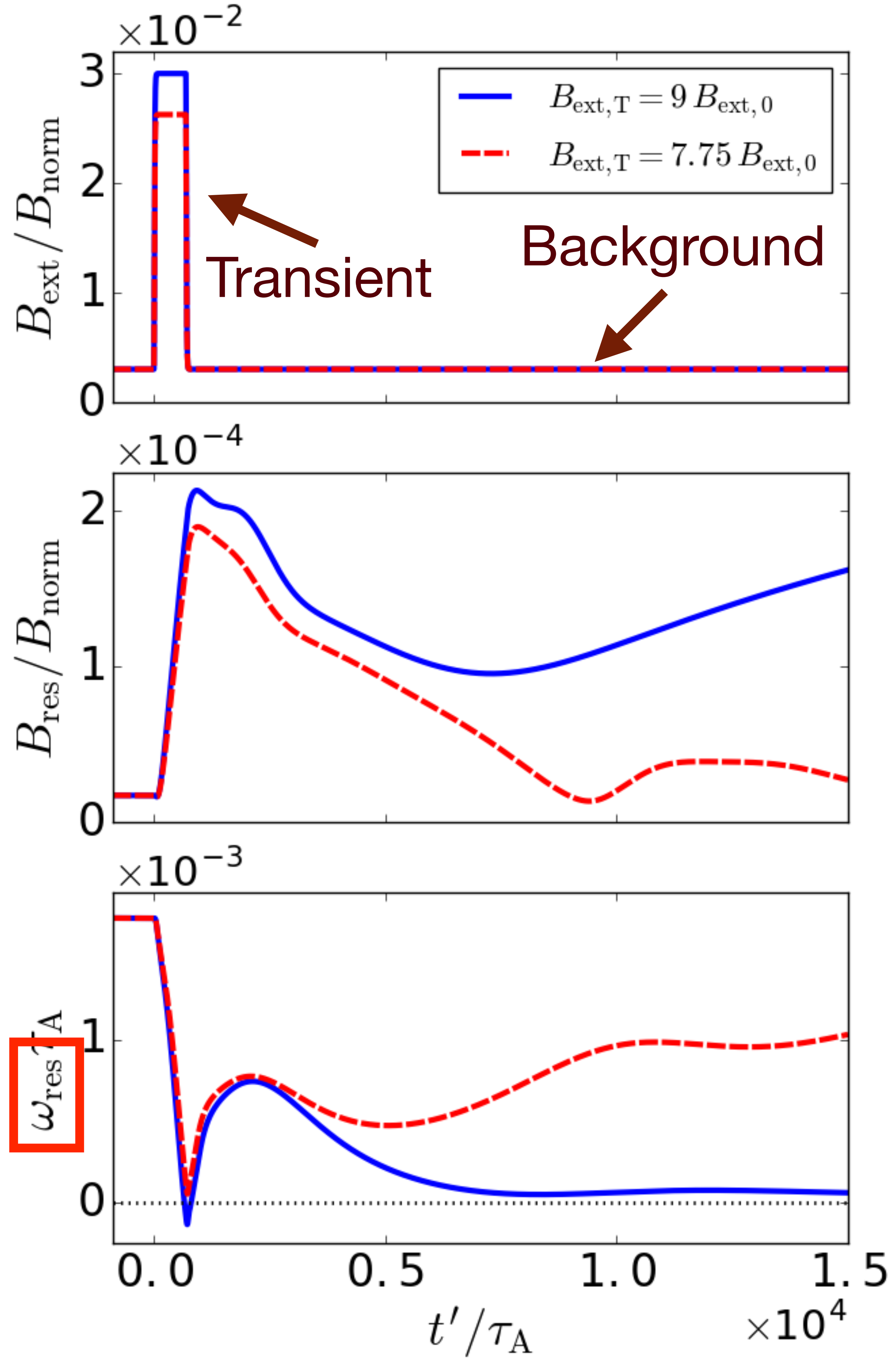

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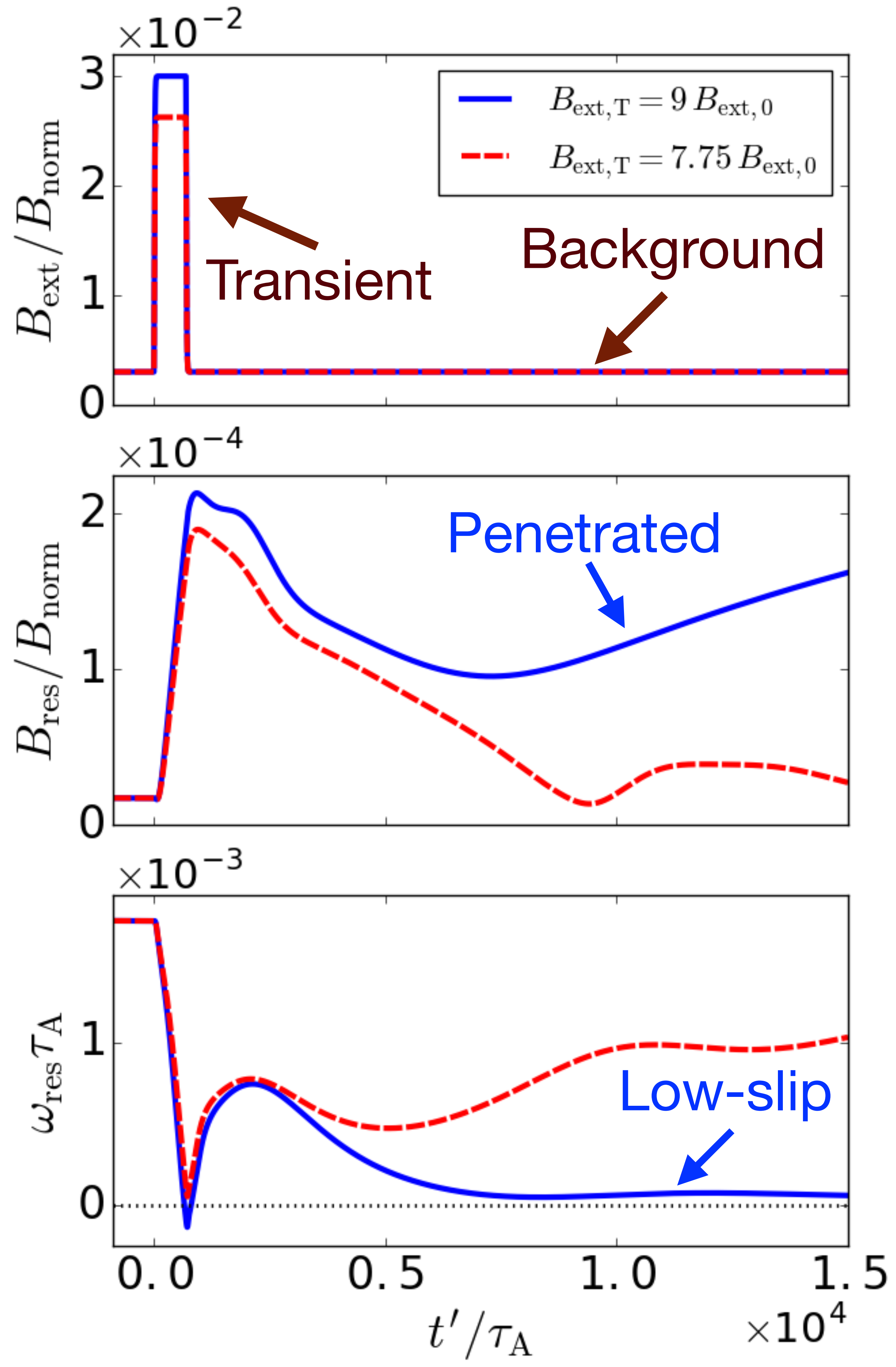

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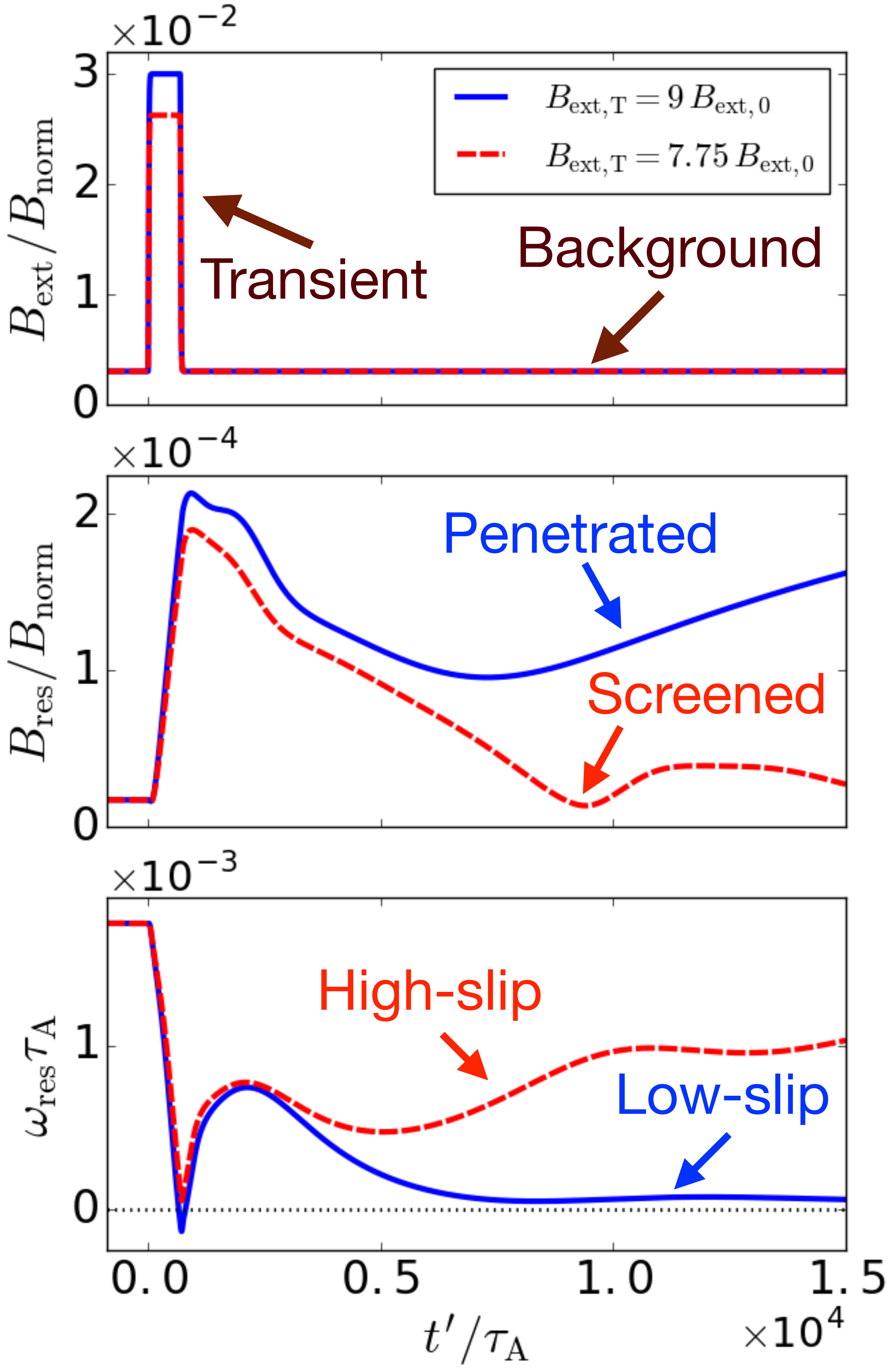
  - Background (0) and transient (T) contributions
- $B_{\rm res}$: **Magnitude of field response at resonant surface**
- $\omega_{\rm res}$: **Flow frequency at resonant surface**
  - Large transient precipitates transition to a low-slip state, with penetrated $B_{\rm res}$
  - Small transient returns to a high-slip state, with screened $B_{\rm res}$



Motivation: External 3D Fields Cause Forced Magnetic Reconnection

- **Externally applied 3D fields force magnetic reconnection (FMR)**
  - Islands can lock plasma to 3D field structure
  - Fundamental physics governed by external forcing, flow, resistivity, and viscosity

Motivation: Transient MHD Events Cause Forced Magnetic Reconnection

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Motivation: ELM Can Precipitate Transition to ELM-Free State

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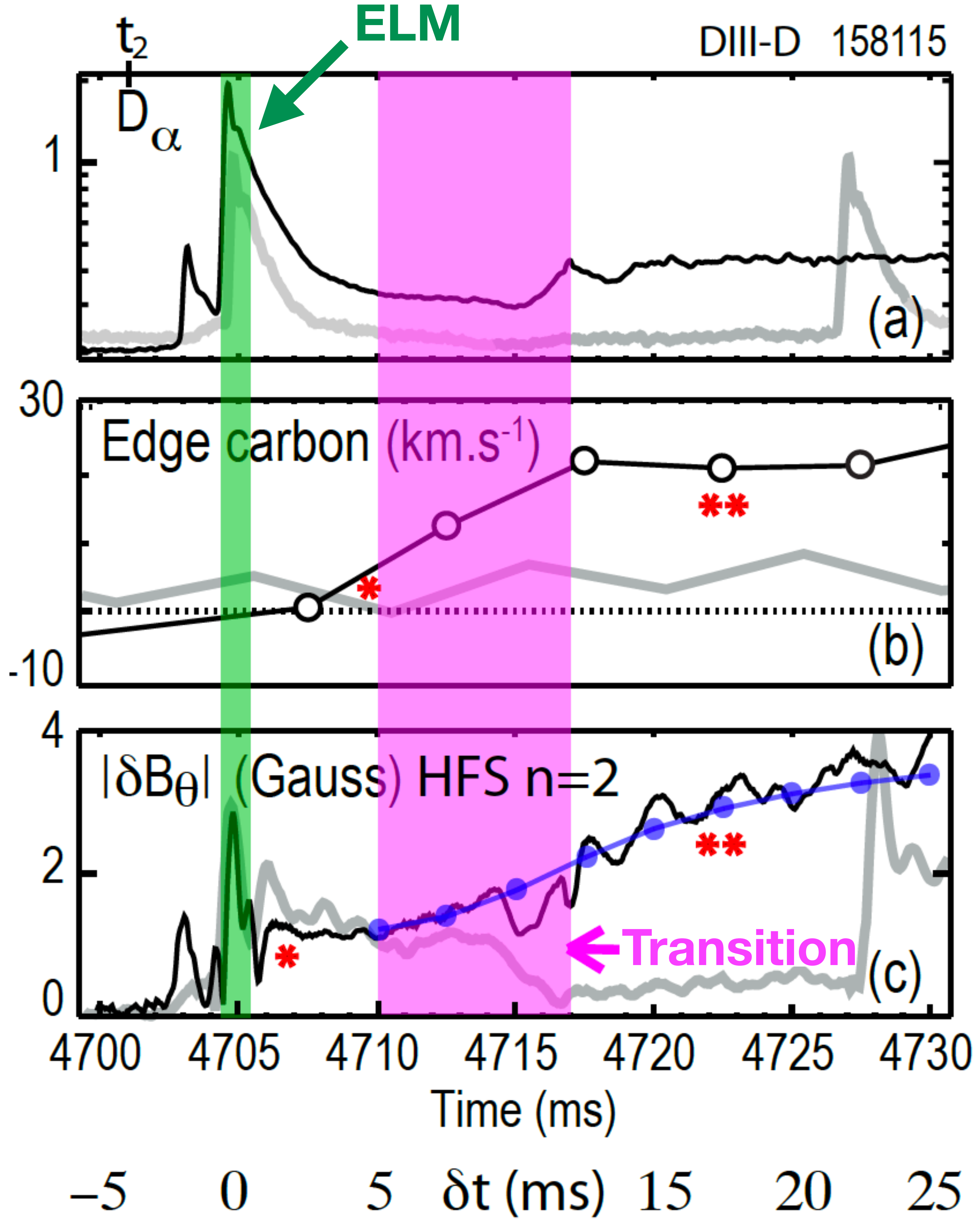
  - Islands can lock plasma to 3D field structure
  - Fundamental physics governed by external forcing, flow, resistivity, and viscosity
- **Transient MHD events are an additional source of 3D fields; can induce transition**
  - ELM can trigger ELM-suppressed state for large resonant magnetic perturbation (RMP)
    - Paz Soldan et al., PRL (2015); Nazikian et al., PRL (2015); Callen et al., UW-CPTC Report 16-4



Motivation: ELMs and Sawteeth Can Precipitate NTM Growth

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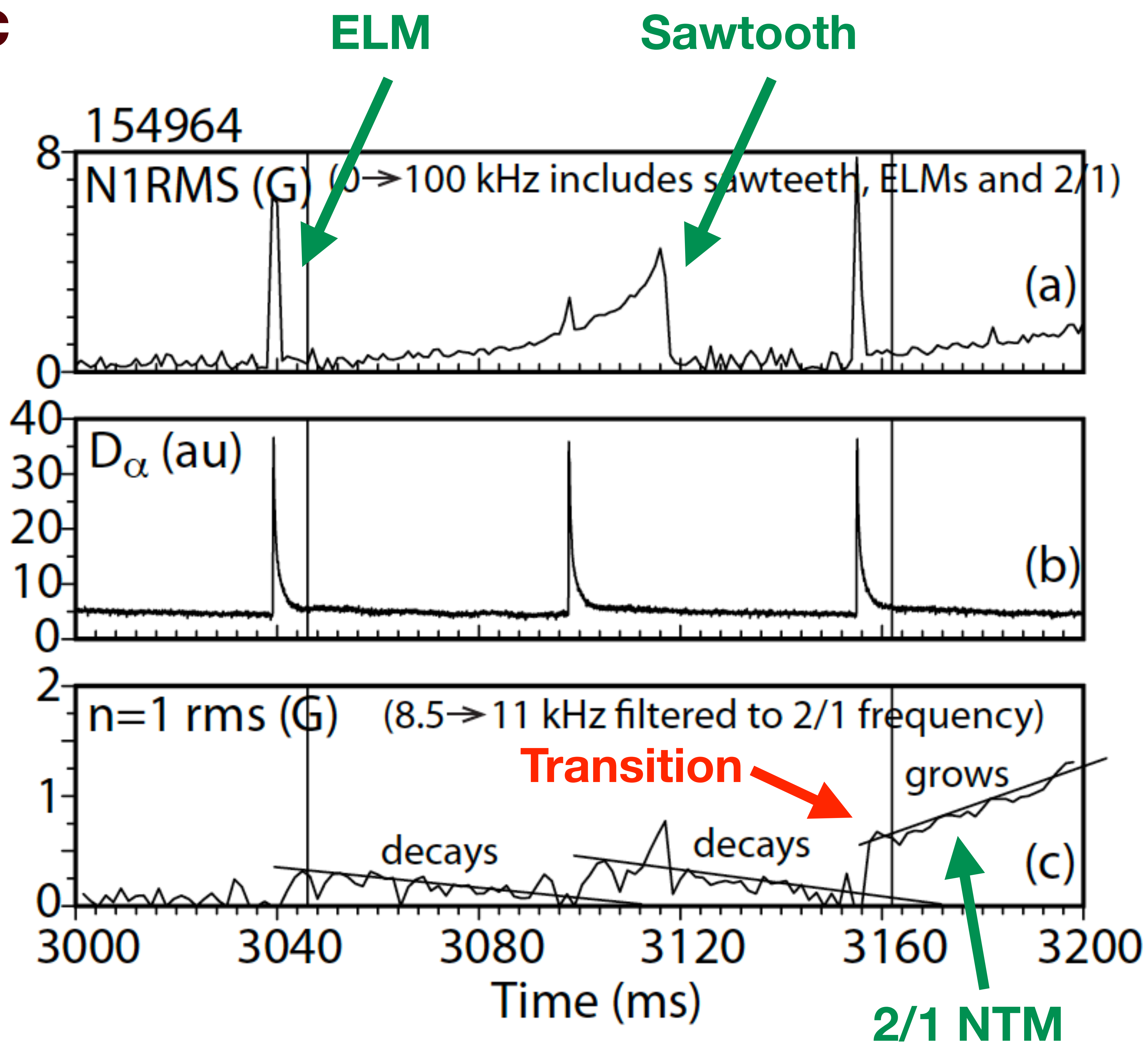
  - Fundamental physics governed by external forcing, flow, resistivity, and viscosity
- **Transient MHD events are an additional source of 3D fields; can induce transition**
  - NTMs can be seeded by ELMs/sawteeth
    - La Haye, private communication (2016)



Mode Penetration Determined by Transient-Induced Force Evolution at Rational Surface

