A Multi-Fluid Analysis of Burning Plasmas

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Introduction

- The next generation of magnetic confinement nuclear fusion experiments aims to achieve burning plasma conditions.
- A clear understanding of performance requirements needed to obtain burning or ignition conditions is desirable.
- Our knowledge to that purpose has not advanced much since Lawson's original work[a].
- We include additional physics in a zero- and one-dimensional analysis of the plasma to improve our estimate of plasma properties relevant to ignition and burning plasma conditions.

In this presentation:

- Modified ignition criterion and burning-plasma analysis:
  - Include two-fluid and α-particle effects.
- Compute and compare $\dot{T}$ vs. T curves for various models.
- Consider one-dimensional, two-parameter density and temperature profiles and evaluate their effect on ignition physics.
- Use the complete model to investigate physics of burning plasmas.

[a] J. D. Lawson, Proc. Phys. Soc. London Sect. B 70, 6 (1957)

Lawson's Time-Independent Analysis

The Lawson criterion is derived starting from the single-fluid zero-dimensional energy balance:

$$\frac{E_\alpha}{16}p^2\frac{<\sigma v>}{T^2}+S_h=\frac{C_B}{4}\frac{p^2}{T^{3/2}}+\frac{3}{2}\frac{p}{\tau_E}\left[+\frac{3}{2}\frac{dp}{dt}\right]. \qquad (1)$$

A straightforward manipulation gives the ignition criterion (with heating power $S_h=0$)

$$p\tau_E=2nT\tau_E\geq\frac{T^2}{\frac{E_\alpha}{24}<\sigma v>-\frac{C_B}{6}T^{1/2}}. \qquad (2)$$

The Starting Equations Are the Time-Dependent Three-Fluid Energy Conservation Equations.

The starting point is the system of zero-dimensional conservation equations for the three species, ions, electrons and αs:

$$\frac{3}{2}n\frac{\partial T_i}{\partial t}=S_{hi}-\frac{3}{2}\frac{p_i}{\tau_{Ei}}+\frac{3}{2}\frac{n(T_e-T_i)}{\tau_{eq}} \qquad (3)$$

$$\frac{3}{2}n\frac{\partial T_e}{\partial t}=S_{he}-\frac{3}{2}\frac{p_e}{\tau_{Ee}}+\frac{n_\alpha}{\tau_\alpha}E_\alpha-C_B\frac{p_e^2}{T_e^{3/2}}+\frac{3}{2}\frac{n(T_i-T_e)}{\tau_{eq}} \qquad (4)$$

$$\frac{\partial n_\alpha}{\partial t}=\frac{n^2}{4}<\sigma v>-\frac{n_\alpha}{\tau_\alpha}-\frac{n_\alpha}{\tau_{E\alpha}}. \qquad (5)$$

Heating terms (S_{hi}, S_{he}) are important in:

1. Transients;
2. Burning-Plasma analysis.

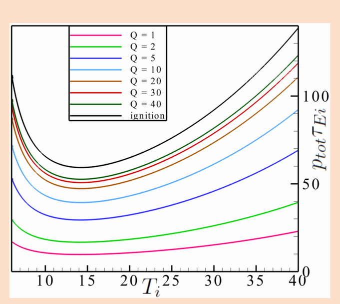
For steady-state burning plasmas, $S_{hi}\equiv f_i\frac{5}{Q}\frac{n^2}{4}<\sigma v>E_\alpha$, similar for S_{he}, $f_i+f_e=1$.

Analysis Is Extended to Burning Plasmas

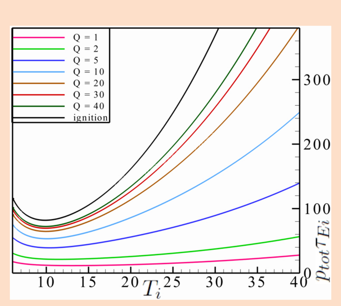
- For future experiments, the burning plasma ($P_\alpha\geq S_h$, $Q\geq5$) state is more relevant than ignition.
- Formally, the only modification needed to extend our analysis is to have heating power on at all times.

"Lawson-Like" Curves for Different Qs are calculated.

- 0D triple product for various values of Q is calculated for the single- (top) and two-fluid (bottom) models.
- In the TF case, heating is equally divided between ions and electrons.
- Two-fluid curves can be lower (low Q) or higher (high Q) than single-fluid curves.



