Nonlinear simulations of locking in the presence of tearing layers with real frequencies

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Abstract

Analysis shows that, for tearing regimes with real frequencies, the Maxwell torque locks the plasma to the tearing mode phase velocity[1]. Real frequencies for tearing modes are due to diamagnetic effects or the Glasser effect[2] in the resistive-inertial (RI) tearing regime, and it has recently been shown[3] that similar propagation frequencies occur in the viscoresistive (VR) regime. In Ref. [1] it was suggested that an effect like that of Ref. [4] might occur for the nonlinear behavior of RI and VR tearing modes: namely, for large island width the sound wave might alter the pressure gradient, causing the propagation to decrease to zero. A case was made in Ref. [1] that this decrease in propagation frequency might reduce the effect of locking to the phase velocity and therefore allow locking to zero velocity. We have performed simulations with NIMROD to investigate this possibility, using cylindrical geometry and a hollow equilibrium pressure prole. For an initial tearing unstable equilibrium with zero pressure we have increased the pressure, causing stabilization due to the outer region as well as favorable curvature in the layer. As pressure is increased, real frequencies are indeed observed and the mode is stabilized. In the presence of a small error field and plasma rotation, the maximum perturbation is observed to be at the phase velocity of the tearing mode, and this response is most peaked when the stable tearing mode is close to marginal stability. For increasing error field a locking bifurcation is expected, with locking to a rotation just above the tearing mode phase velocity. Simulations with larger error fields, or with modes closer to marginal stability, will be shown.

[1]. Finn, A. Cole, D. Brennan, PoP Letters 22, 120701 (2015).
[2]. A. Glasser, J. Greene, J. Johnson, Phys. Fluids 18, 875 (1975).
[3]. J. Finn, A. Cole, D. Brennan, arXiv:1708.04700 (2017)
[4]. B. Scott, A. Hassam, J. Drake, Phys Fluids, 28, 275 (1985).

For tearing regimes with real frequencies, the Maxwell torque induced by a static error field locks the plasma to the tearing mode phase velocity (v_{ph})[1]

- Real frequencies for tearing modes are due to diamagnetic effects or the Glasser effect[2] in the resistive-inertial regime and as seen recently in the viscoresistive (VR) regime[3,4].☑
- In the presence of a small error field and plasma rotation, the maximum perturbation occurs at the v_{ph} of the tearing mode, and this response peaks when the stable tearing mode ($\gamma \leq 0$) is close to marginal stability
  - $\Rightarrow$ Reconnection driven by an error field ☑
- For large island width the sound wave can flatten the pressure gradient, slowing down the propagation (**Scott effect** for ω_*[5]) ✗
- This decrease in propagation frequency might reduce the effect of locking to the phase velocity and allow locking to zero velocity.☒
- We perform simulations with NIMROD to investigate all of the above in a large-aspect ratio periodic cylinder with a hollow pressure profile.

[1]. Finn, A. Cole, D. Brennan, PoP Letters 22, 120701 (2015).
[2]. A. Glasser, J. Greene, J. Johnson, Phys. Fluids 18, 875 (1975).
[3]. J. Finn, A. Cole, D. Brennan, arXiv:1708.04700 (2017)
[4]. Poster **P2.020** by A. Cole on Monday Apr 23
[5]. B. Scott, A. Hassam, J. Drake, Phys Fluids, 28, 275 (1985).

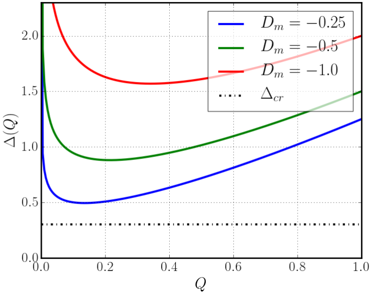
- Dispersion relation in resistive-inertial (RI) regime[2]

$$\Delta' = \Delta(\gamma) = (\gamma\tau)^{5/4} - \frac{D_m}{(\gamma\tau)^{1/4}} \tag{1}$$

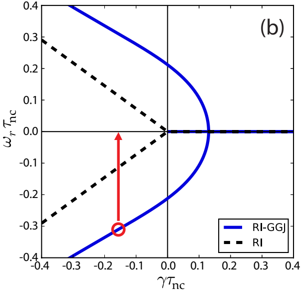
where $D_m = \left(1 - q(r_t)^2\right) D_s$ is the Mercier parameter and $D_s = -\frac{2 r_t p'(r_t)}{B_\theta^2 R^2 q'(r_t)^2}$ the Suydam parameter.