- **Begin in time-asymptotic, metastable state**
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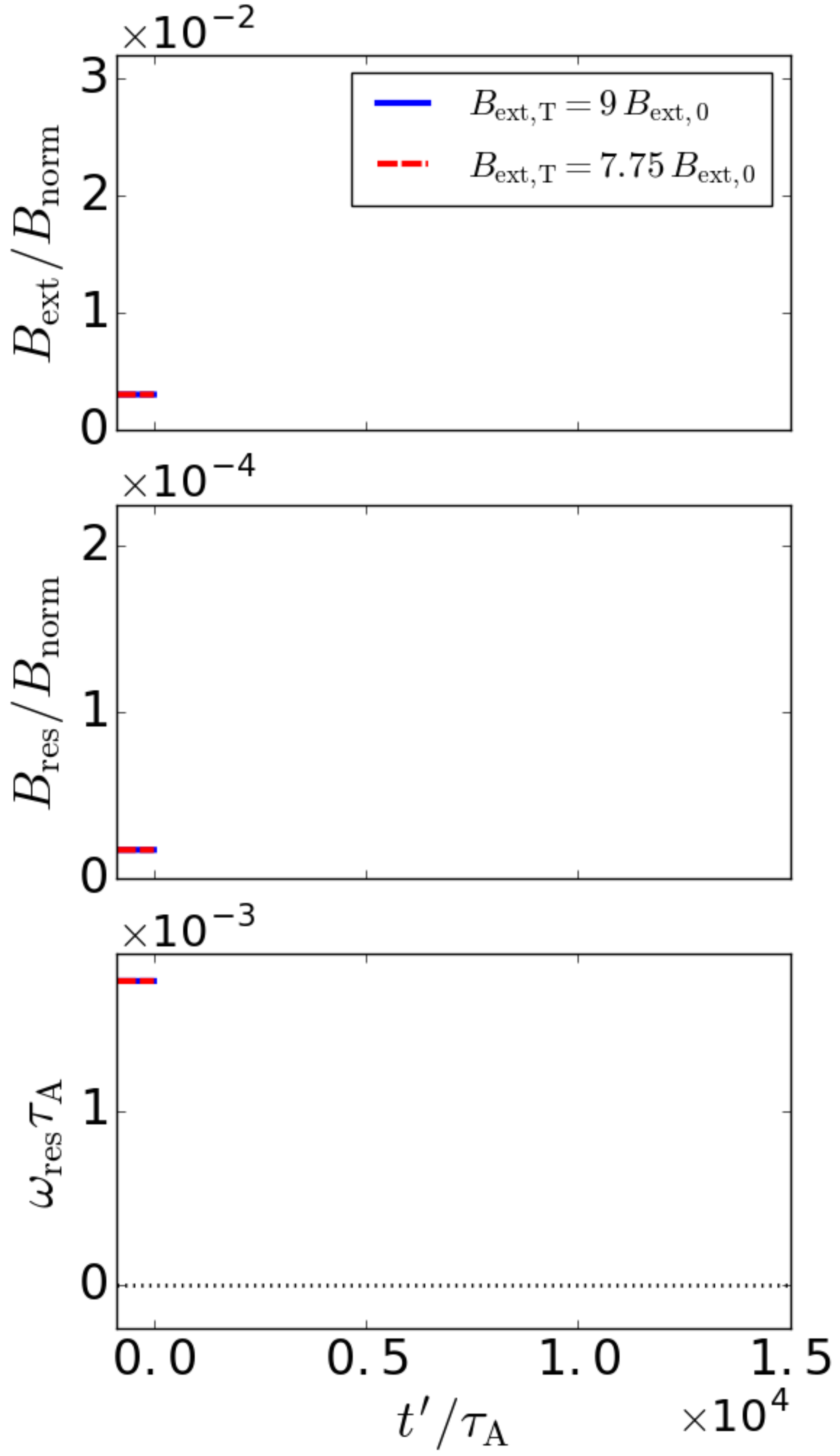

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  - Forcing decreases flow locally
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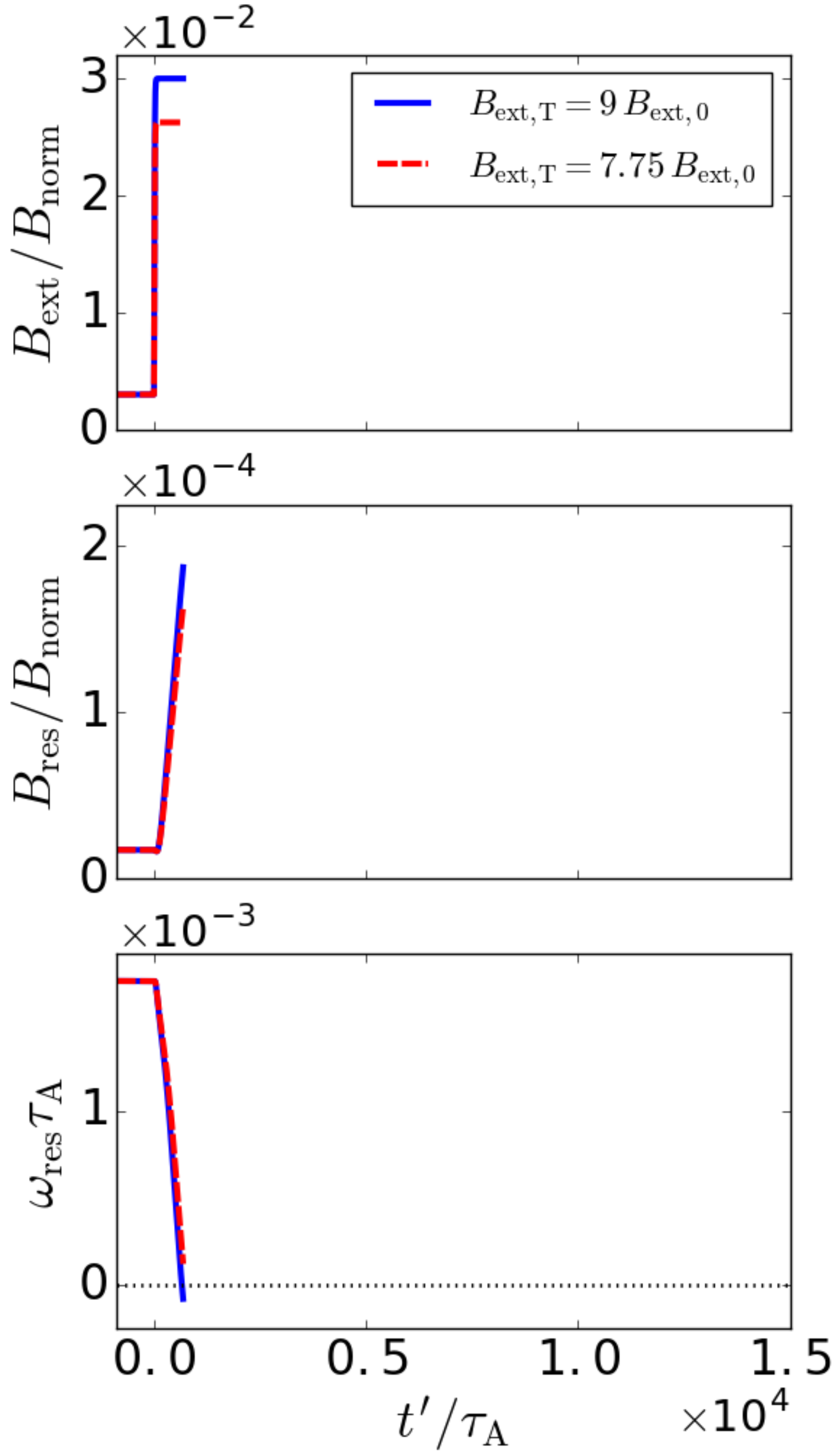

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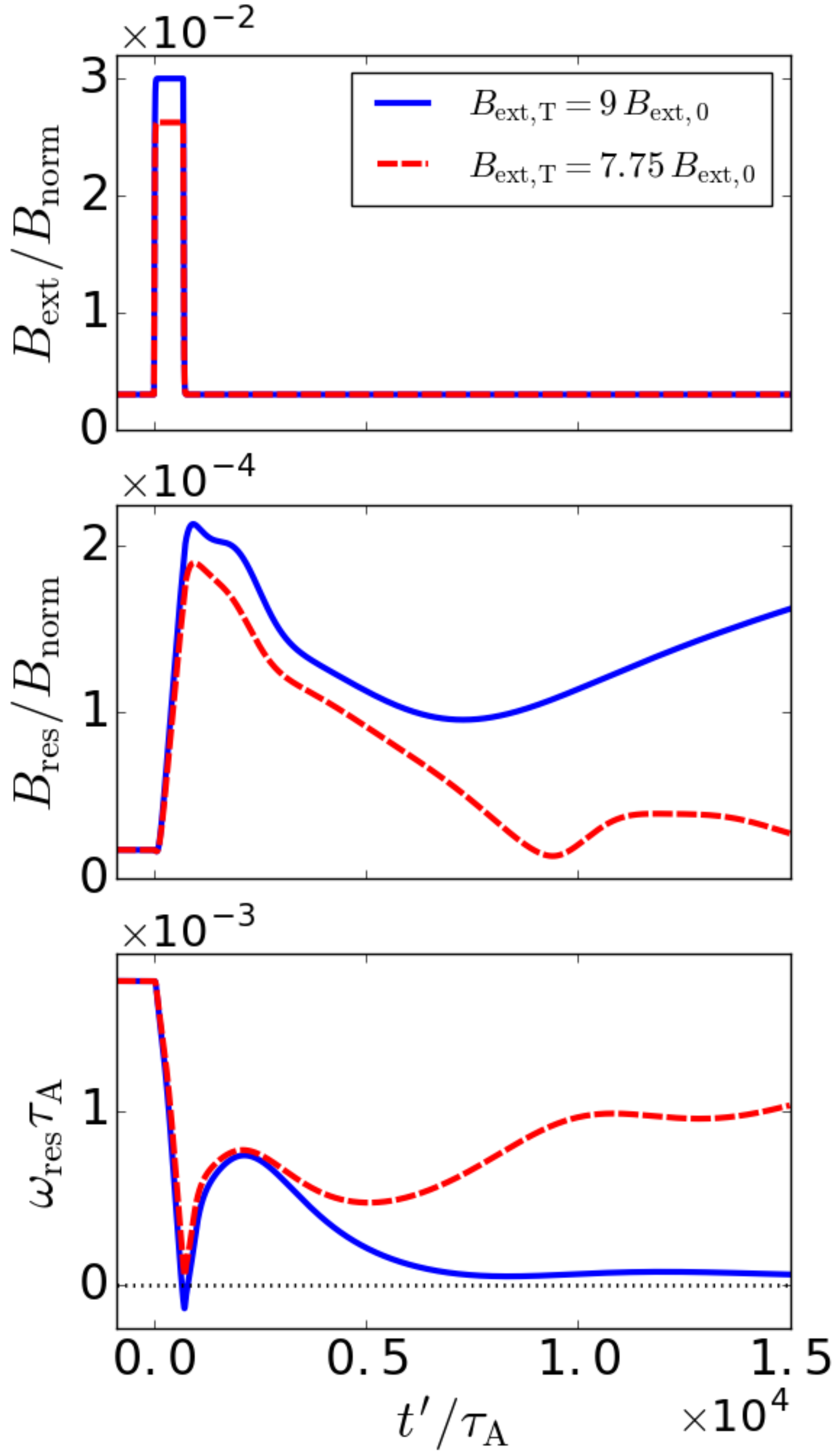
- **Transient turns off and system continues to evolve**
  - Mutual evolution of forces determines final state



Outline

- Explore dynamics of transient perturbation in slab geometry
- Computational results elucidate mode penetration dynamics
- Develop analytic model of mode penetration dynamics

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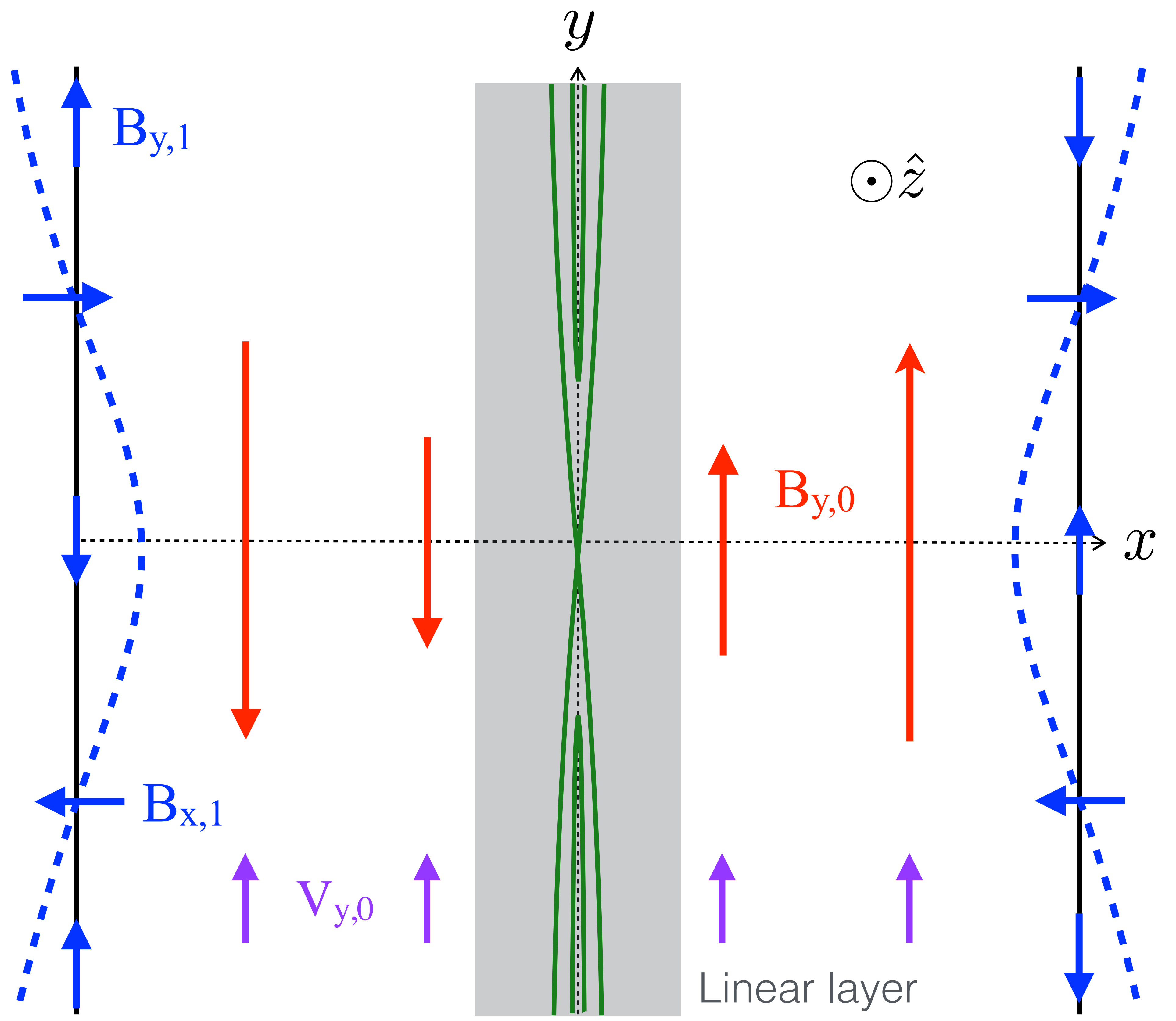
Slab Model Employed to Simplify Computations and Analytics

- **Slab geometry with uniform out-of-plane current density**
  - Stable equilibrium with $\Delta' a \cong -2k_y a < 0$



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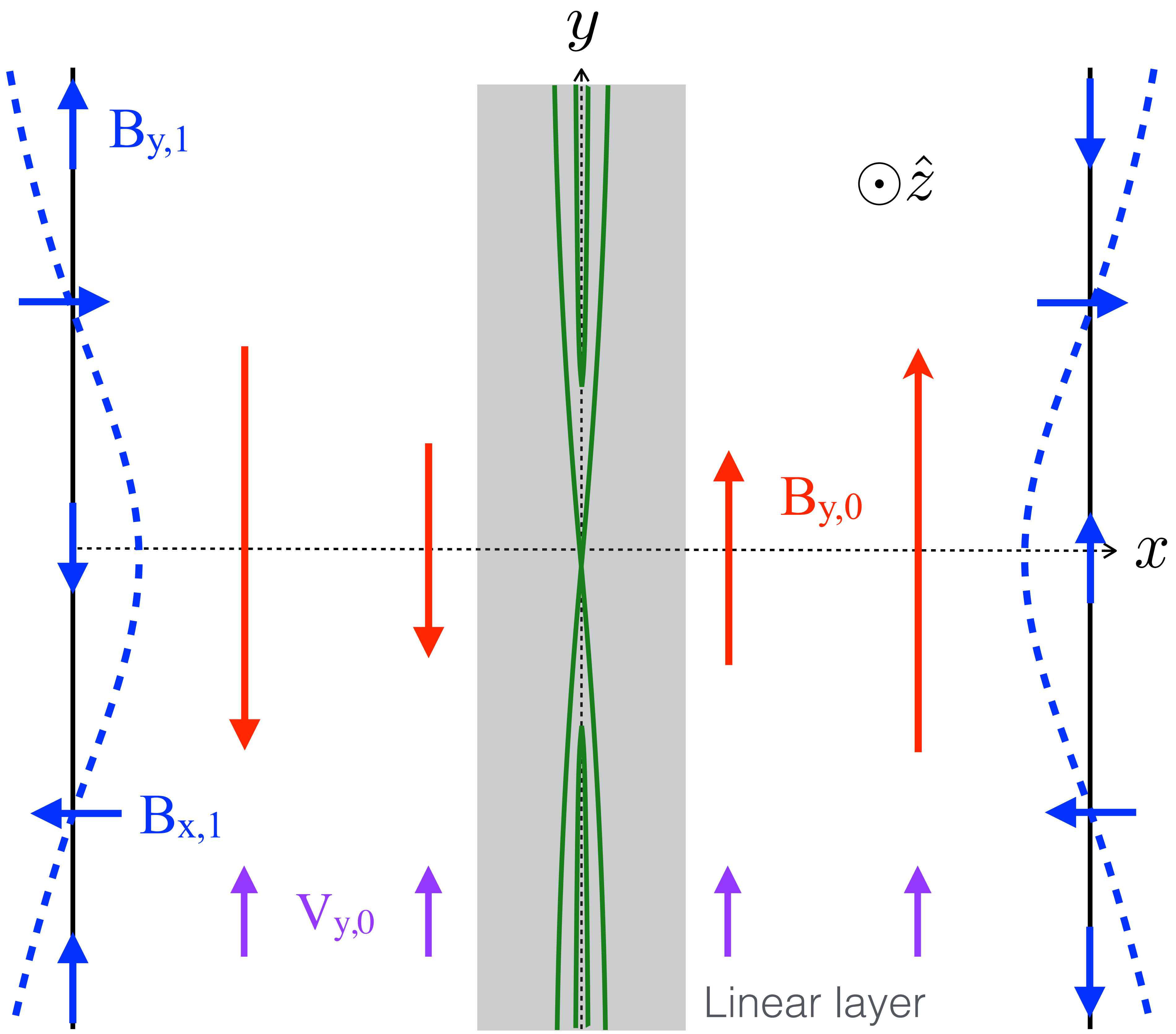

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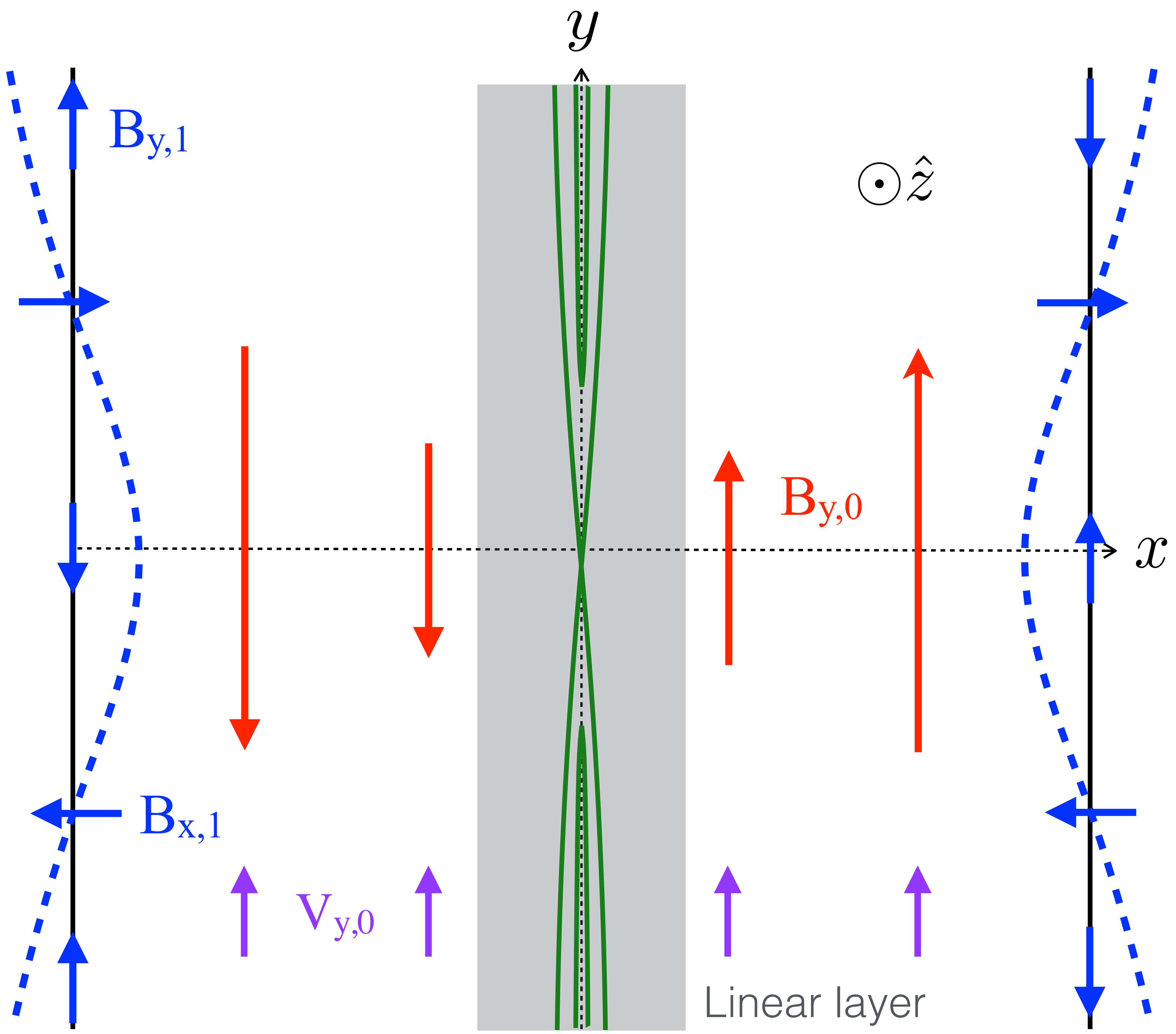