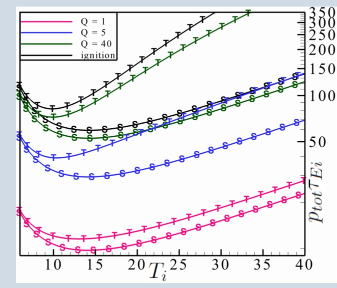
$p_{tot}\tau_E$ vs. T, SF case



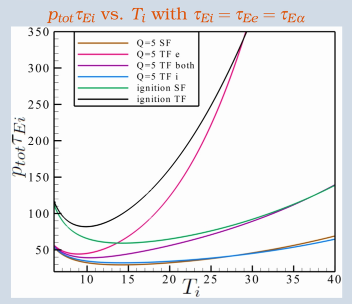
TF: $p_{tot}\tau_{Ei}$ vs. T_i with $\tau_{Ei}=\tau_{Ee}=\tau_{E\alpha}$

Single- and Two-Fluid Models Are Compared.

- SF and TF models are compared with heating divided between ions and electrons (top).
- If all heating goes to ions, triple product curves are higher for TF for $Q\gtrsim5$, higher for SF if $Q<5$.
- If some or all heating goes to the electrons, curves become significantly higher.
- For $Q=5$, TF curves can be higher than the SF ignition curve and even than the TF ignition curve.



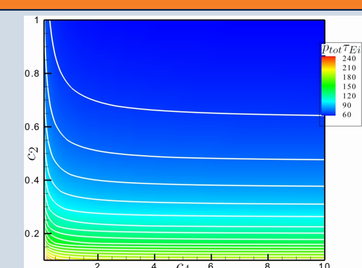
$p_{tot}\tau_{Ei}$ vs. T_i with $\tau_{Ei}=\tau_{Ee}=\tau_{E\alpha}$



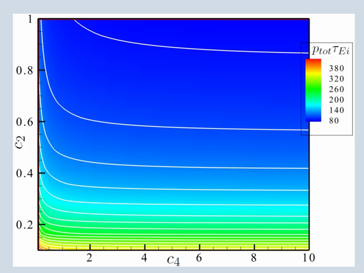
$p_{tot}\tau_{Ei}$ vs. T_i with $\tau_{Ei}=\tau_{Ee}=\tau_{E\alpha}$

Triple Product for Ignition and Finite Q Strongly Depends on Electron Confinement.

- The effect of electron and α confinement is considered.
- For each point, the minimum triple product for $Q=10$ (top) and ignition (bottom) is computed.
- Heating power is evenly split between ions and electrons for the $Q=10$ plot.
- Poorly confined electrons (low $c_2\equiv\tau_{Ee}/\tau_{Ei}$) require higher triple products.
- Results depend moderately on α confinement ($c_4\equiv\tau_{E\alpha}/\tau_{Ei}$).



$p_{tot}\tau_{Ei}$ vs. T_i with $Q=10$, $S_{hi}=S_{he}$



$p_{tot}\tau_{Ei}$ vs. T_i for ignition

One-Dimensional Parameters Are Introduced

- We introduce the density and temperature profiles:

$$n(r,t)=n_0(t)\left(1-r^\theta\right)^\eta \qquad T_{i,e}(r,t)=T_{0;i,e}(t)\left(1-r^\nu\right)^\mu,$$
$$\text{with}\ \ 0.1\leq(\mu;\eta)\leq2 \qquad \text{and}\ \ 1.1\leq(\nu;\theta)\leq4.$$

- Spatial profiles are fixed in time even during time-dependent simulations: We assume that profile equilibration is faster than transients (i.e., time evolution of n_0 etc.).
- Ion and electron temperature profiles are kept identical for ignition analysis, but are different in burning-plasma analysis. Note that $T_{0;i}\neq T_{0;e}$!

One-Dimensional Problem Setup

- For n_α the "equilibrium" spatial profile is used, obtained from

$$\frac{\partial n_\alpha(r,t)}{\partial t}=\frac{n(r,t)^2}{4}<\sigma v>(r,t)-\frac{n_\alpha(r,t)}{\tau_\alpha(r,t)}-\frac{n_\alpha(r,t)}{\tau_{E\alpha}} \qquad (6)$$

and normalized to 1 at $r=0$.