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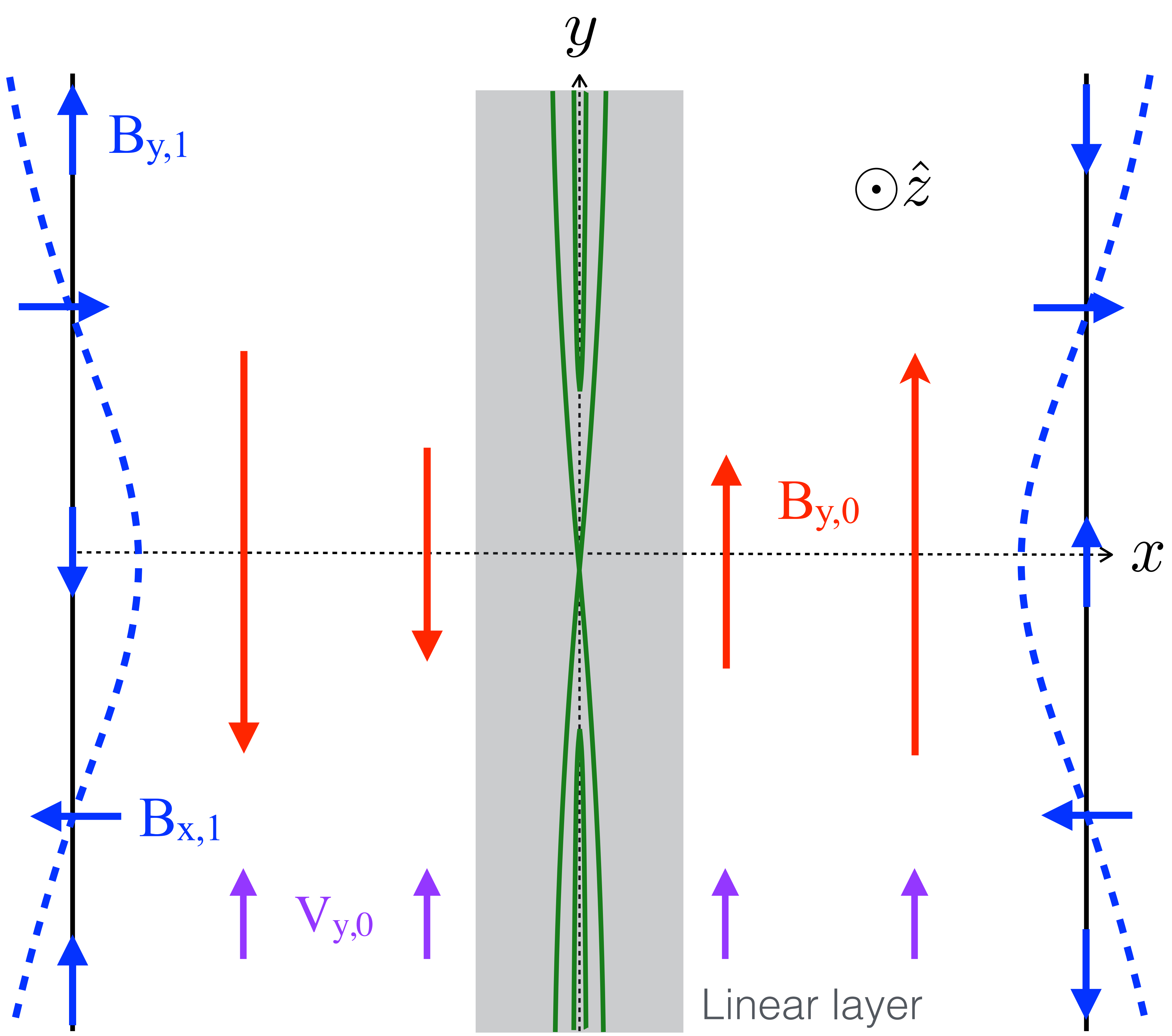
- **Visco-Resistive dissipation parameters**
  - $S = 1.1\times10^7$, $P_m = 20$
  - Linear layer width: $\delta_{\text{VR}} = S^{-1/3} P_m^{1/6} a = 7.4\times10^{-3} a$



Time-Asymptotic Electromagnetic and Viscous Forces Balance At Rational Surface

- **n=0 EM force per unit length in z at x=0**

$$\hat{F}_{y,EM} = \int_{-\delta_{\mathrm{VR}}/2}^{\delta_{\mathrm{VR}}/2} dx \int_{-L_{y/2}}^{L_y/2} dy (\mathbf{J} \times \mathbf{B}) \cdot \hat{y} = \boxed{-\frac{n\pi}{\mu_0 k_y^2} \mathrm{Im}\left\{ B^*_{\mathrm{res}} [\![\partial_x B_{\mathrm{res}}]\!]_{x=0} \right\}}$$

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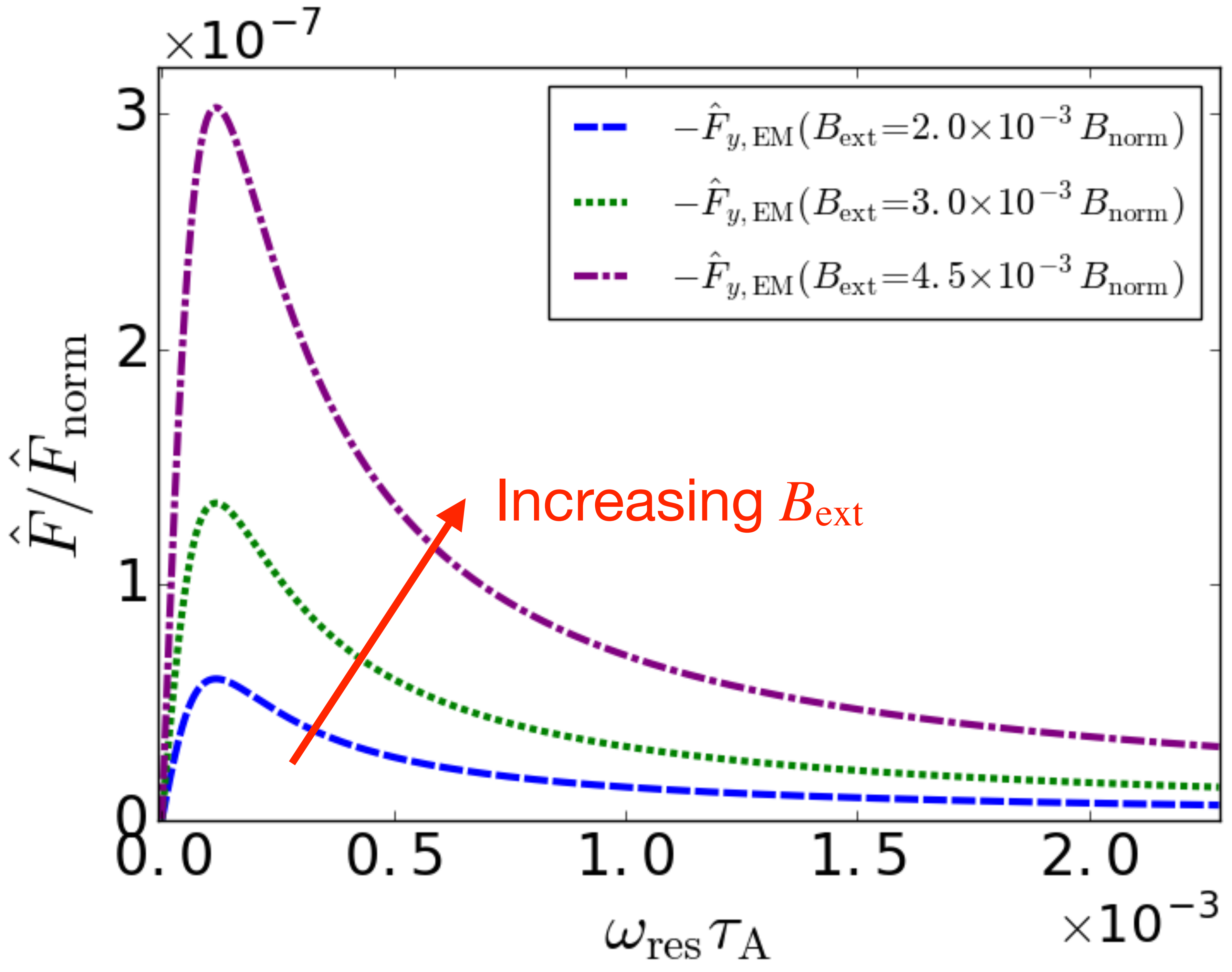
- Linear, time-asymptotic, visco-resistive response:

$$B_{res} = \frac{a\Delta'_{ext}}{-a\Delta' + i\omega_{res}\tau_{VR}} B_{ext} \rightarrow \boxed{\hat{F}_{y,EM} = -\frac{\omega_{res}\tau_{VR}}{(-a\Delta')^2 + (\omega_{res}\tau_{VR})^2}\frac{n\pi(a\Delta'_{ext})^2}{ak_y^2}\frac{B_{ext}^2}{\mu_0}}$$

  - R. Fitzpatrick, NF **33**, 1049 (1993)
  - Quasilinear $F_{y,\mathrm{EM}}$ localized at x=0



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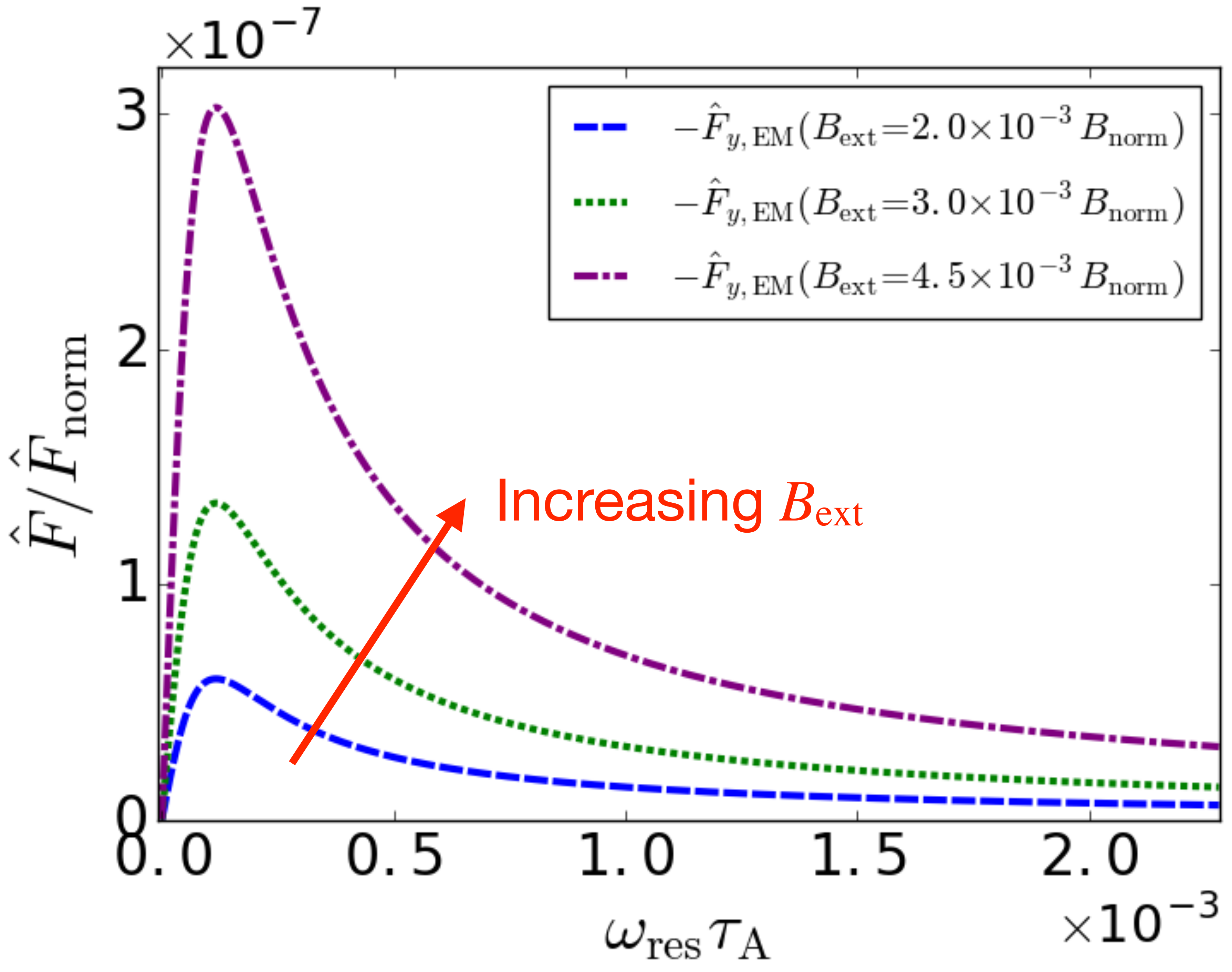

- **Viscous force per unit length in z at x=0**

$$\hat{F}_{y,VS} = \int_{-\delta_\delta/2}^{\delta_\delta/2} dx \int_{-L_y/2}^{L_y/2} dy [\boldsymbol{\nabla}\cdot\rho\nu\boldsymbol{\nabla}\mathbf{V}]\cdot\hat{y} = \boxed{L_y\rho\nu_0[\![\partial_x V(x,t)]\!]_{x=0}}$$

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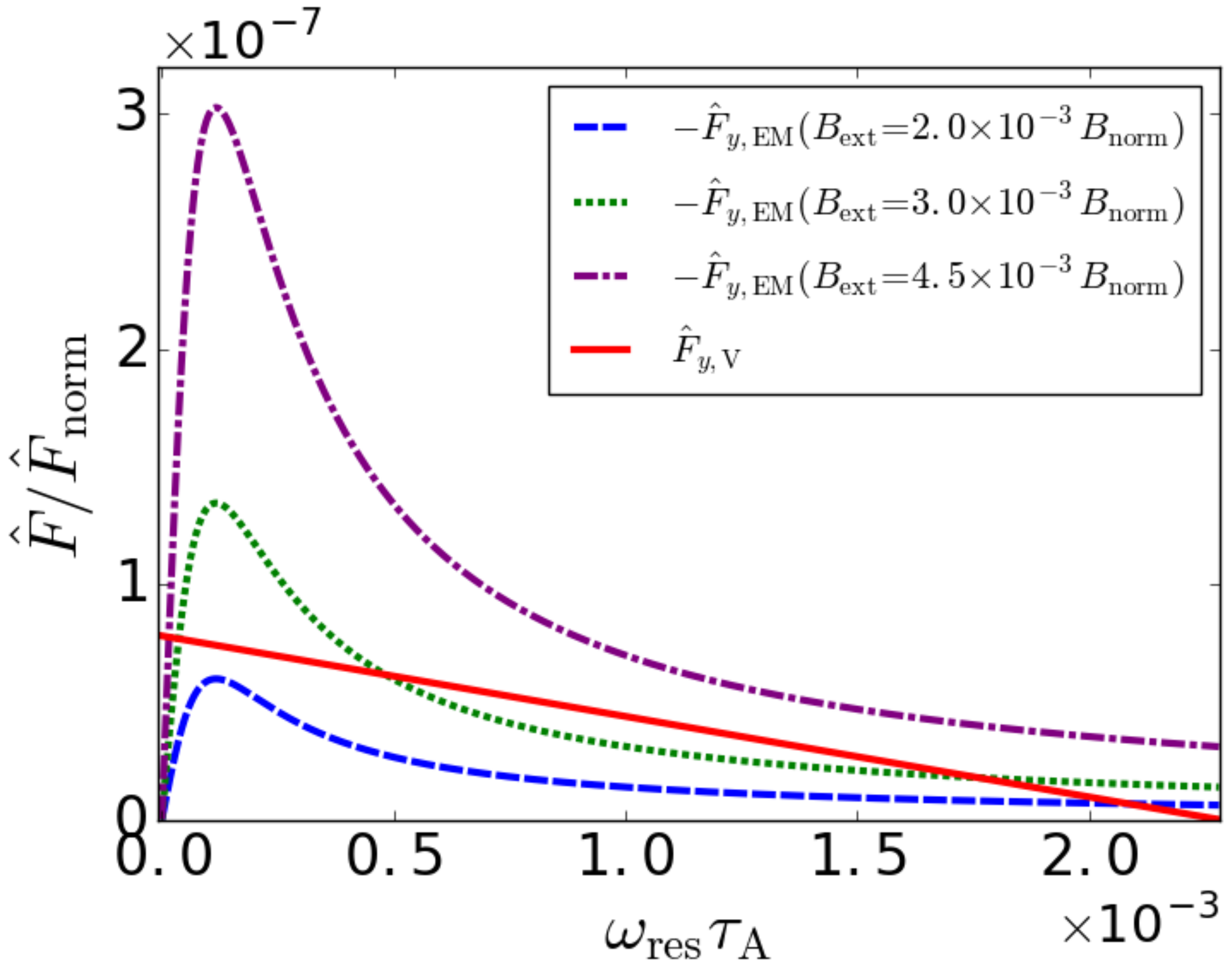

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- Because EM force is localized at x=0, $\partial^2_{xx}V_y$=0 in $0<x<|a_\nu|$ yields ➝ $\boxed{\hat{F}_{y,V} = \frac{2L_y\rho\nu_0}{k_y a_\nu}(\omega_0 - \omega_{res})}$

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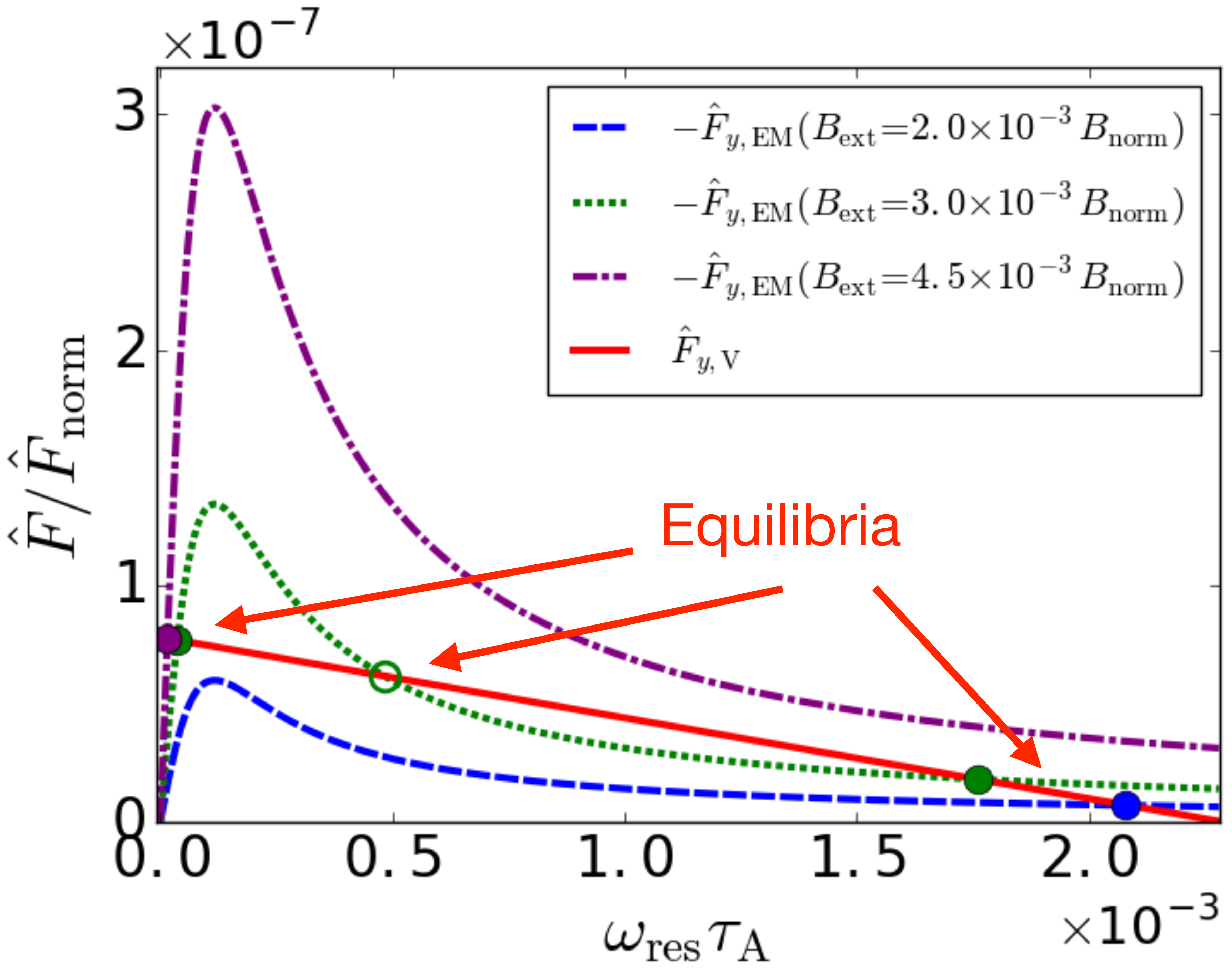

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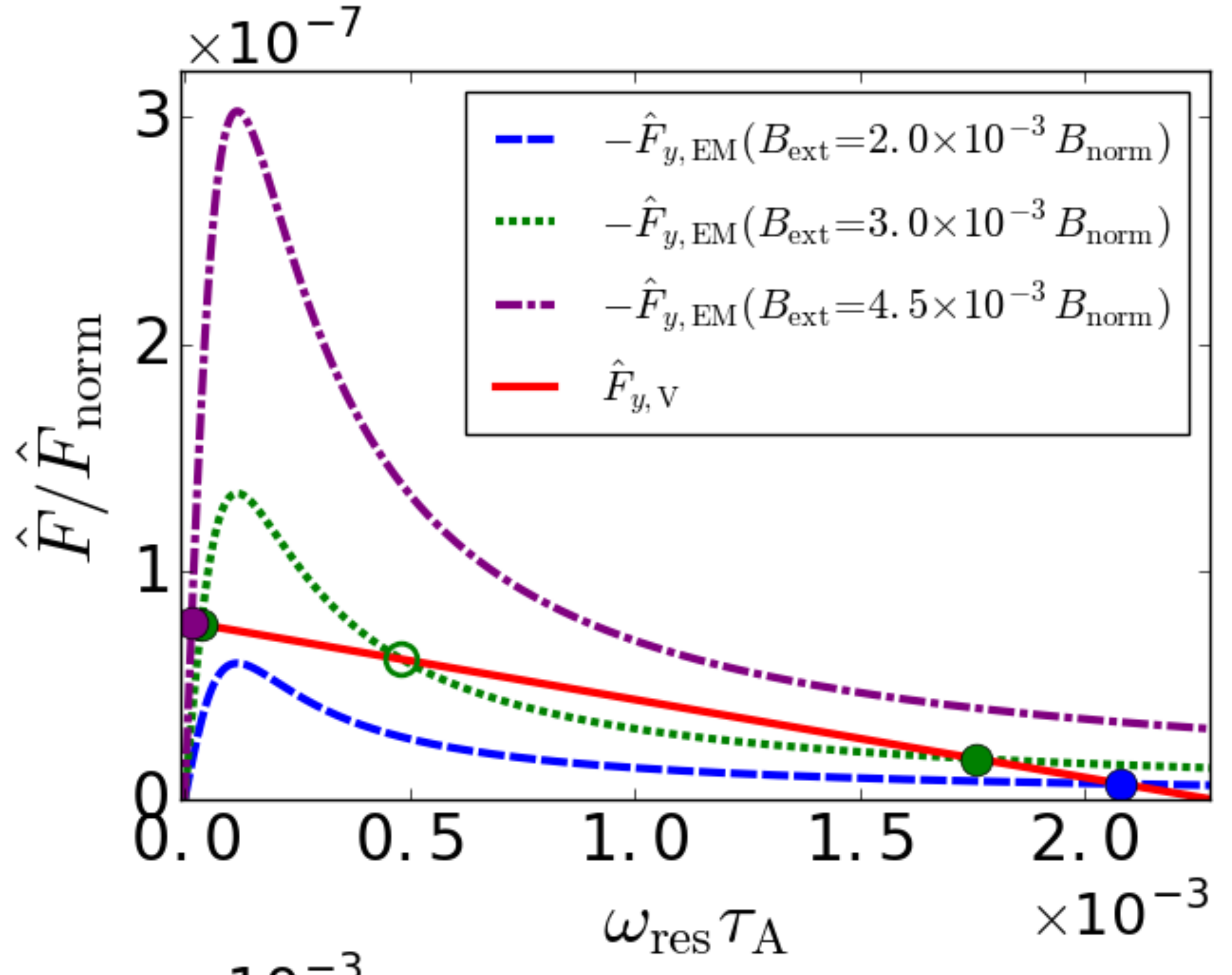
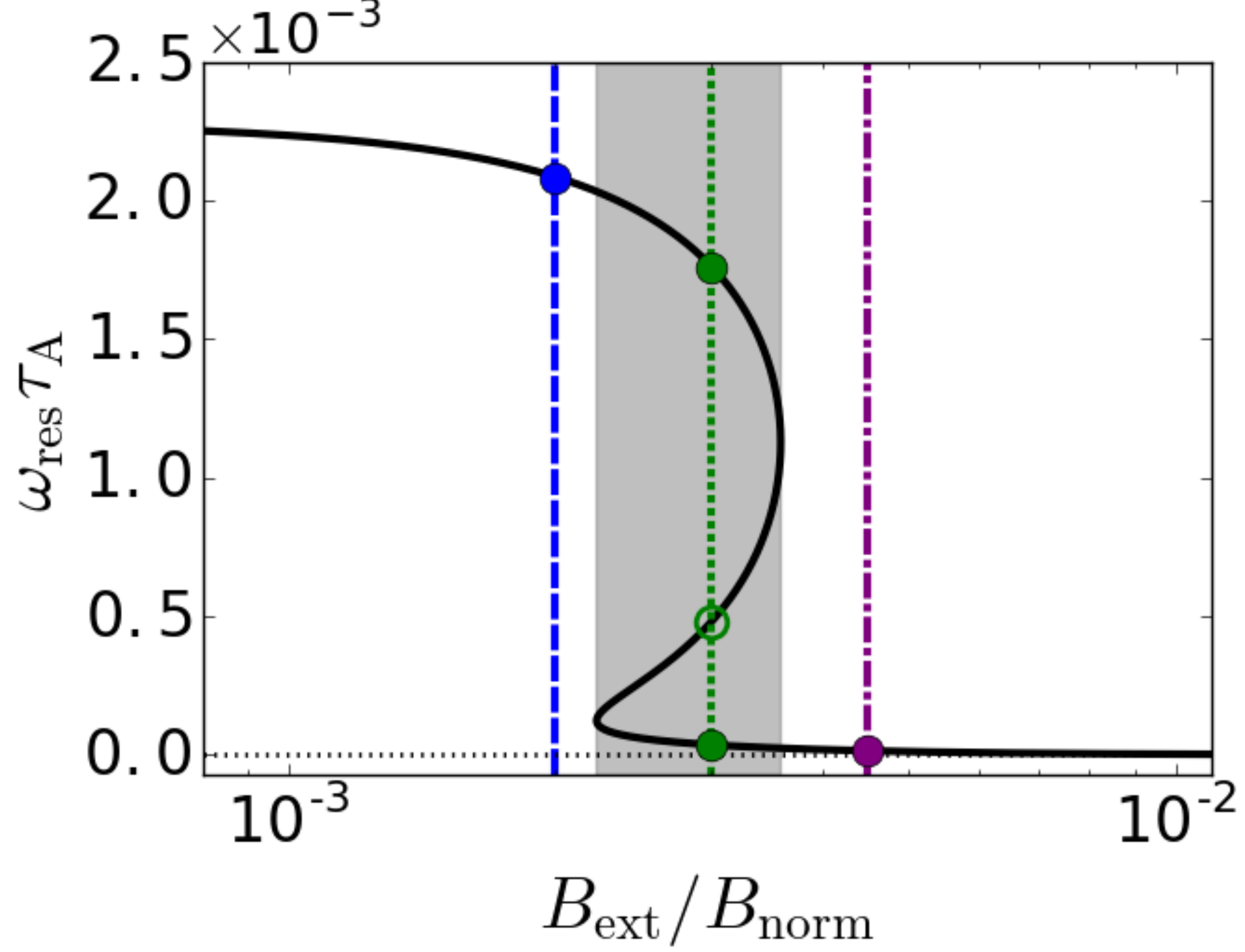
EM and Viscous Force Balance Gives Rise to Bifurcated, Metastable Equilibria

- **Force balance gives cubic relation for ω_{res}**

$$\boxed{\frac{\omega_0}{\omega_{\mathrm{res}}} - 1 + \omega_0\omega_{\mathrm{res}}\tau_{\mathrm{VR}}'^2 - \omega_{\mathrm{res}}^2\tau_{\mathrm{VR}}'^2 = \frac{a_\nu \tau_{\mathrm{VR}}}{4a\rho\nu_0}\left(\frac{\Delta'_{\mathrm{ext}}}{-\Delta'}\right)^2 \frac{B_{\mathrm{ext}}^2}{\mu_0}}$$

- Here, $\tau_{\mathrm{VR}} = 2.104\tau_{\mathrm{A}} S^{2/3} {P_m}^{1/6}$ and $\tau_{\mathrm{VR}}' \equiv \tau_{\mathrm{VR}}/(-a\Delta')$





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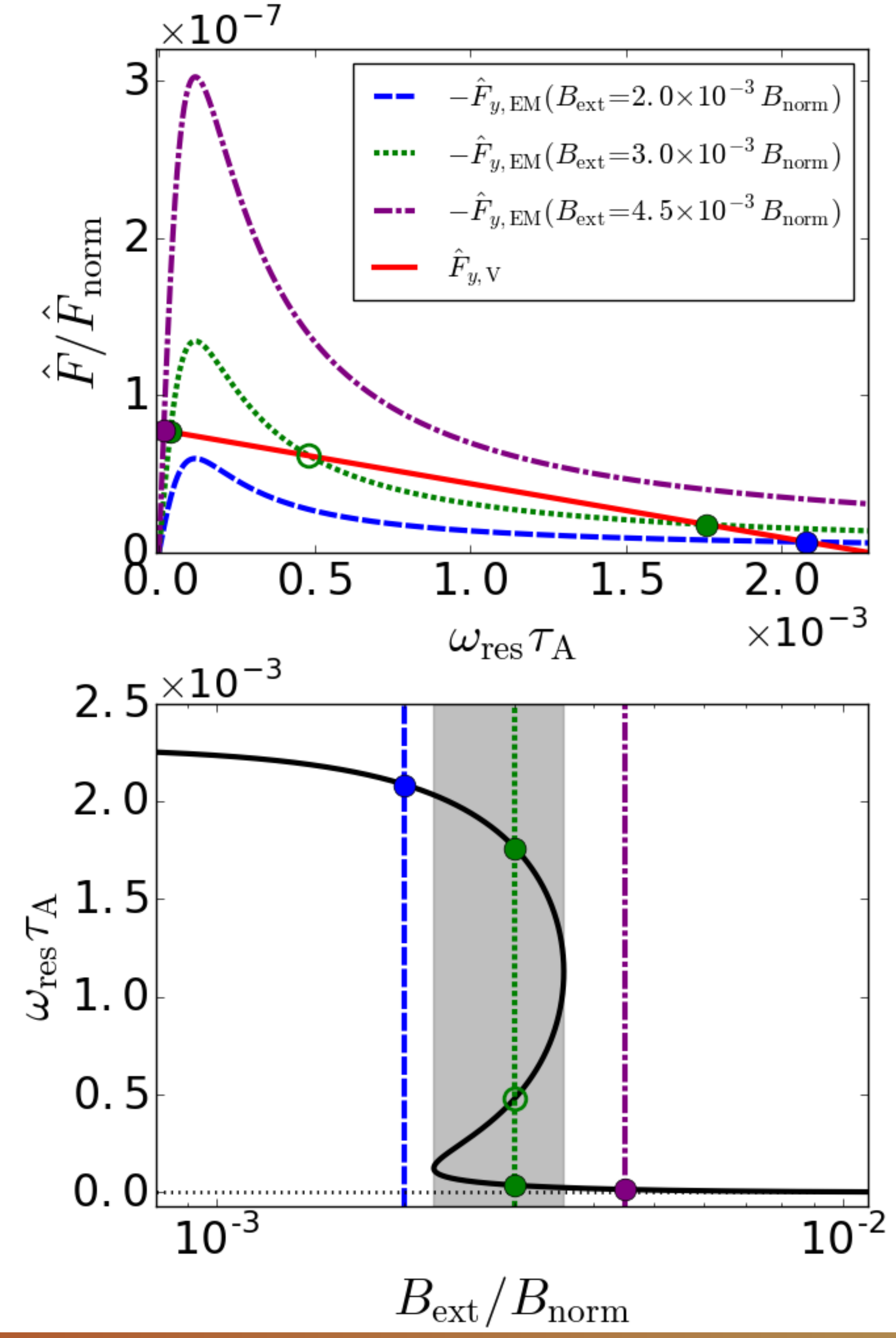

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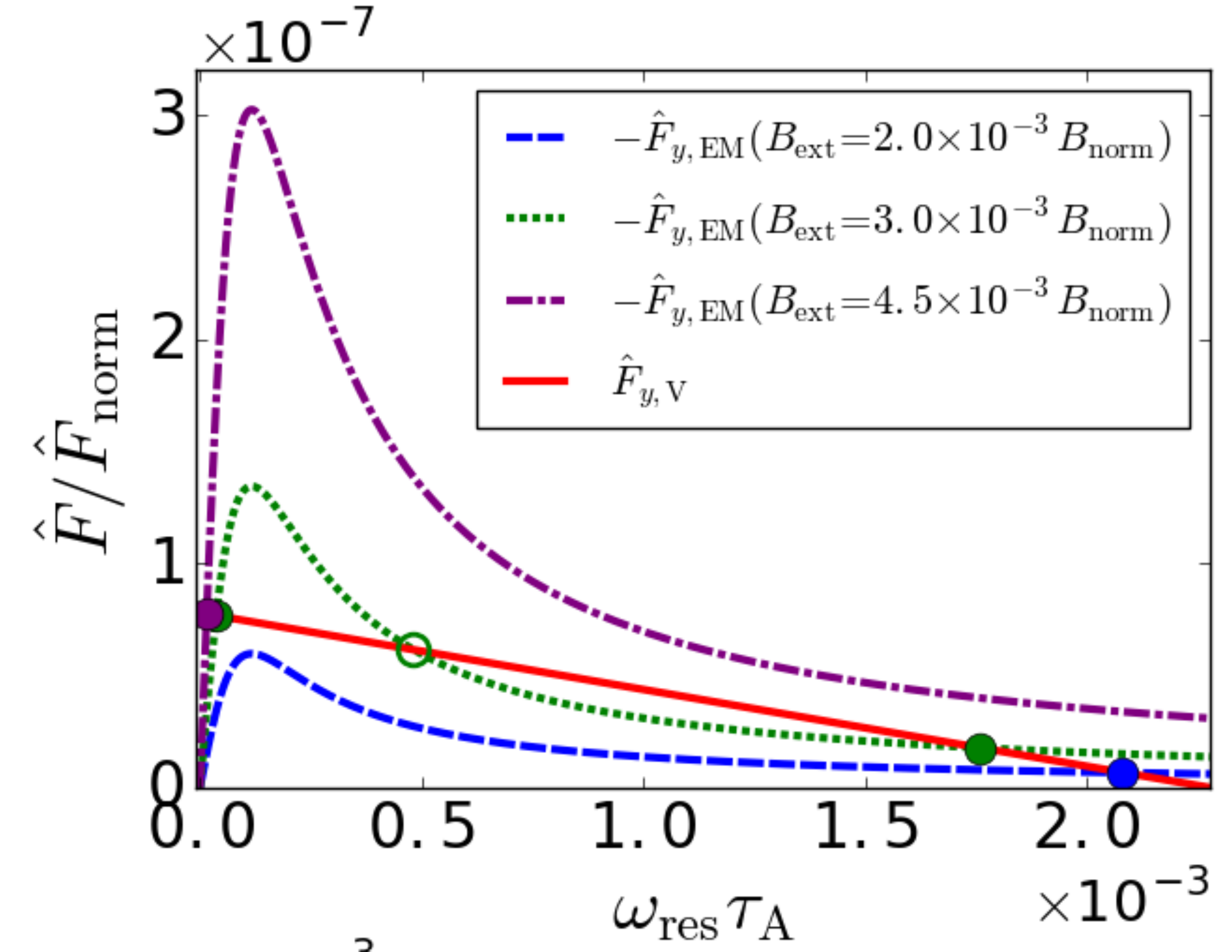
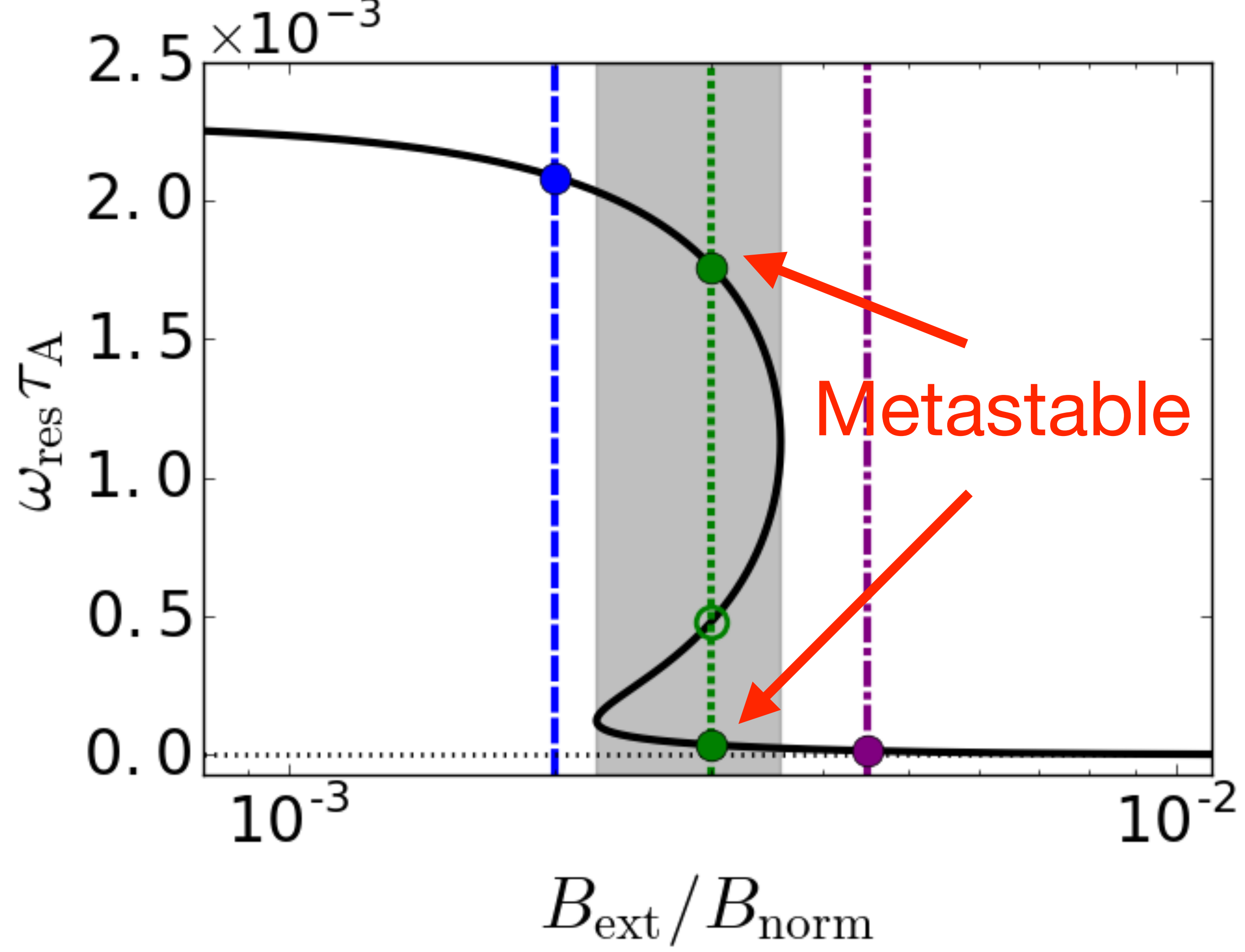
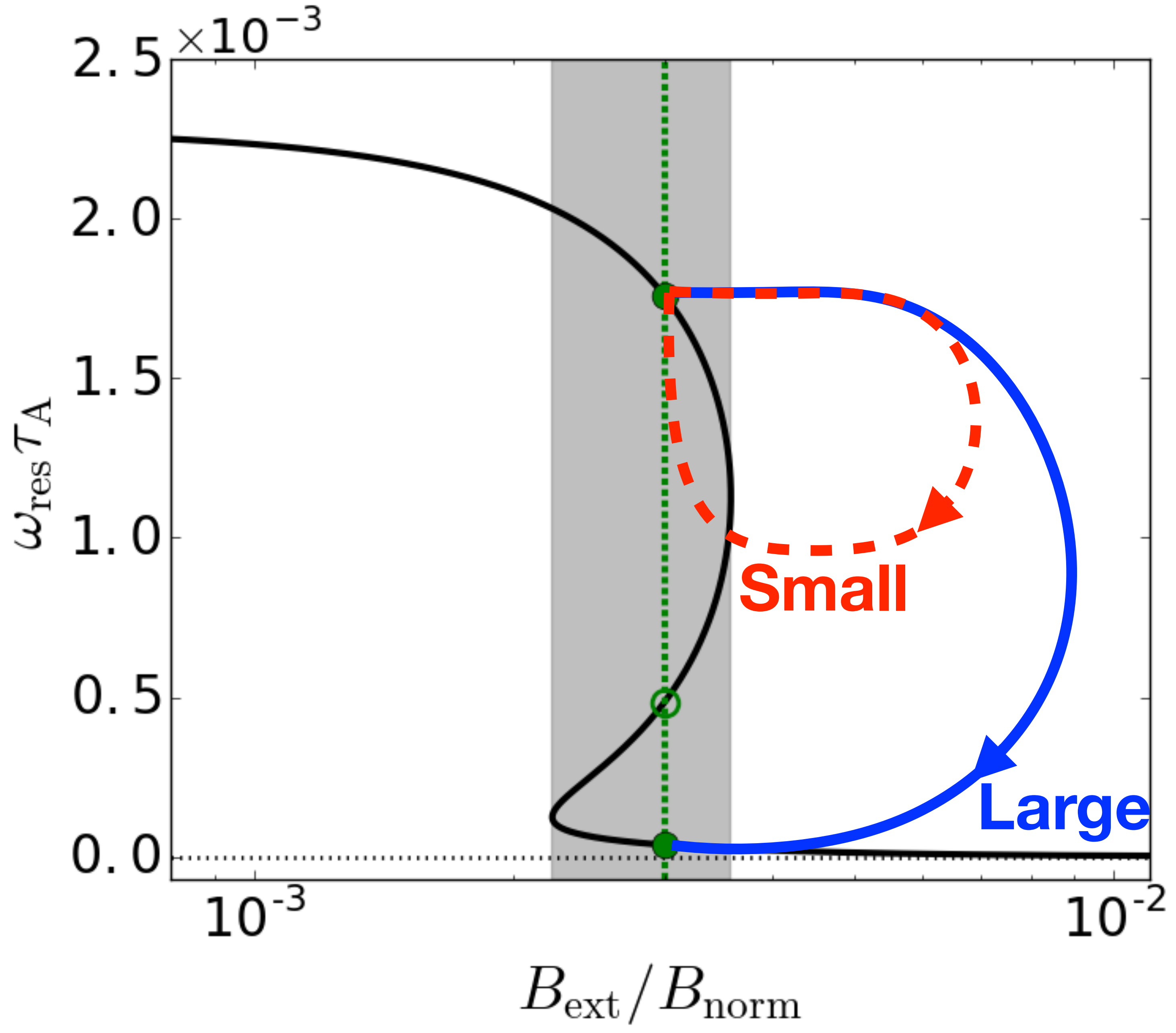
- **Two metastable equilibria: flow-screened and mode-penetrated**
  - Shaded region is metastable
  - Existence of metastable equilibria enables transient-induced mode penetration





Transient Can Precipitate Transition Between Metastable Equilibria

- **Hypothesis: If transient causes enough flow evolution, mode penetration occurs**