- Keep in mind that $<\sigma v>=<\sigma v>(T_i(r,t))$ and $\tau_\alpha=\tau_\alpha(n(r,t),T_e(r,t,))$.
- The ion-electron equilibration time τ_{eq} also depends on profiles, but energy confinement times τ_{Ei}, τ_{Ee}, $\tau_{E\alpha}$ are entered as constant values for each case.
- The full set of equations for both ignition and burning-plasma analysis:

$$\frac{3}{2}n(r,t)\frac{\partial T_i(r,t)}{\partial t}=S_{hi}-\frac{3}{2}\frac{n(r,t)T_i(r,t)}{\tau_{Ei}}+\frac{3}{2}\frac{n(r,t)\left(T_e(r,t)-T_i(r,t)\right)}{\tau_{eq}}, \qquad (7)$$

$$\frac{3}{2}n(r,t)\frac{\partial T_e(r,t)}{\partial t}=S_{he}-\frac{3}{2}\frac{n(r,t)T_e(r,t)}{\tau_{Ee}}+\frac{n_\alpha(r,t)}{\tau_\alpha}E_\alpha-C_B\frac{\left(n(r,t)T_e^2(r,t)\right)}{T_e^{3/2}(r,t)}+\frac{3}{2}\frac{n(r,t)\left(T_i(r,t)-T_e(r,t)\right)}{\tau_{eq}}, \qquad (8)$$

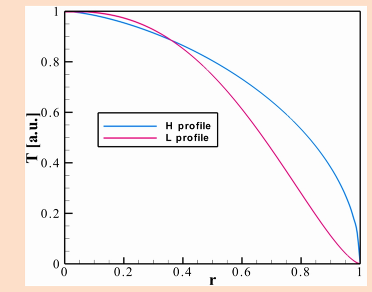
$$\frac{\partial n_\alpha(r,t)}{\partial t}=\frac{n^2(r,t)}{4}<\sigma v>-\frac{n_\alpha(r,t)}{\tau_\alpha}-\frac{n_\alpha(r,t)}{\tau_{E\alpha}} \qquad (9)$$

is first integrated (i.e., averaged) in space at each time step, then advanced in time.

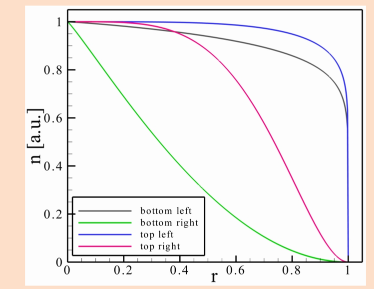
- Note that $n_\alpha'(t)\neq0$ since only the shape (and not the numerical value) of n_α is assigned.

The Importance of Profiles Is Studied

- The 1D profile definitions allow in principle for a 4D (η,θ,μ,ν) space to be explored for profile optimization (6D if one allows for different profiles for T_i and T_e).
- In practice, temperature profiles are determined by transport and are less amenable to external control than density profile.
- In most cases, we assign either $T_{i,e}(r)\equiv T_L(r)$ or $T_{i,e}(r)\equiv T_H(r)$ (L- or H-mode-like profiles).
- For some of the burning-plasma analysis experimental profiles (for e.g. ITER scenarios or DIII-D shots) are introduced as arbitrary expressions, fits or interpolations and different spatial profiles are used for ion and electron temperatures.

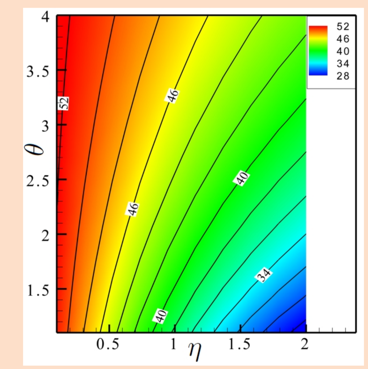


L- ($\mu=1.5$, $\nu=2.5$) and H- ($\mu=0.5$, $\nu=1.5$) mode temperature profiles

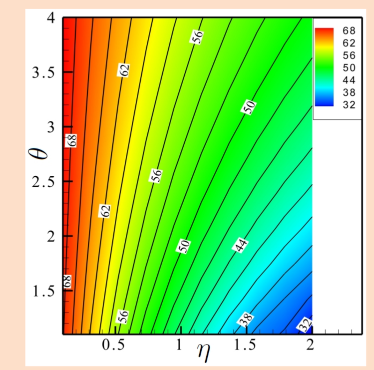


Limiting profiles for parametric scan

Minimum $p_{tot}\tau_{Ei}$ for Ignition Depends on Profiles.