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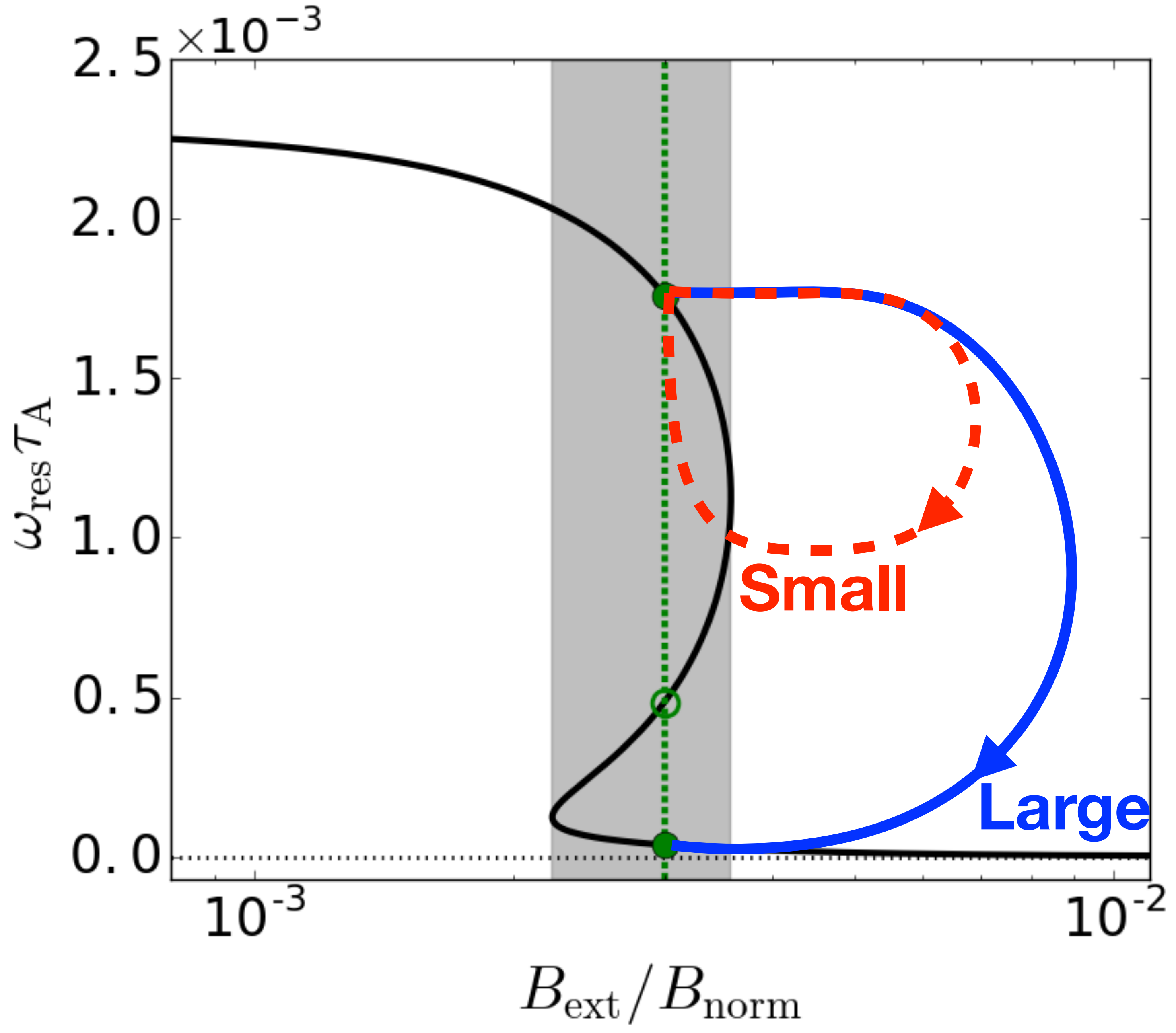

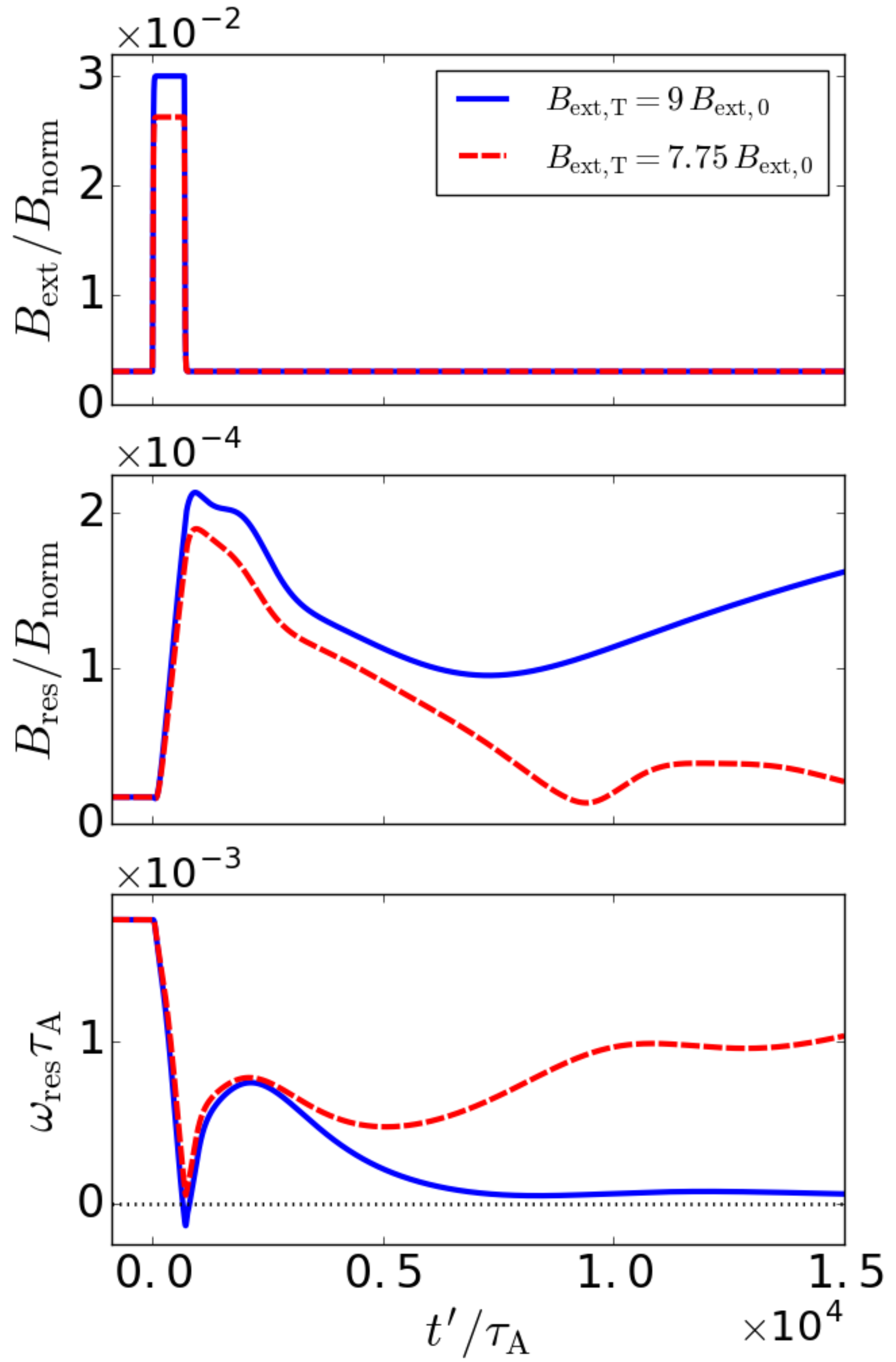

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- **Explore dynamics of transient perturbation in slab geometry**
  - Time-asymptotic EM and viscous force balance
  - Transient induced mode penetration needs metastable equilibrium
- **Computational results elucidate mode penetration dynamics**
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NIMROD Code Employed to Solve Visco-Resistive MHD Equations

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  - Presently, assume cold plasma and ignore two-fluid effects

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = \mathbf{J}\times\mathbf{B} - \nabla\cdot\mathbf{\Pi}_i\,,$$

$$\mathbf{\Pi}_i \equiv -\rho\nu\left[\nabla\mathbf{V} + \nabla\mathbf{V}^T - \frac{2}{3}\nabla\cdot\mathbf{V}\right],$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}, \quad \mu_0\mathbf{J} = \nabla\times\mathbf{B},$$

$$\mathbf{E} = -\mathbf{V}\times\mathbf{B} + \eta\mathbf{J}$$

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- **Time discretization uses finite difference**
  - Implicit leapfrog time evolution

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\boldsymbol{\nabla}\mathbf{V}\right) = \mathbf{J}\times\mathbf{B} - \boldsymbol{\nabla}\cdot\boldsymbol{\Pi}_i\,,$$

$$\boldsymbol{\Pi}_i \equiv -\rho\nu\left[\boldsymbol{\nabla}\mathbf{V} + \boldsymbol{\nabla}\mathbf{V}^T - \frac{2}{3}\boldsymbol{\nabla}\cdot\mathbf{V}\right],$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\boldsymbol{\nabla}\times\mathbf{E},\quad \mu_0\mathbf{J} = \boldsymbol{\nabla}\times\mathbf{B},$$

$$\mathbf{E} = -\mathbf{V}\times\mathbf{B} + \eta\mathbf{J}$$

- Evolve perturbation fields about a fixed equilibrium

NIMROD Code Employed to Solve Visco-Resistive MHD Equations

- **NIMROD capable of solving extended-MHD equations**
  - Presently, assume cold plasma and ignore two-fluid effects
- **Time discretization uses finite difference**
  - Implicit leapfrog time evolution
  - Evolve perturbation fields about a fixed equilibrium
- **Spatial discretization uses 2D, C^0, spectral elements**
  - Employ mesh packing at rational surface and edge

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\boldsymbol{\nabla}\mathbf{V}\right) = \mathbf{J}\times\mathbf{B} - \boldsymbol{\nabla}\cdot\boldsymbol{\Pi}_i\,,$$

$$\boldsymbol{\Pi}_i \equiv -\rho\nu\left[\boldsymbol{\nabla}\mathbf{V} + \boldsymbol{\nabla}\mathbf{V}^T - \frac{2}{3}\boldsymbol{\nabla}\cdot\mathbf{V}\right],$$

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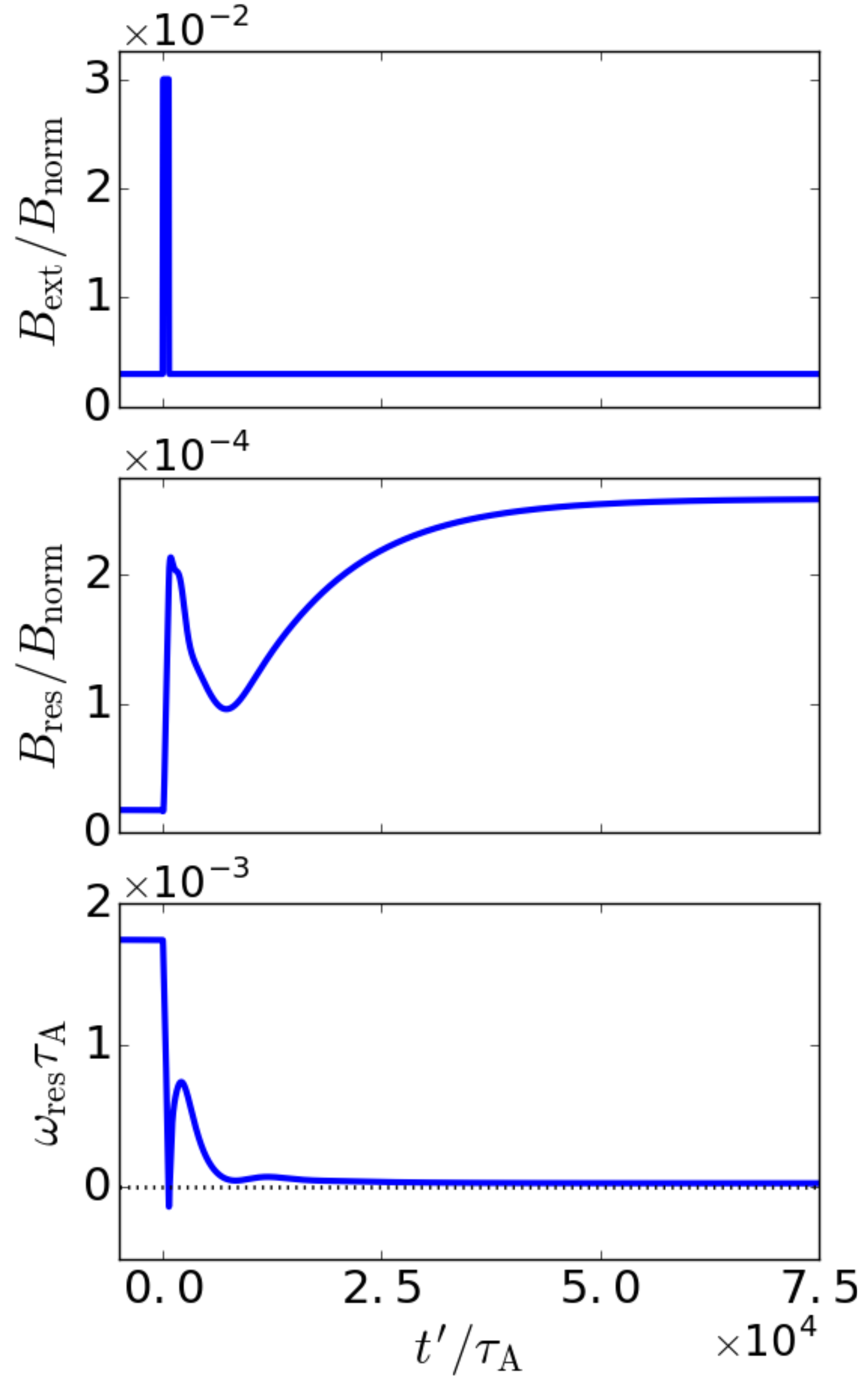
Large Transient Induces Mode Penetration

- **System properties**
  - $S = 1.1\times10^{7}$
  - $P_m = 20$
  - $V_0 = 500$ m/s
  - $B_{ext,0} = 3\times10^{-4}$ T
- **Transient properties**
  - $B_{ext,T} = 9\ B_{ext,0}$
  - $\Delta t_T = 690\ \tau_A$ duration
  - Approximately square



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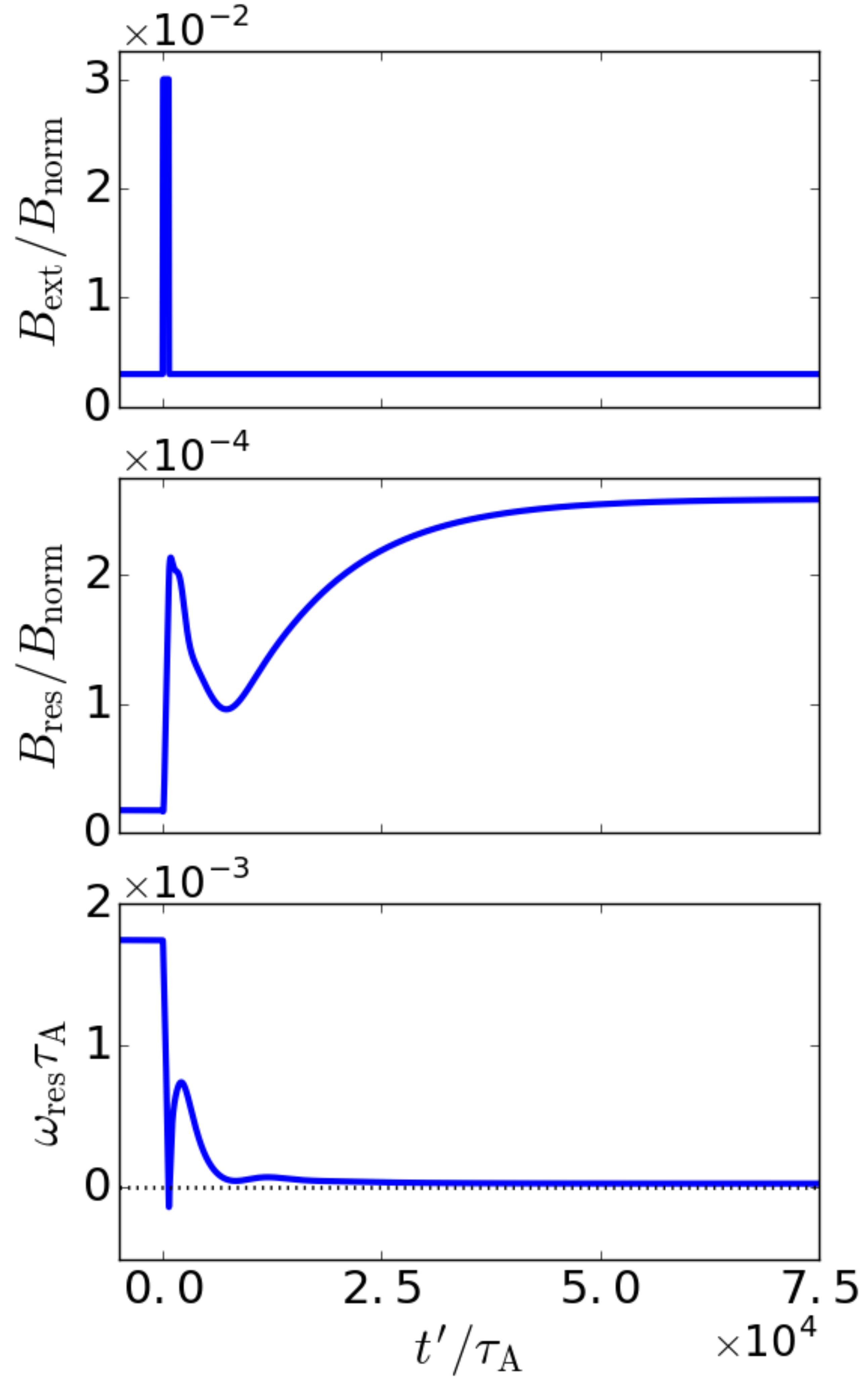
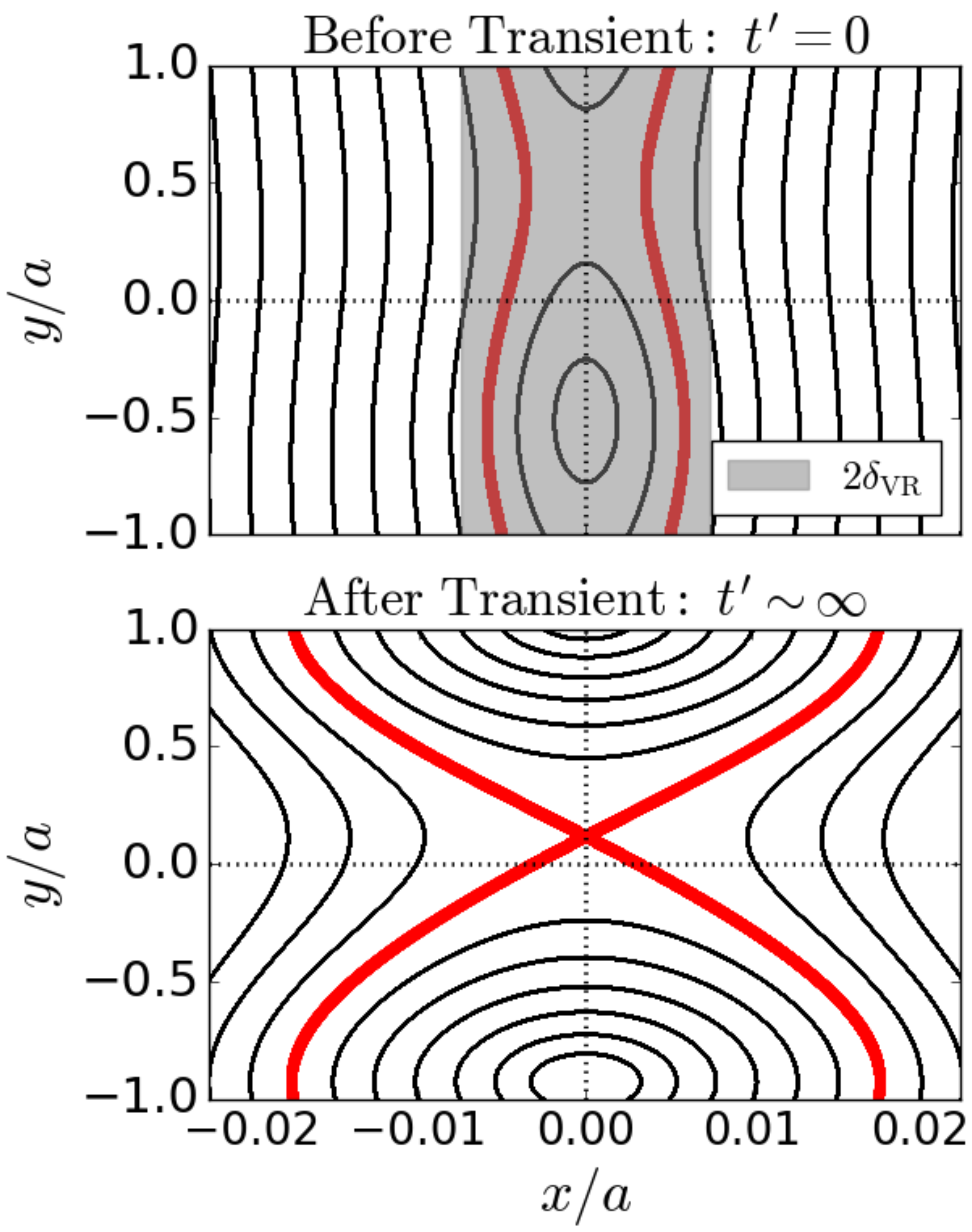
- **Mode penetration forms nonlinear magnetic island**

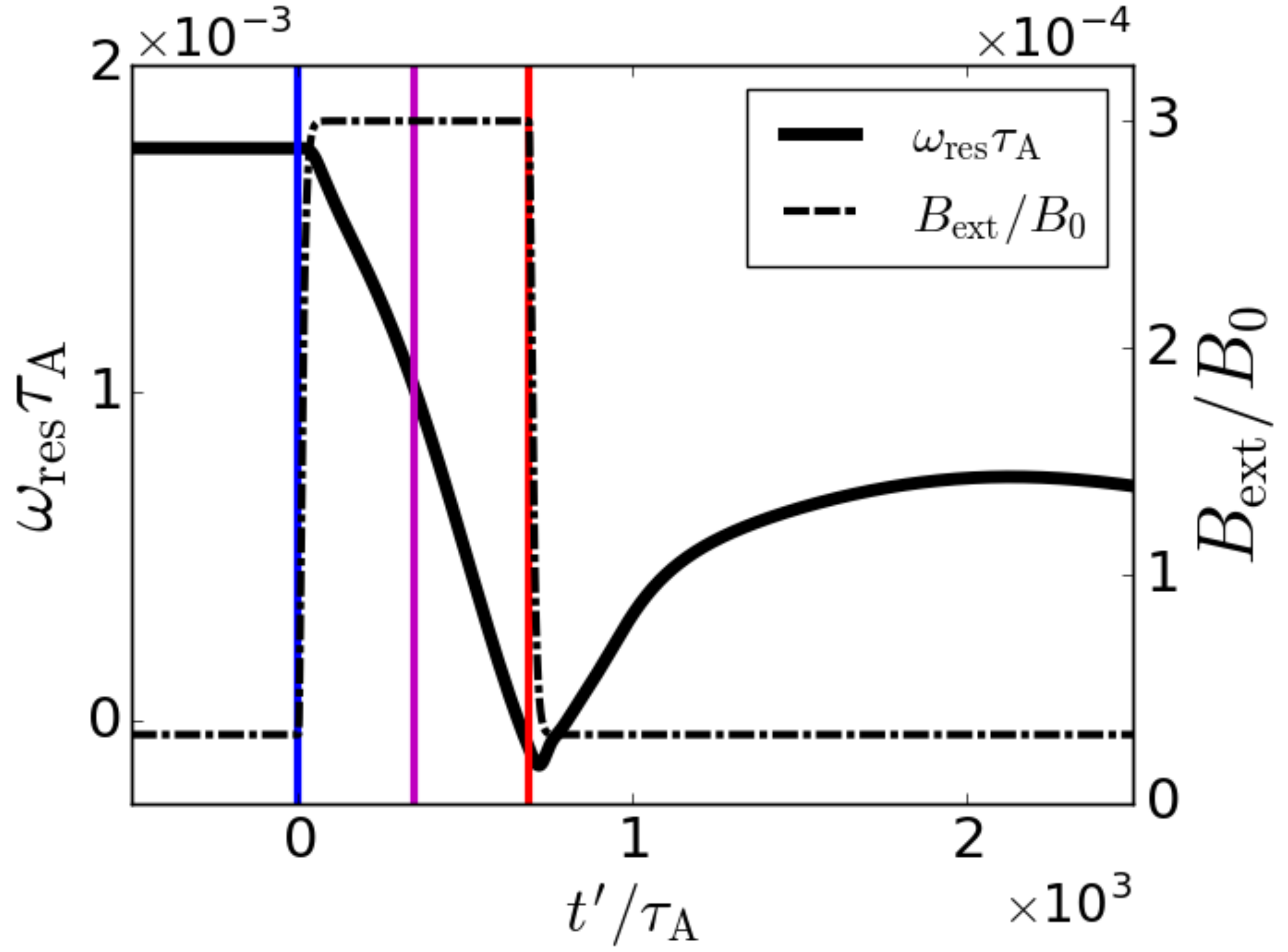


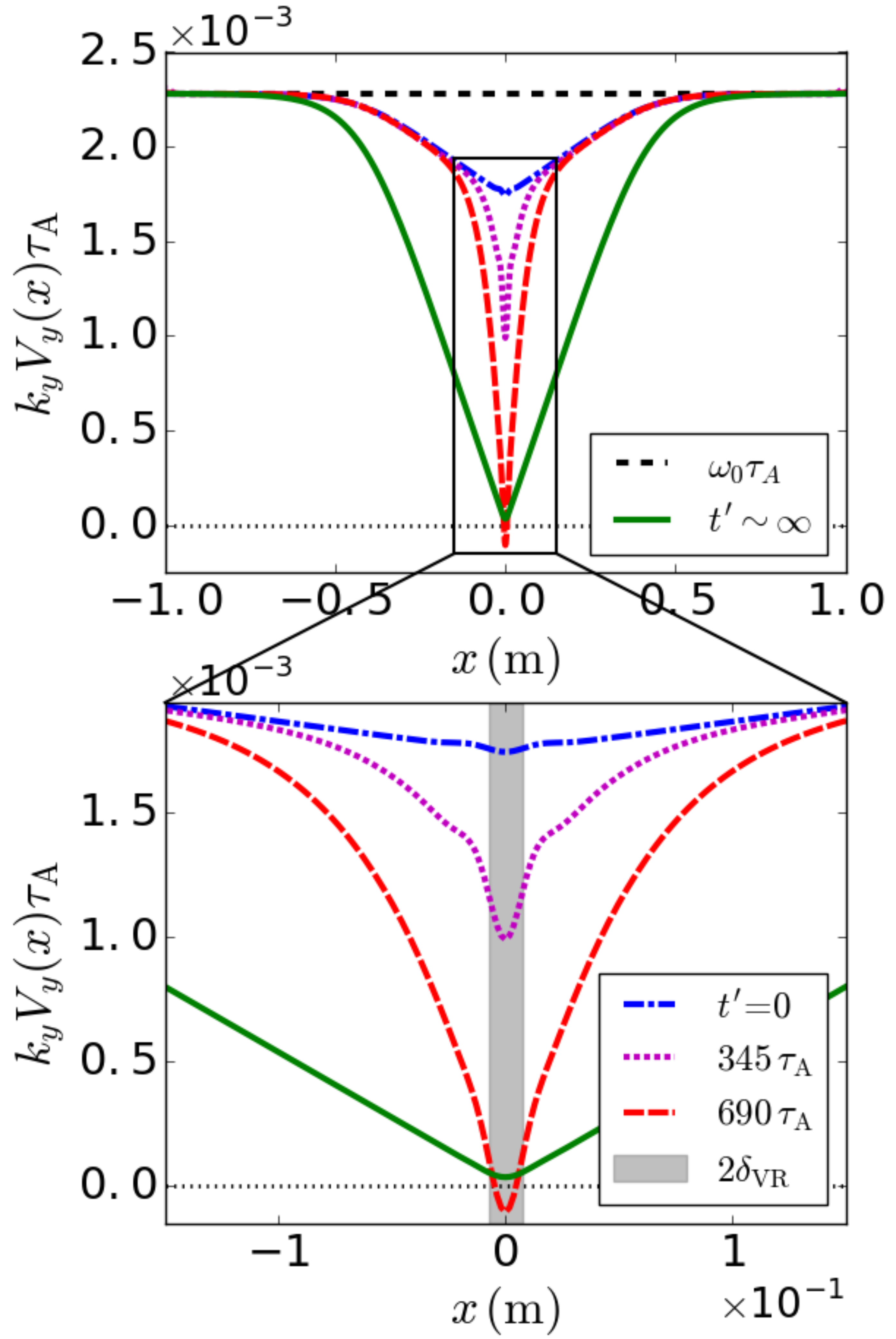


Flow Profile Evolution Determines Magnitude of Viscous Force

- **Transient enhancement of viscous force due to local flow evolution**

$$\hat{F}_{y,VS} = L_y \rho \nu_0 \left[\!\left[\partial_x V(x,t') \right]\!\right]_{x=0}$$





Magnitude of Transient Is Critical For Flow Response and Mode Penetration

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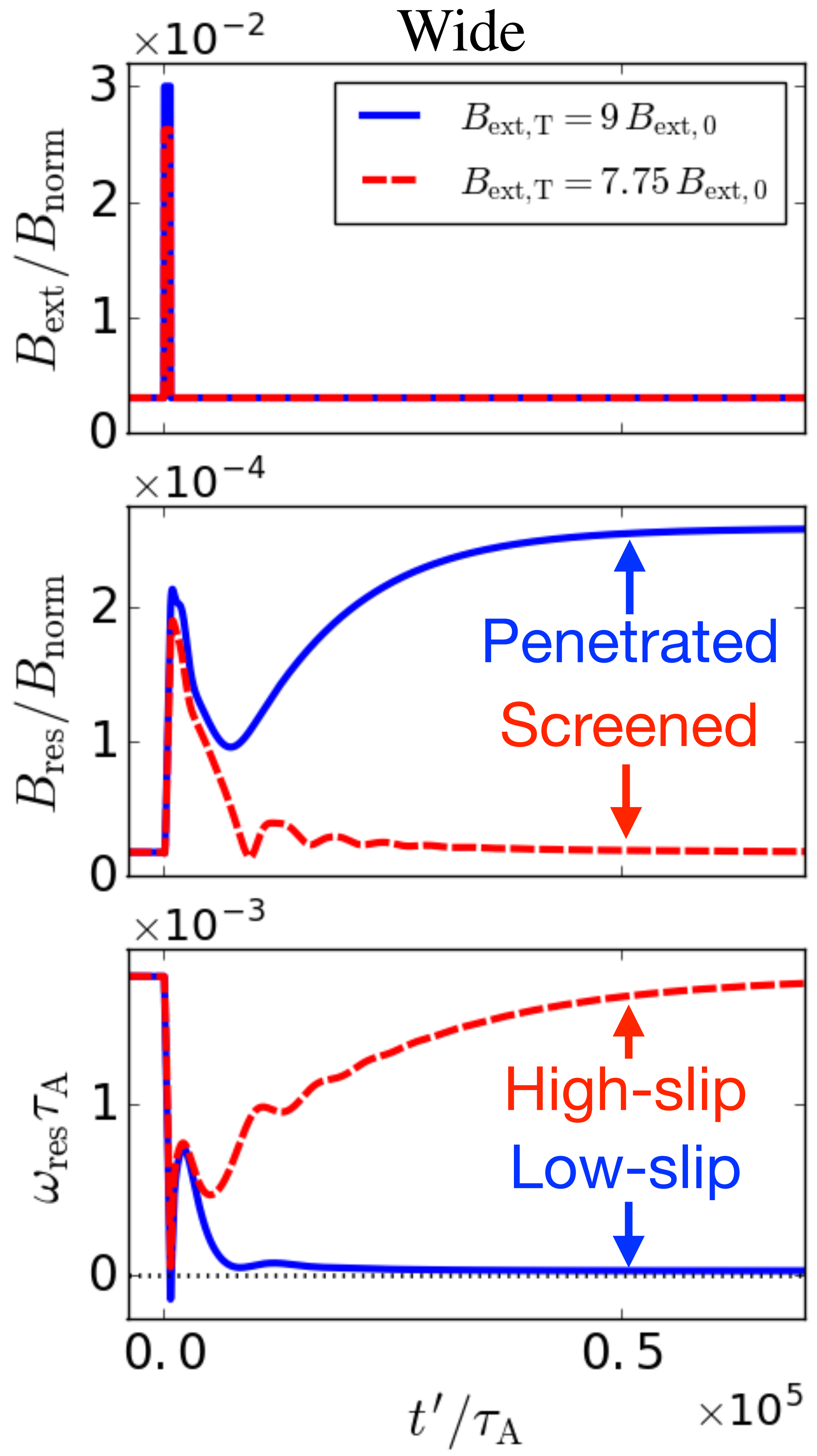
- **$B_{ext,T} = 9\ B_{ext,0}$ mode penetrates**
  - $B_{ext,T} = 7.75\ B_{ext,0}$ returns to high slip state



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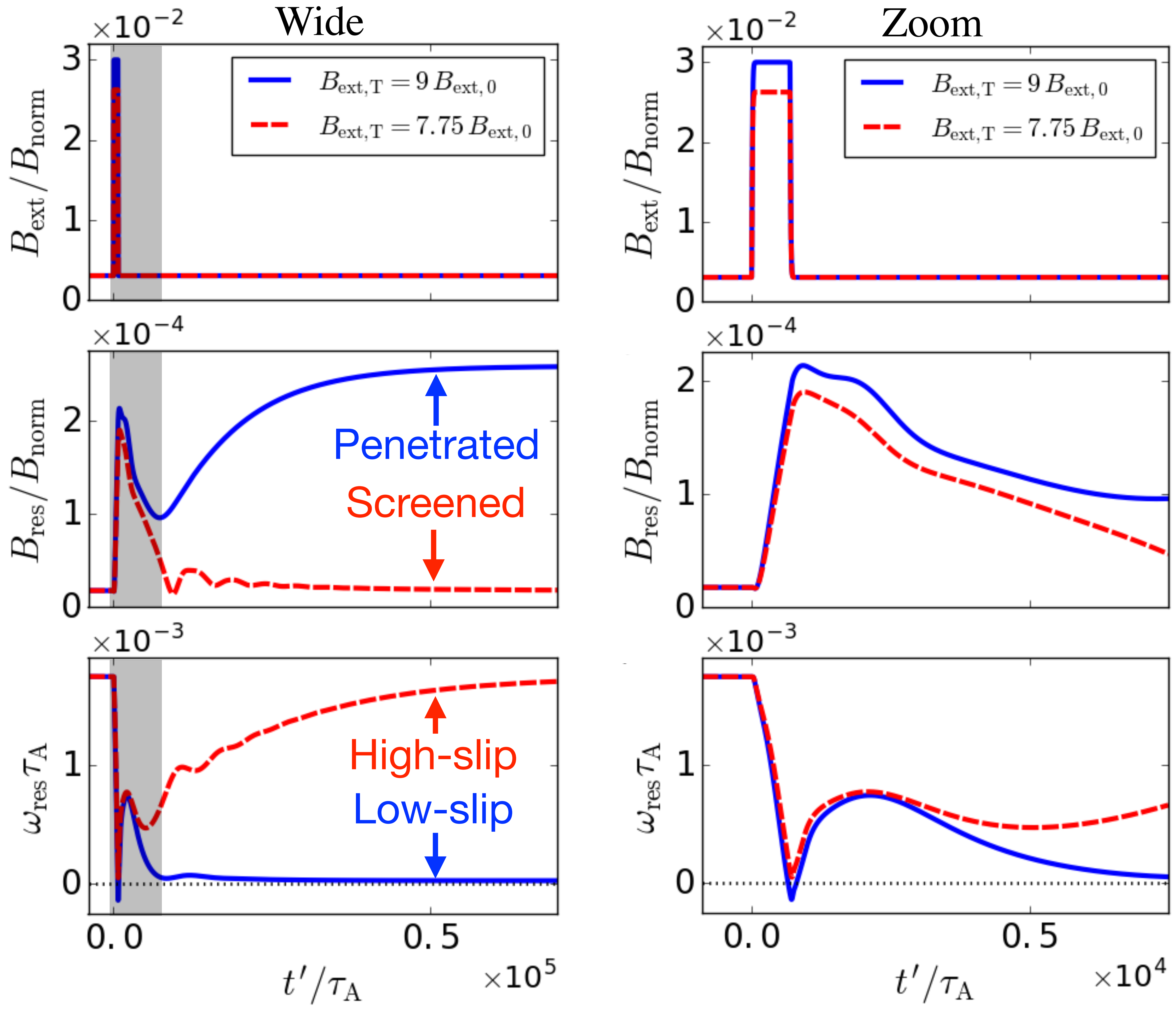
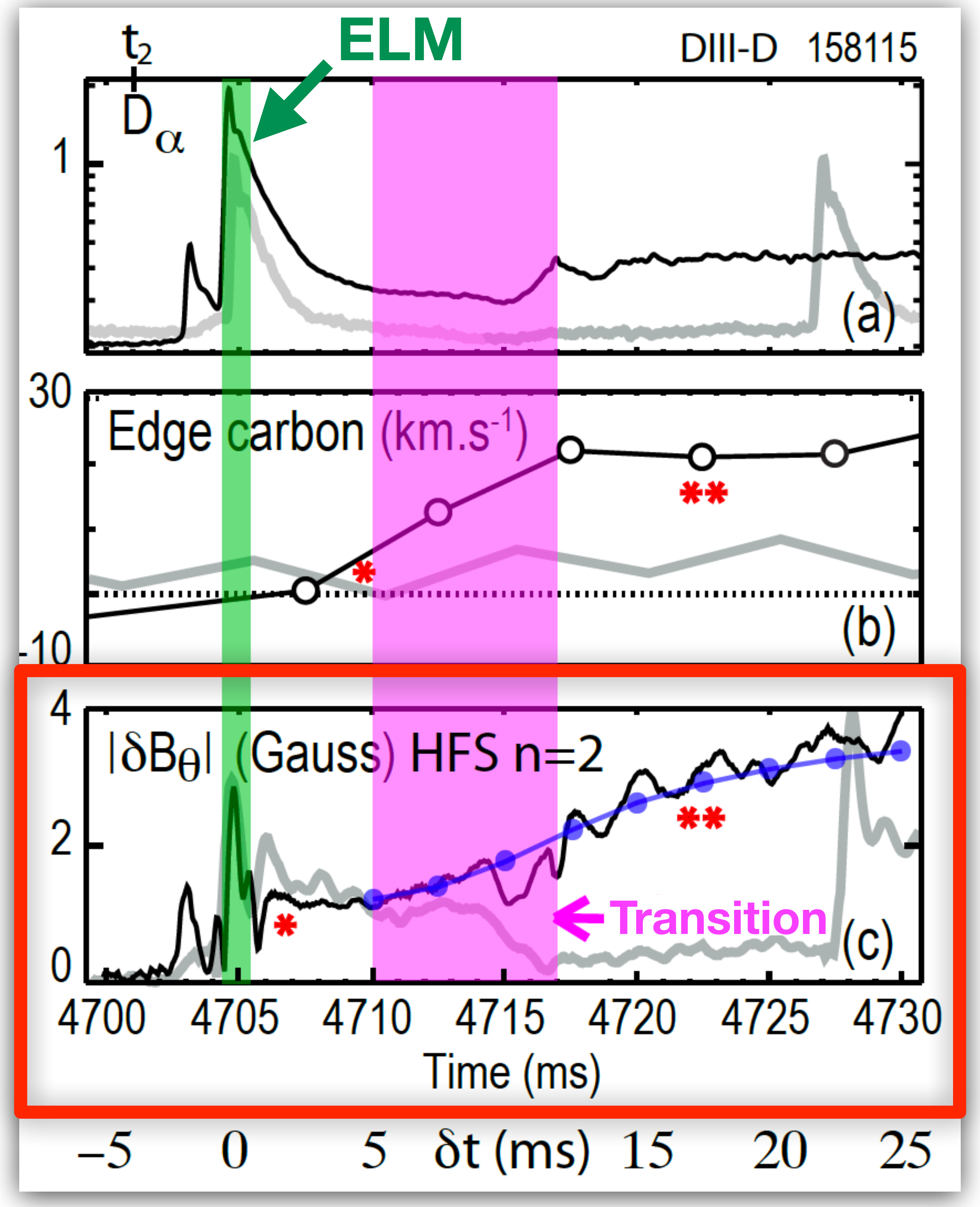
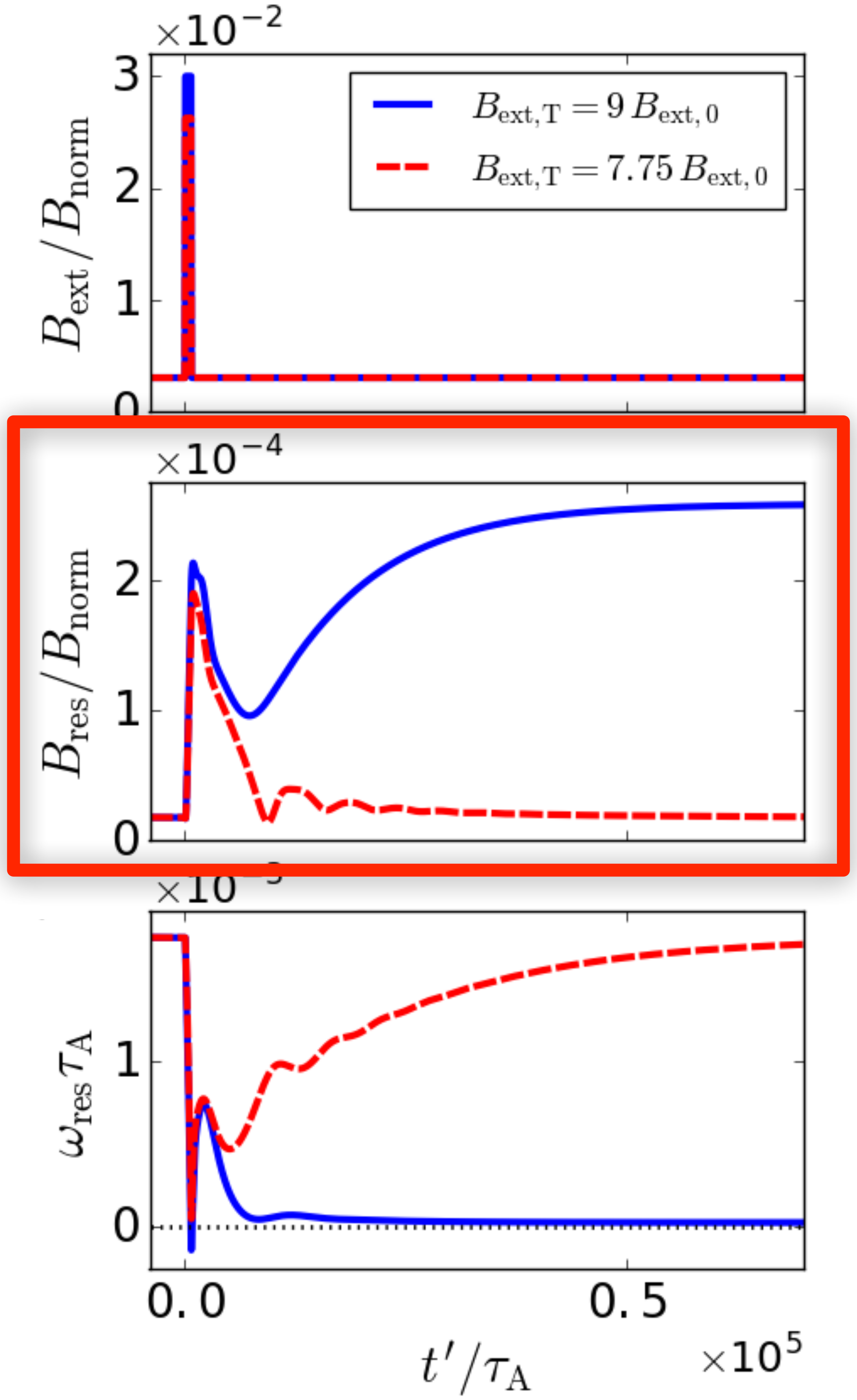