Minimum $p_{tot}\tau_{Ei}$ for ignition, L-mode temperature profiles

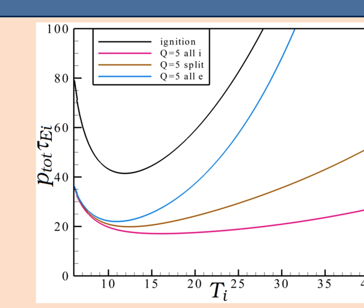


Minimum $p_{tot}\tau_{Ei}$ for ignition, H-mode temperature profiles

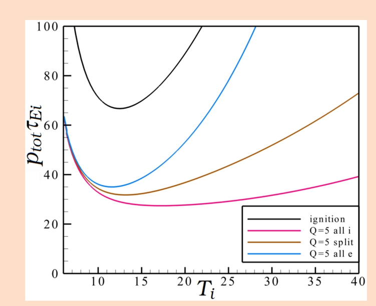
- Density profiles are varied keeping temperature profiles fixed.
- Average n is fixed for all runs.
- The energy confinement time needed for ignition depends on the density and temperature profiles.
- For reference, the SF and MF 0D values are $\simeq59$ and 82 $[10^{20}\mathrm{m}^{-3}\ \mathrm{keV\ s}]$.

Triple Product for Burning Plasma Depends on Profiles.

- $p_{tot}\tau_{Ei}$ for $Q=5$ is obtained.
- Both L-mode and H-mode profiles are used.
- Heating is distributed between ions and electrons with different fractions.
- Curves are qualitatively similar, but numerically different.
- Higher triple product is needed when electrons are heated.



$p_{tot}\tau_{Ei}$ vs. T_i with $Q=5$, L-mode profiles



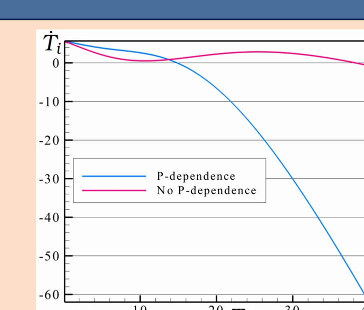
$p_{tot}\tau_{Ei}$ vs. T_i with $Q=5$, H-mode profiles

Non-Constant Energy Confinement Time Is Considered.

- It is more realistic to consider

$$\tau_E=\tau_{E0}\frac{S_h}{S_h+P_\alpha}. \qquad (10)$$

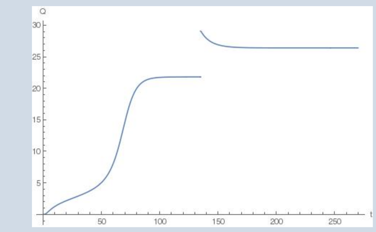
- This may result in a $\dot{T}$ curve without minimum.
- On the right, ITER cases with $\tau_{Ej}=2.8$s (top), $\tau_{E0j}=5.6$s (bottom).
- In both cases, $S_h=25$MW for both ions and electrons.



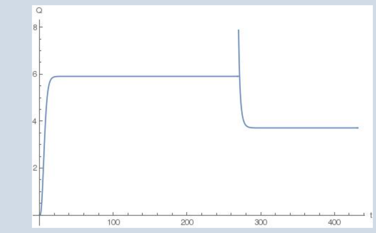
$\dot{T}$ vs. T with fixed and variable $\tau_{Ei}=\tau_{Ee}=\tau_{E\alpha}$

Output Power Strongly Depends on τ_E Model.

- The gain factor $Q(t)$ is calculated numerically for the fixed and variable τ_E models.
- It can be expected that a larger Q will be obtained if the heating power is reduced once the burning-plasma state is reached.
- This is verified for the fixed τ_E case only.
- Heating power is reduced by 25% halfway through the simulation.



Q vs. t with fixed $\tau_{Ei}=\tau_{Ee}=\tau_{E\alpha}=2.7$s



Q vs. t with fixed $\tau_{E0i}=\tau_{E0e}=\tau_{E0\alpha}=5.4$s

Experimental Performance Can Be Evaluated In Terms of $p_{no-\alpha}$.