Computed Field Response Is Similar to Experimental Observations



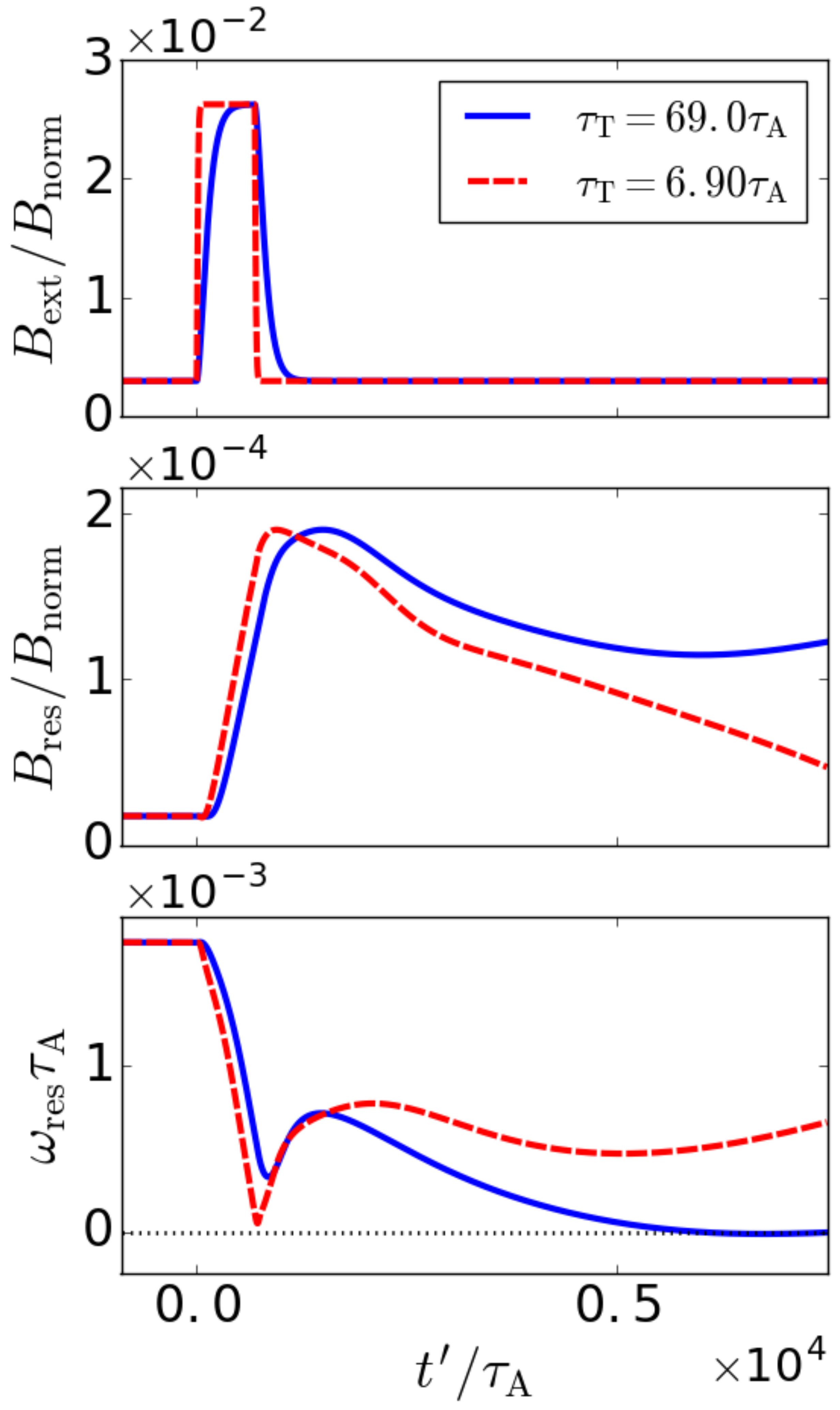


Mode Penetration Threshold Is Sensitive to Transient Shape

- **Rise and fall time of transient parameterized as**

$$T(t) = 1 - e^{-t/\tau_T} - \frac{t}{\tau_T} e^{-t/\tau_T}$$

- **Transient properties**
  - $B_{ext,T} = 7.75\ B_{ext,0}$
  - $\Delta t_T = 690\ \tau_A$ duration
- **$\tau_T = 69\ \tau_A$ mode penetrates**
  - $\tau_T = 6.9\ \tau_A$ returns to high slip state



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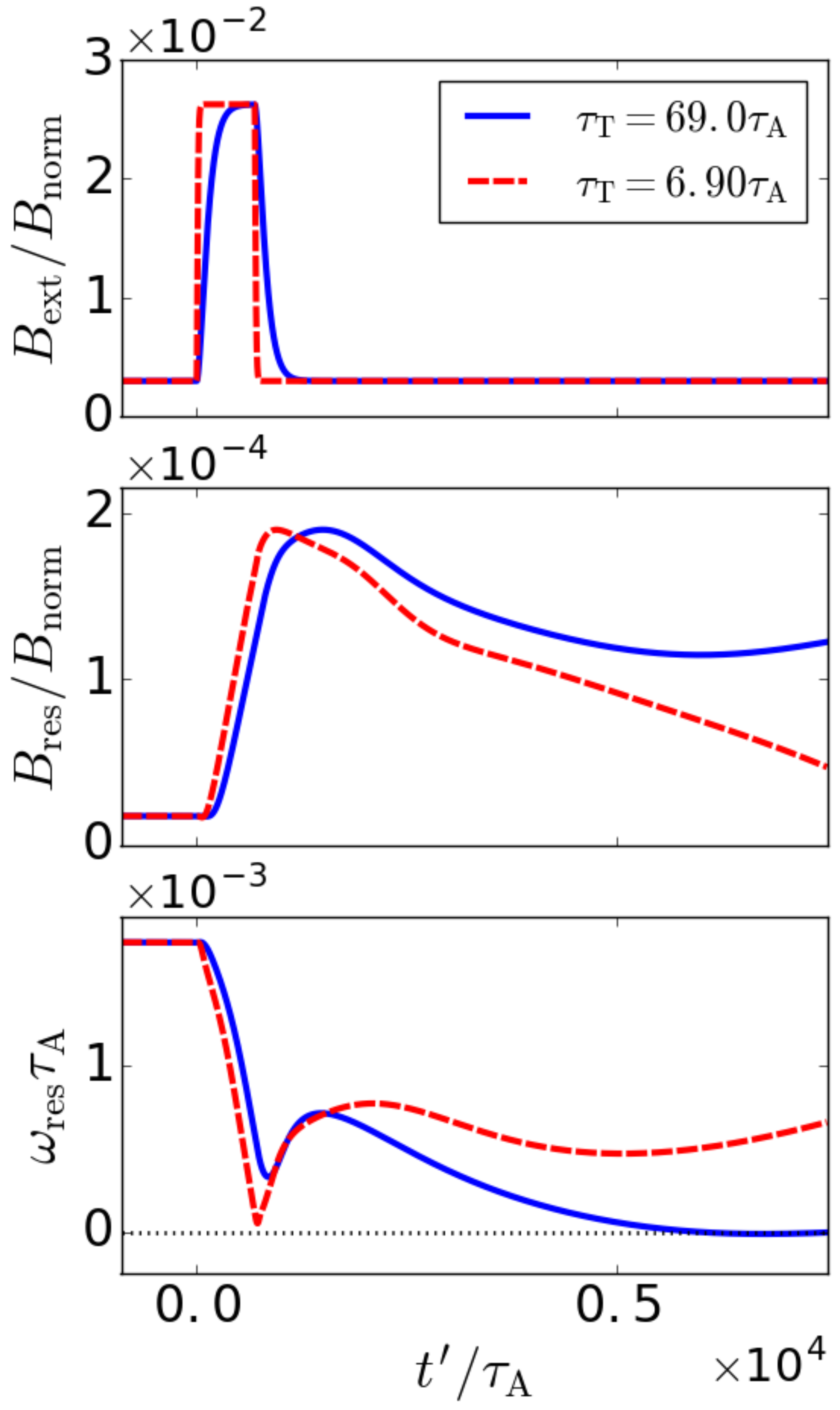
  - $B_{\mathrm{ext,T}} = 7.75\ B_{\mathrm{ext,0}}$
  - $\Delta t_\mathrm{T} = 690\ \tau_\mathrm{A}$ duration
- $\boldsymbol{\tau_\mathrm{T} = 69\ \tau_\mathrm{A}}$ **mode penetrates**
  - $\tau_\mathrm{T} = 6.9\ \tau_\mathrm{A}$ returns to high slip state
- **Same time-integrated transient RMP can yield different final state!**



Outline

- Explore dynamics of transient perturbation in slab geometry

- **Computational results elucidate mode penetration dynamics**
  - Effects of transient perturbation on metastable equilibrium
  - Parametric tests illustrate sensitivity of mode penetration

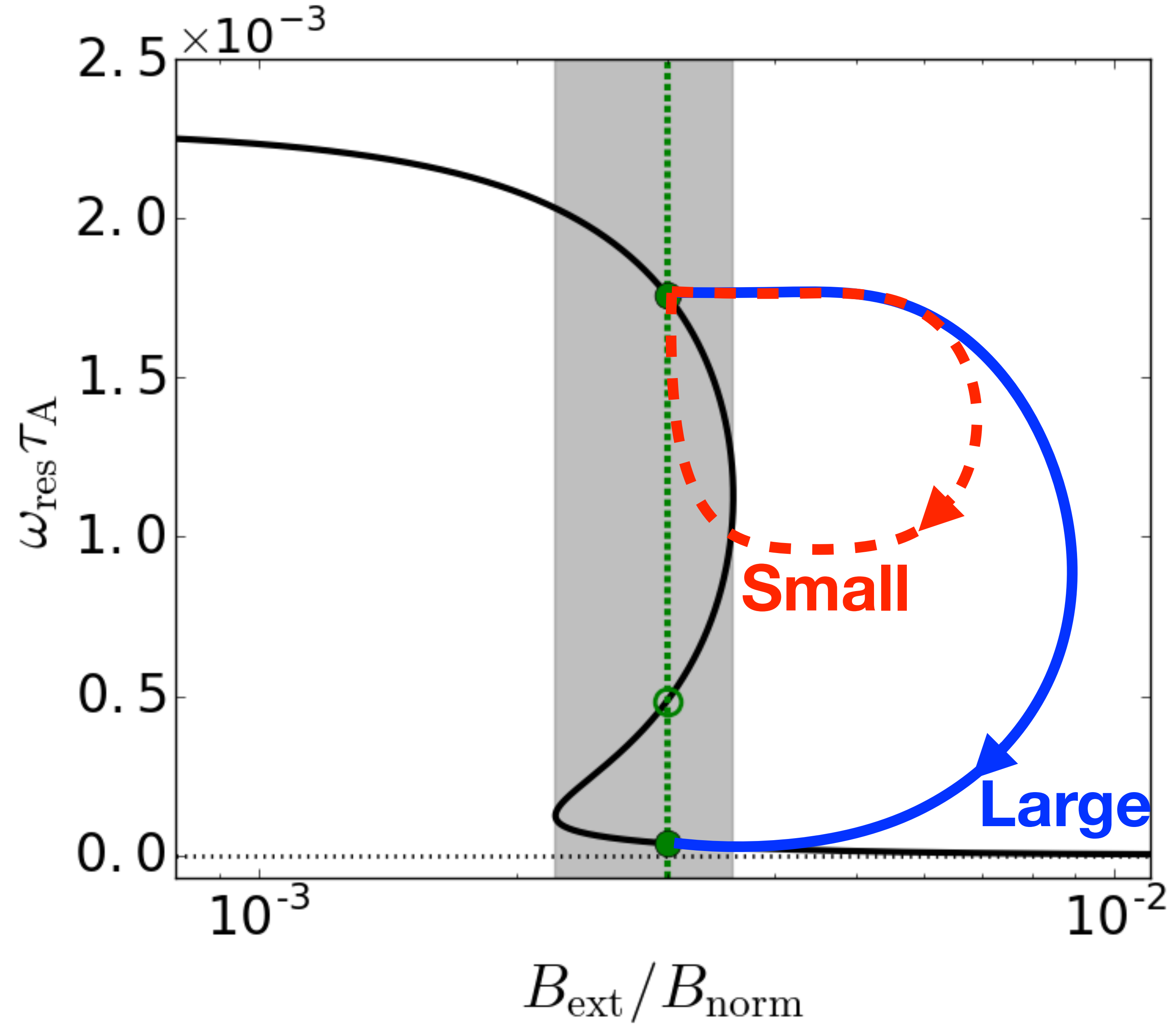
- Develop analytic model of mode penetration dynamics

Outline

- Explore dynamics of transient perturbation in slab geometry
- Computational results elucidate mode penetration dynamics
- **Develop analytic model of mode penetration dynamics**

Effect of Magnetic Transient Depends on EM and Viscous Force Evolution

- **Hypothesis: If transient causes enough flow evolution, mode penetration occurs**

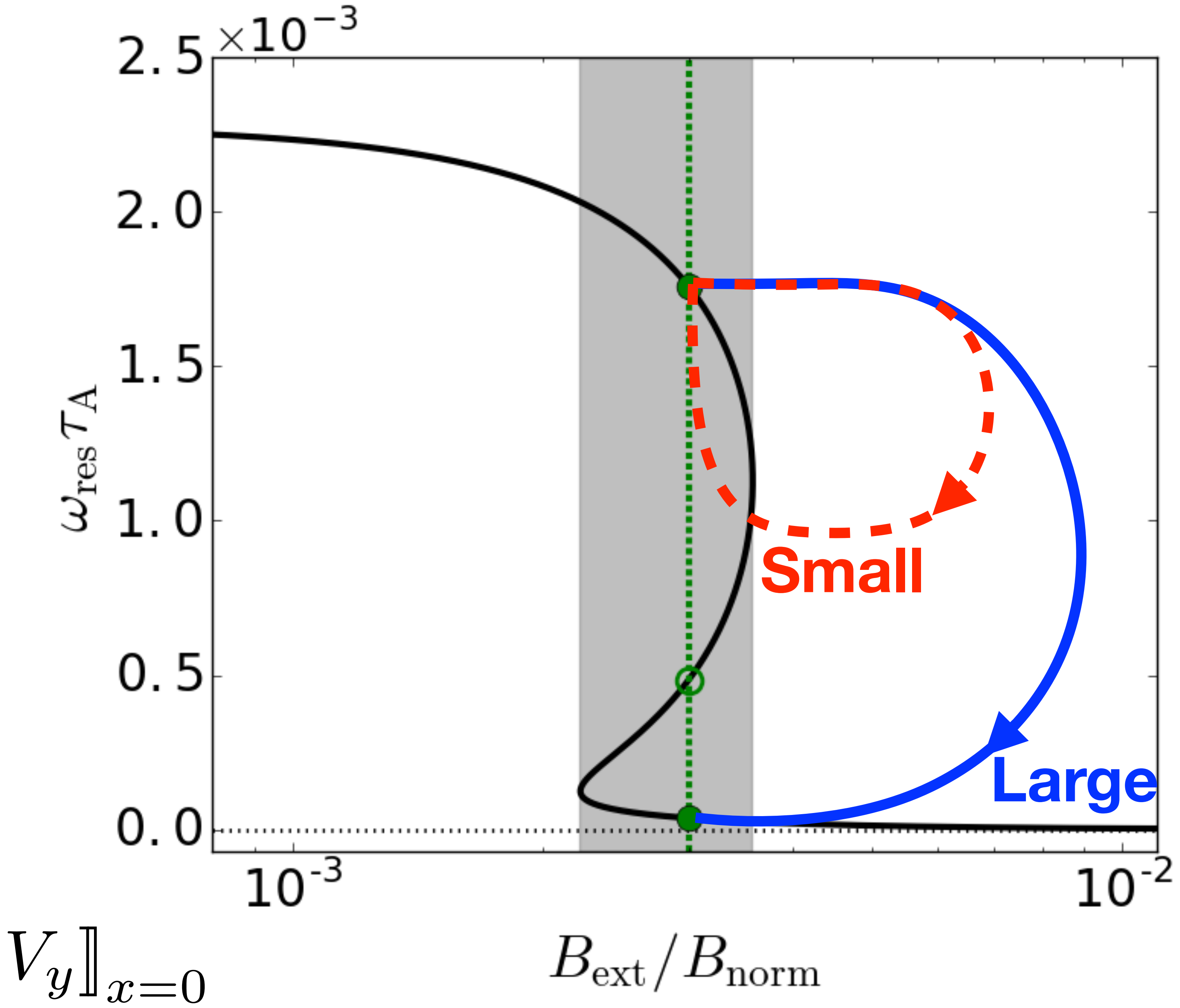


Effect of Magnetic Transient Depends on EM and Viscous Force Evolution

- **Hypothesis: If transient causes enough flow evolution, mode penetration occurs**
- **Flow evolution equation with EM and viscous forces:**

$$\frac{\delta_{\mathrm{VR}} L_y \rho}{k_y}\frac{d\omega_{\mathrm{res}}}{dt} = \hat{F}_{\mathrm{EM}} + \hat{F}_{\mathrm{V}}$$

$$= -\frac{n\pi}{\mu_0 k_y^2}\mathrm{Im}\left\{B^*_{\mathrm{res}}\,[\![\partial_x B_{\mathrm{res}}]\!]_{x=0}\right\} + L_y \rho \nu_0\,[\![\partial_x V_y]\!]_{x=0}$$



- **Transient magnetic perturbation causes forces to evolve**
  - Directly increases EM force local to the rational surface
  - Local change in flow profile increases viscous force

Model For Accurate EM Force Depends on History of Flow Evolution

- **Evolution of penetrated field governed by asymptotic matching of induction equation**

$$\frac{dB_{\mathrm{res}}(t')}{dt'} + \left[i\omega_{\mathrm{res}}(t') - \frac{a\Delta'}{\tau_{\mathrm{VR}}} \right] B_{\mathrm{res}}(t') = \frac{a\Delta'_{\mathrm{ext}}}{\tau_{\mathrm{VR}}} \left[B_{\mathrm{ext,0}} + B_{\mathrm{ext,T}} T(t') \right]$$

  - Background $B_{\mathrm{ext,0}}$ is constant in time; transient $B_{\mathrm{ext,T}}$ is applied with time-dependence $T(t')$

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- **Solve ODE for B_{res} by using integration factor**

$$B_{\text{res}}(t') = \frac{\Delta'_{\text{ext}}}{-\Delta'} \exp\left[-\frac{t'}{\tau'_{\text{VR}}} - i\varphi_{\text{res}}(t')\right] \left\{ \frac{B_{\text{ext,0}}}{1 + i\omega_{\text{res}}(0)\tau'_{\text{VR}}} + \frac{B_{\text{ext,0}}}{\tau'_{\text{VR}}} \int_0^{t'} ds\, \exp\left[\frac{s}{\tau'_{\text{VR}}} + i\varphi_{\text{res}}(s)\right] + \frac{B_{\text{ext,T}}}{\tau'_{\text{VR}}} \int_0^{t'} ds\, \exp\left[\frac{s}{\tau'_{\text{VR}}} + i\varphi_{\text{res}}(s)\right] T(s) \right\}$$

  - History of flow frequency evolution quantified by $\boxed{\varphi_{\text{res}}(t') \equiv \int_0^{t'} ds\, \omega_{\text{res}}(s)}$

Model For Accurate EM Force Depends on History of Flow Evolution

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- **Solve ODE for $B_{\rm res}$ by using integration factor**

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$$\underline{+ \frac{B_{\rm ext,T}}{\tau'_{\rm VR}} \int_0^{t'} ds \exp\left[\frac{s}{\tau'_{\rm VR}} + i\varphi_{\rm res}(s)\right] T(s)} \Bigg\}$$

  - History of flow frequency evolution quantified by $\boxed{\varphi_{\rm res}(t') \equiv \int_0^{t'} ds\, \omega_{\rm res}(s)}$
  - Separate contributions due to initially penetrated field and transient RMP penetration

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$$\left. + \underline{\frac{B_{\rm ext,T}}{\tau'_{\rm VR}}\int_0^{t'} ds \exp\left[\frac{s}{\tau'_{\rm VR}} + i\varphi_{\rm res}(s)\right] T(s)}\right\}$$

  - History of flow frequency evolution quantified by $\boxed{\varphi_{\rm res}(t') \equiv \int_0^{t'} ds\, \omega_{\rm res}(s)}$
  - Separate contributions due to initially penetrated field and transient RMP penetration

- **Quasilinear EM force** $\hat{F}_{y,\rm EM}(t') = -\frac{n\pi}{\mu_0 k_y^2}\Delta'_{\rm ext}\left[B_{\rm ext,0} + B_{\rm ext,T}T(t')\right]\mathrm{Im}\left\{B^*_{\rm res}(t')\right\}$
  - Separate contributions of $B_{\rm res}$ interact with transiently-induced current

Model For Viscous Force Evolution Depends on Evolution of Flow Profile

- **Evaluate** $\hat{F}_{y,V} = L_y \rho \nu_0 \left[\!\left[\partial_x V(x,t') \right]\!\right]_{x=0}$ **with evolving** $\boldsymbol{V(x,t')}$

Model For Viscous Force Evolution Depends on Evolution of Flow Profile

- **Evaluate** $\hat{F}_{y,V} = L_y \rho \nu_0 \left[\!\left[\partial_x V(x,t') \right]\!\right]_{x=0}$ **with evolving** $\boldsymbol{V(x,t')}$
- **Because EM force is localized at** $\boldsymbol{x=0}$**, solve for flow profile in** $\boldsymbol{0<|x|<a_\nu}$
  - $\partial_t V(x,t') = \nu_0 \partial_{xx}^2 V(x,t')$ with time-dependent BCs solved using infinite series expansion

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Model For Viscous Force Evolution Depends on Evolution of Flow Profile

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- **Because EM force is localized at** $\boldsymbol{x}$**=0, solve for flow profile in** $\boldsymbol{0 < |x| < a_\nu}$
  - $\partial_t V(x,t') = \nu_0\, \partial_{xx}^2 V(x,t')$ with time-dependent BCs solved using infinite series expansion

$$V(x,t') = \underline{V_{\text{res}}(t') + [V_0 - V_{\text{res}}(t')]\left(\frac{|x|}{a_\nu}\right)} - \sum_{n=1}^{\infty} \sin\left(\frac{n\pi|x|}{a_\nu}\right) e^{-(n\pi)^2 \frac{t'}{\tau_\nu}}$$
$$\times \left\{ \underline{\frac{2}{a_\nu}\int_0^{a_\nu} dx \sin\left(\frac{n\pi|x|}{a_\nu}\right)\left[V(x,0) - \left\{V_{\text{res}}(0) + [V_0 - V_{\text{res}}(0)]\left(\frac{|x|}{a_\nu}\right)\right\}\right] - \frac{2}{n\pi}\int_0^{t'} ds \frac{dV_{\text{res}}(s)}{ds} e^{(n\pi)^2 \frac{s}{\tau_\nu}}} \right\}$$

  - Separate contributions due to time-asymptotic and transient $V_{\text{res}}(t)$

Model For Viscous Force Evolution Depends on Evolution of Flow Profile

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- **Because EM force is localized at x=0, solve for flow profile in $0 < |x| < a_\nu$**
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$$V(x,t') = \underline{V_{\text{res}}(t') + [V_0 - V_{\text{res}}(t')]\left(\frac{|x|}{a_\nu}\right)} - \sum_{n=1}^{\infty} \sin\left(\frac{n\pi|x|}{a_\nu}\right) e^{-(n\pi)^2 \frac{t'}{\tau_\nu}}$$

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  - Separate contributions due to time-asymptotic and transient $V_{\text{res}}(t)$

- **Using derived flow profile yields viscous force:**

$$\hat{F}_{y,V}(t') = \frac{2L_y \rho \nu_0}{k_y a_\nu}\left\{ \underline{[\omega_0 - \omega_{\text{res}}(t')]} + \underline{2\left[\omega_{\text{res}}(0) - \omega_{\text{res}}(t')\right]\sum_{n=1}^{\infty} \exp\left[-(n\pi)^2 \frac{t'}{\tau_\nu}\right]} \right\}$$

Model Of Self-Consistent Force Balance Exhibits Mode Penetration

- **Balancing EM and viscous forces against inertia yields system of coupled PDEs:**

$$\frac{\delta_{\mathrm{VR}} L_y \rho}{k_y} \frac{d\omega_{\mathrm{res}}(t')}{dt'} = \hat{F}_{\mathrm{EM}}(t') + \hat{F}_{\mathrm{V}}(t')\,, \quad \frac{d\varphi_{\mathrm{res}}(t')}{dt'} = \omega_{\mathrm{res}}(t')$$

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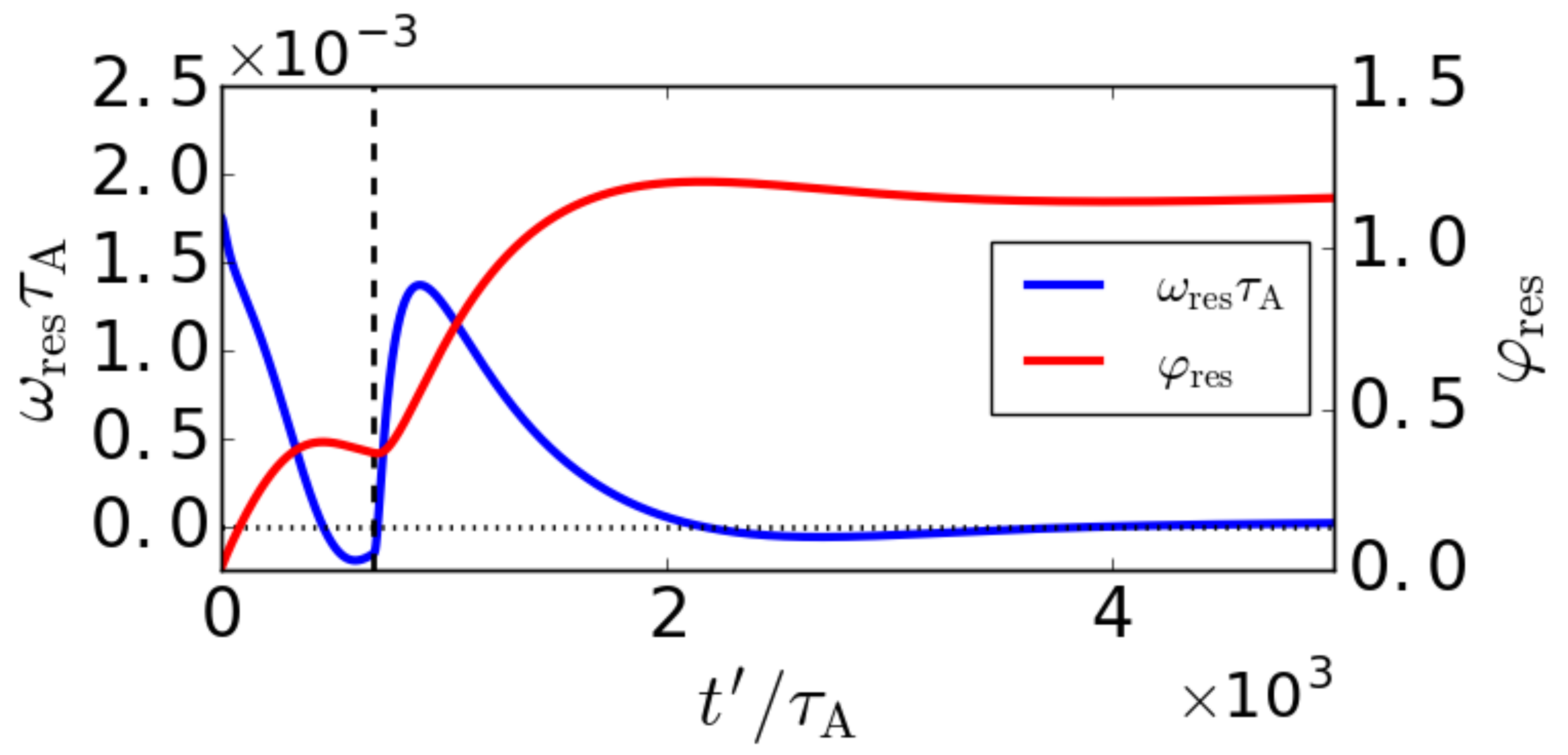
- **Numerically solve system**
  - Solution shown for transient with $B_{\mathrm{ext,T}} = 9\ B_{\mathrm{ext,0}}$, $\Delta t_{\mathrm{T}} = 690\ \tau_{\mathrm{A}}$, $\tau_{\mathrm{T}} = 6.9\ \tau_{\mathrm{A}}$



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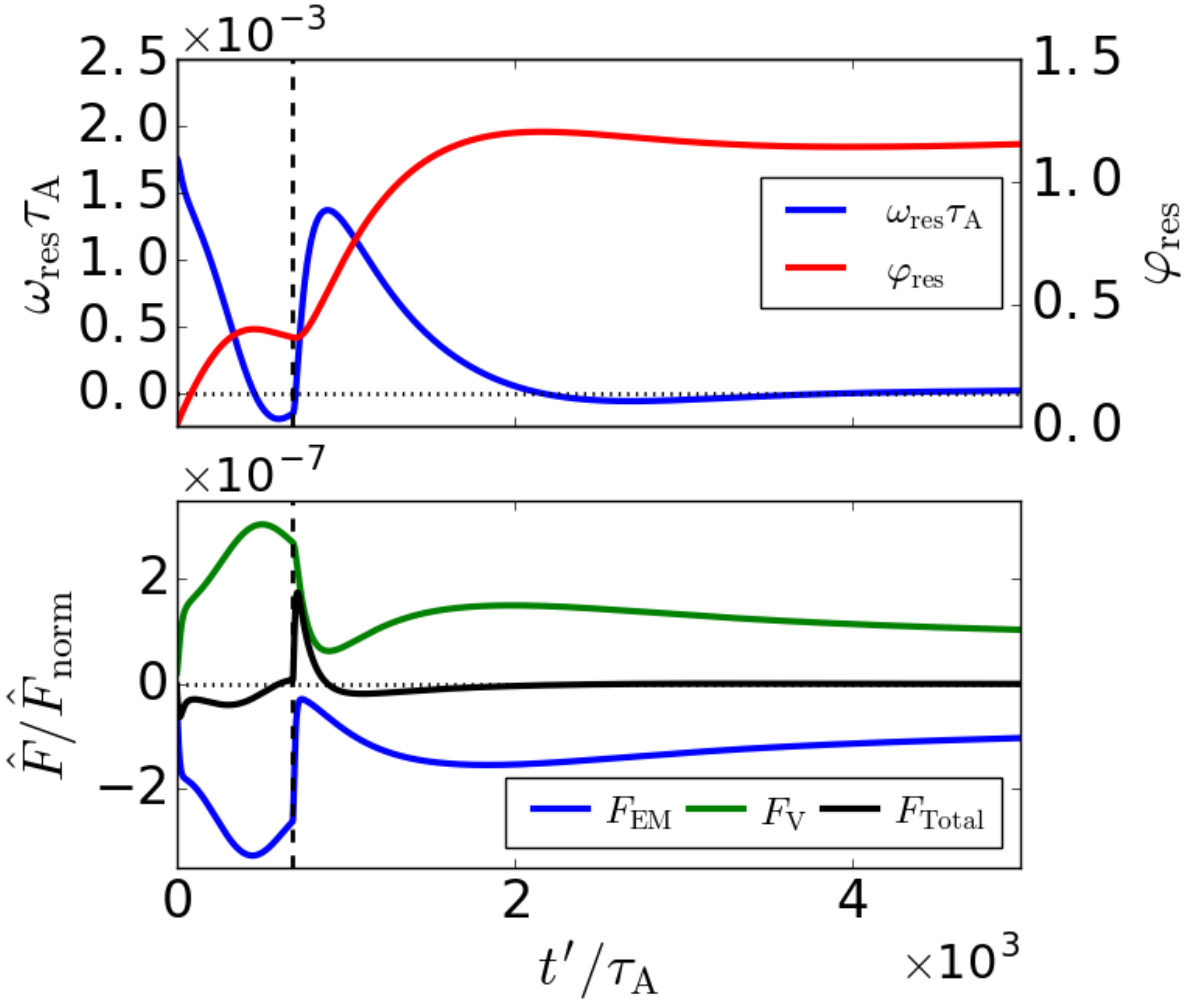
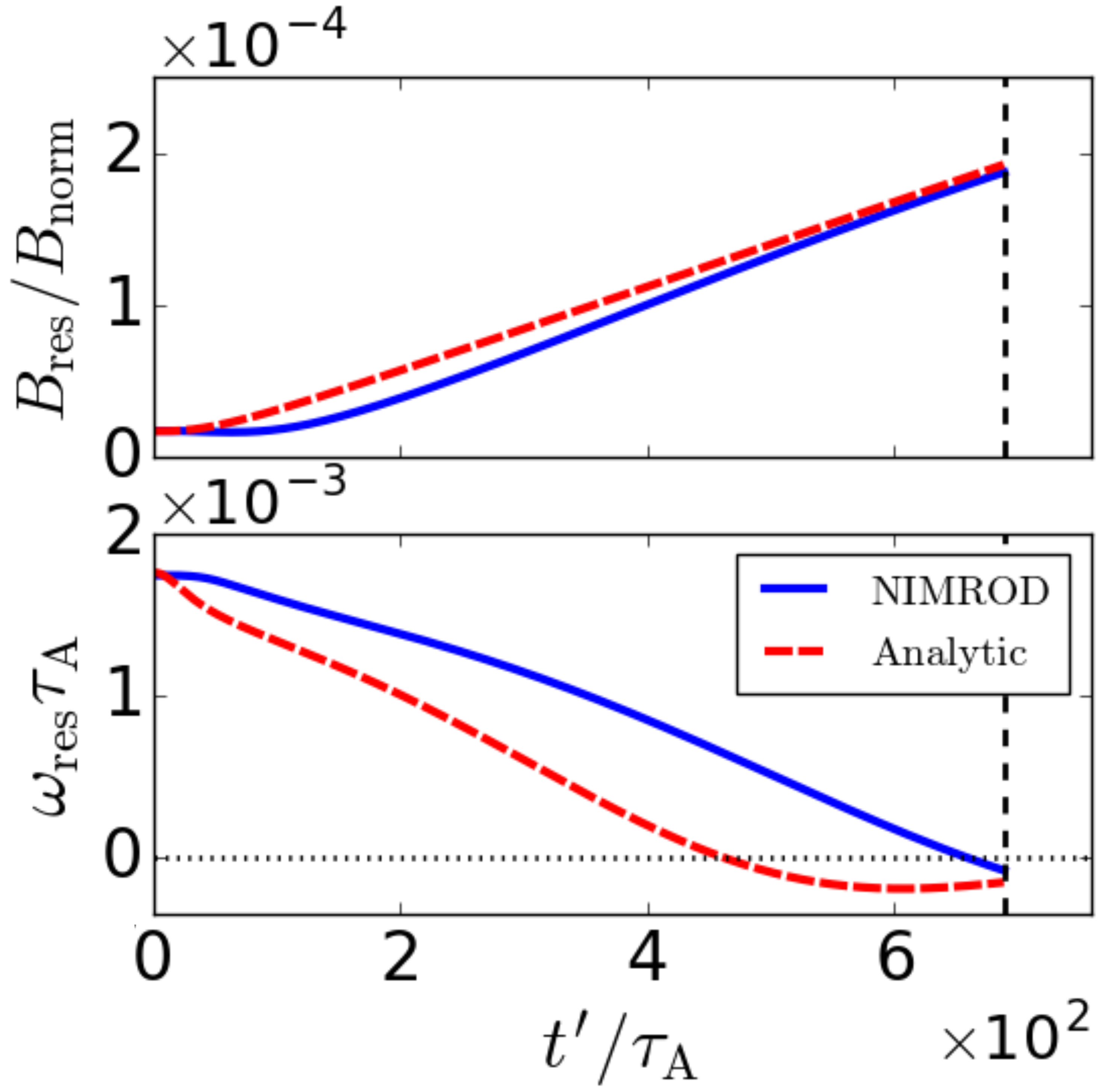
- **EM and viscous forces balance in time-asymptotic, mode penetrated state**
  - Recoil directly following transient due to slow response of viscous force



Qualitative Agreement Between Analytical Model and Computational Results

- **Agreement with NIMROD during transient**
  - Analytics yield mode penetration threshold at $B_{\mathrm{ext,T}} = 4.75\ B_{\mathrm{ext,0}}$ (not shown)
    - Within factor ~ 2 of NIMROD results
  - In line with analytics from J.D.Callen poster P3.017



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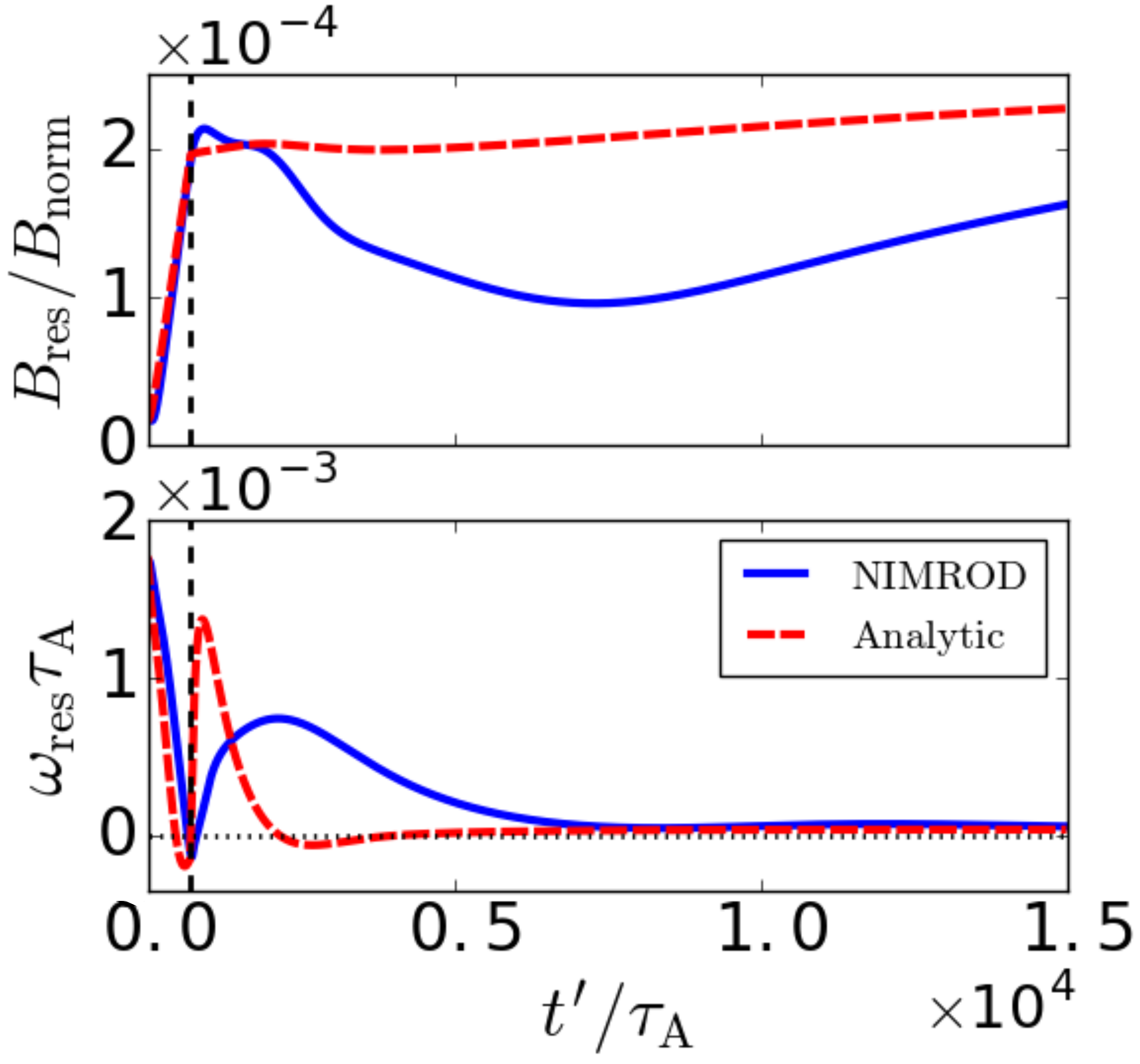
  - Linear asymptotic matching procedure inadequate during nonlinear Rutherford evolution (after $t' \simeq 3{\times}10^2 \tau_A$)



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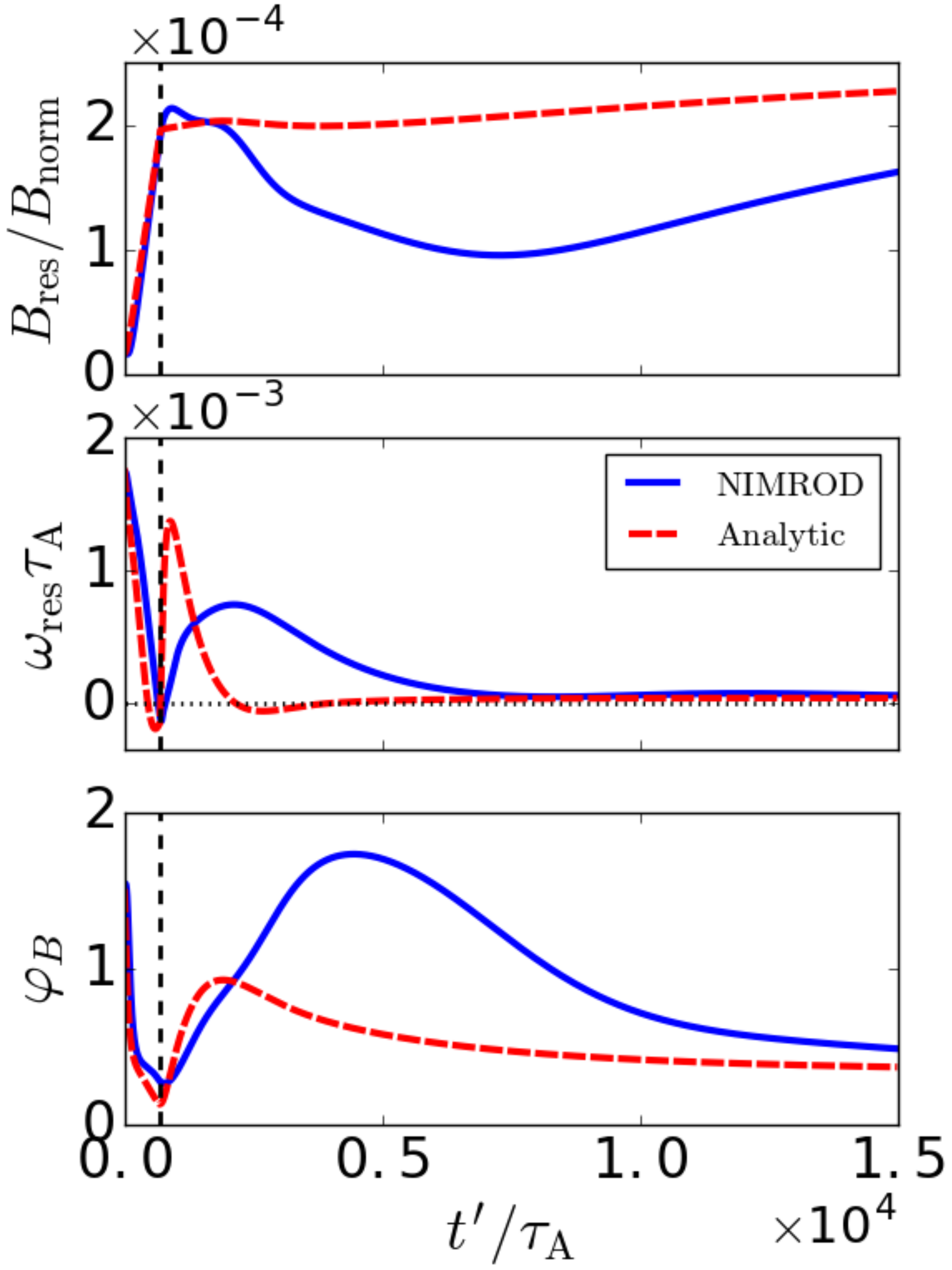
- **Agreement with NIMROD for island phase shift φ_B during transient**



Conclusions

- **Transient RMP can precipitate mode penetration**
  - Initial state must satisfy threshold for metastable state to exist $\omega_0 > 3\sqrt{3}/\tau'_{\mathrm{VR}}$
  - $B_{\mathrm{ext},0}$ must be in metastable region, and $B_{\mathrm{ext},\mathrm{T}}$ must cause enough flow evolution

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  - Initial state must satisfy threshold for metastable state to exist $\omega_0 > 3\sqrt{3}/\tau'_{\mathrm{VR}}$
  - $B_{\mathrm{ext,0}}$ must be in metastable region, and $B_{\mathrm{ext,T}}$ must cause enough flow evolution
- **Computational results explore mode penetration dynamics**
  - Threshold sensitive to transient parameterization
  - Sensitivity to shape suggests importance of the time history of evolution
- **Qualitative agreement between analytical and computational results**

Take-away: While analytic models provide rough criteria for mode penetration due to transient RMPs, computational models are necessary for accurate dynamical predictions