- To fix ideas, start from he single fluid, 0D case:

$$\frac{3}{2}n\frac{\partial T}{\partial t}=\frac{E_\alpha}{16}p^2\frac{<\sigma v>}{T^2}+S_h-\frac{C_B}{4}\frac{p^2}{T^{3/2}}-\frac{3}{2}\frac{p}{\tau_E}. \qquad (11)$$

- Solve

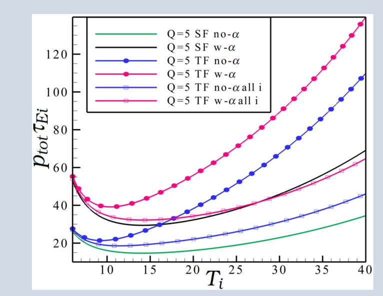
$$\dot{T}=0 \qquad \text{and} \qquad \frac{\partial\dot{T}}{\partial T}=0 \qquad (12)$$

for T and *one* of S_h, n and τ_E.

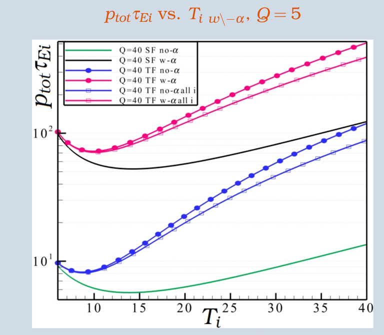
- Given the values, turn off the α heating and calculate p at steady state. This is $p_{no-\alpha}$.
- A similar procedure can be performed numerically for the full MF, 1D case.

$P_{no-\alpha}$ Is Estimated for Burning Plasmas.

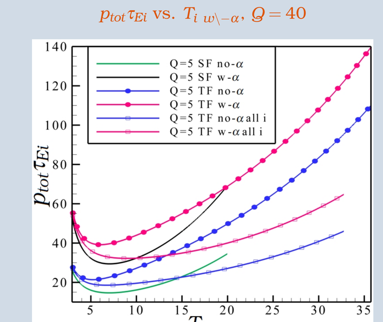
- To estimate $P_{no-\alpha}$:
  - Given Q, τ_{Ei} and S_h are calculated for different values of T_i.
  - The same values of τ_{Ei} and S_h are used setting $P_\alpha=0$.
  - New T_i, T_e are calculated and a new $P_{tot}\tau_{Ei}$ is obtained.
- The calculation is repeated for single- and two-fluid models.
- For the TF case, heating is evenly split between ions and electrons or all applied to the ions.
- Results for $Q=5,\ 40$ are plotted vs. $T_{i\ w\backslash\alpha}$ (top two figures).
- The plot is repeated vs. $T_{i\ no-\alpha}$ (bottom two figures).
- Notice the log scale in the $Q=40$ plot.
- In all cases $\tau_{Ei}=\tau_{Ee}=\tau_{E\alpha}$.



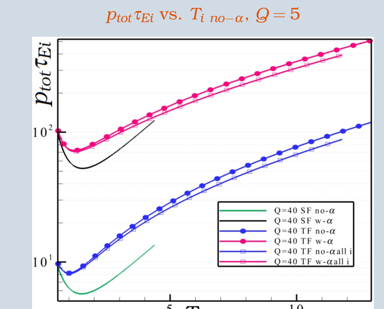
$p_{tot}\tau_{Ei}$ vs. $T_{i\ w\backslash-\alpha}$, $Q=5$



$p_{tot}\tau_{Ei}$ vs. $T_{i\ w\backslash-\alpha}$, $Q=40$



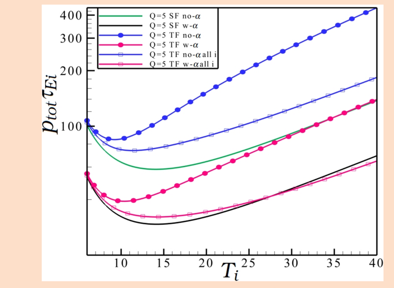
$p_{tot}\tau_{Ei}$ vs. $T_{i\ no-\alpha}$, $Q=5$



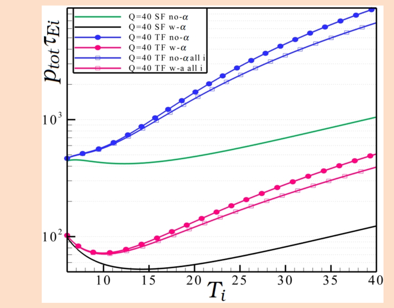
$p_{tot}\tau_{Ei}$ vs. $T_{i\ no-\alpha}$, $Q=40$

$P_{no-\alpha}$ Is Estimated for Burning Plasmas: Variable τ_E

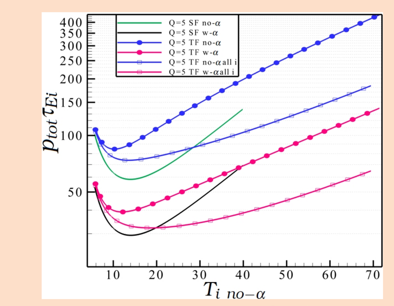
- In the previous plots, we assumed τ_E does not depend on heating power.
- We repeat the calculation with $\tau_E=\tau_{E0}\frac{S_h}{S_h+P_\alpha}$ as in Eq. (10).
- No-α calculations have higher $\tau_E\rightarrow$
  - No-α temperatures may be higher even with lower heating;
  - The no-α $nT\tau$ may be *higher* than the one $w\backslash\alpha$!
  - Direct calculation confirms that $p_{tot}\tau_{Ei}$ is higher in the no-α case, even for $Q=1$ (not shown).
  - The difference become larger for large Q (small no-α heating power).
  - A minimum value for τ_E may need to be implemented.
  - Results for $Q=5,\ 40$ are plotted vs. $T_{i\ w\backslash\alpha}$ (top two figures).
  - The plot is repeated vs. $T_{i\ no-\alpha}$ (bottom two figures).
  - Notice the log scales in all plots.
  - In all cases $\tau_{Ei}=\tau_{Ee}=\tau_{E\alpha}$.



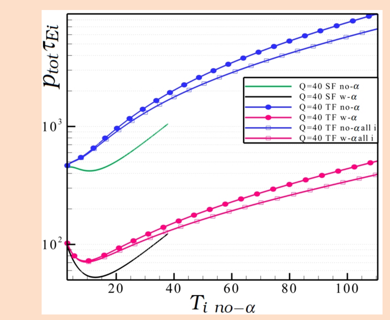
$p_{tot}\tau_{Ei}$ vs. $T_{i\ w\backslash-\alpha}$, $Q=5$



$p_{tot}\tau_{Ei}$ vs. $T_{i\ w\backslash-\alpha}$, $Q=40$



$p_{tot}\tau_{Ei}$ vs. $T_{i\ no-\alpha}$, $Q=5$



$p_{tot}\tau_{Ei}$ vs. $T_{i\ no-\alpha}$, $Q=40$

Power Law Is Implemented for τ_E.

- The ITER database power law fit was implemented in our Mathematica notebooks:

$$\tau_{E\ IPB98}=0.145M^{\alpha M}I^{\alpha I}a^{\alpha a}B^{\alpha B}\kappa^{\alpha\kappa}n^{\alpha n}p^{\alpha P}R^{\alpha R}$$
$$=0.145M^{0.19}I^{0.93}a^{0.58}B^{0.15}\kappa^{0.78}n^{0.41}p^{-0.69}R^{1.39}$$

- Test runs were made with with ITER data $M=2.5$, $I=12$, $a=2$, $B=5.3$, $\kappa=1.8$, $R=6.2$.
- Results are qualitatively similar to the previous slide (not shown).
- However, τ_E diverges for small power and best performance is found for small heating power. An upper limit to τ_E needs to be implemented in future work.

Analytical Formulas Approximate Lawson's Product In 1D

Interpolation formulas were derived to approximate Lawson's Product in 1D.

A fair interpolation was found for the Lawson product needed for any Q with one-dimensional profiles:

$$\frac{p_{tot}\tau_{Ei}}{(p_{tot}\tau_{Ei})_{0D}}\simeq g_0\left((1+g_1\eta)(1+g_2\theta)+g_3\eta^2\theta^2\right). \qquad (13)$$

where

Coefficient	L-mode	H-mode
g_0	$0.63-0.84/\sqrt{Q}+0.30/Q$	$0.83-1.1/\sqrt{Q}+0.39/Q$
g_1	-0.22	-0.28
g_2	0.039	0.035
g_3	4.5×10^{-3}	4.9×10^{-3}

References

1. J. D. Lawson, Proc. Phys. Soc. London **Sect. B 70**, 6 (1957)
2. J. P. Freidberg, *Plasma Physics and Fusion Energy*, Cambridge University Press, Cambridge UK, 2007
3. http://www.auburn.edu/cosam/faculty/physics/guazzotto/research/TF_Lawson_main.html (google "Guazzotto Lawson")
4. L. Guazzotto and R. Betti, *Tokamak Two-Fluid Ignition Conditions*, Phys. Plasmas **24**, 082504 (2017)