- For $\Delta_{cr} < \Delta' < \Delta_{min}$, **a pair of complex roots**, *i.e.*, $\gamma = \gamma_r \pm i\omega_r$.
- For $0 < \Delta' < \Delta_{cr}$, **stabilization/damping**, *i.e.*, $\gamma_r \leq 0$.

Dispersion relation for $D_m \neq 0$

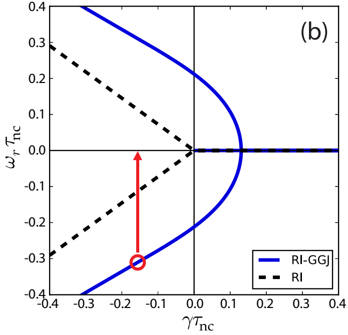


Locus of roots

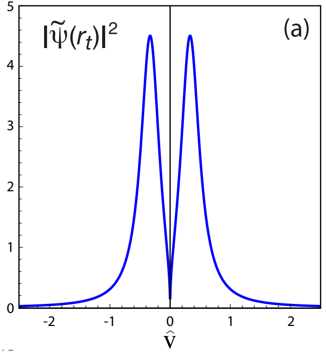


Backward propagating wave ($\omega_r < 0$) with a zero frequency in the lab frame if plasma rotates at $-\omega_r/k$

Locus of roots and Doppler shift



Reconnected flux $|\tilde{\psi}(r_t)|^2$ *vs* plasma rotation



- Maximum response of $\tilde{\psi}(r_t)$ to static error field at $v = -\omega_r/k$ with $v \to -v$ symmetry, unlike diamagnetic drift stabilization

  $\Rightarrow$ ω_* does not come in a complex conjugate pair.
- This ω_r effect also occurs in the viscoresistive (VR) regime[1].

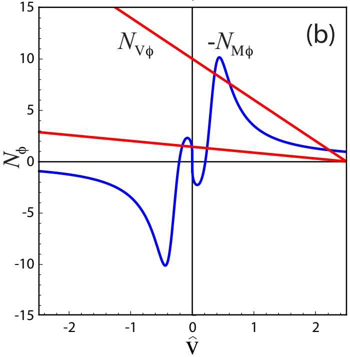
Quasilinear theory for locking to a static error field

- **Linear theory**: The reconnecting flux is proportional to the error field ϵ:

$$\tilde{\psi}(r_t) \propto \frac{\tilde{\psi}(r_w)}{\Delta' - \Delta(\gamma + ikv)} \propto \frac{\epsilon}{\Delta' - \Delta(\gamma_d)} \quad (2)$$

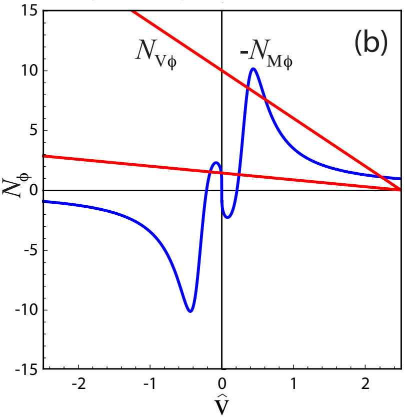
- $\Delta(\gamma_d) \to \Delta(ikv)$ – Doppler shifted from the error field ($\gamma = 0$) because of the plasma rotation at $r = r_t$
- Plasma rotation at $r = r_t$ is determined by the balance between:
  1. The EM torque $\propto R \int rdr \langle j_z b_r \rangle_\theta$ by the error field
     $$N_M \propto \frac{|\psi(r_w)|^2 (\Delta_{imag}(ikv))}{|\Delta' - \Delta(ikv)|^2}, \quad (3)$$
  2. Viscous torque with a momentum source v_0:
     $$N_V = N_0(v - v_0). \quad (4)$$
- N_M is largest when mode is closest to marginal stability in the complex plane: both $(\Delta' - \Delta_{real}(ikv))^2$ and $\Delta_{imag}(ikv)^2$ small.

Quasilinear Maxwell and viscous torques vs plasma rotation

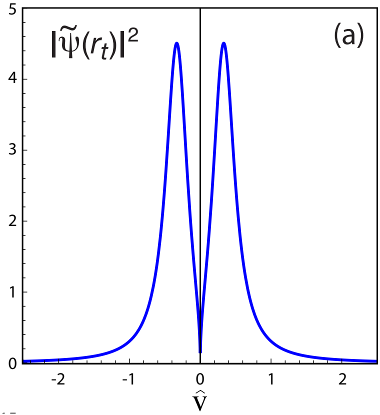


Fields lock to the static error field while the plasma flow locks to a finite frequency/velocity $\gtrsim \omega_r/k$.

Quasilinear Maxwell and viscous torques vs plasma rotation

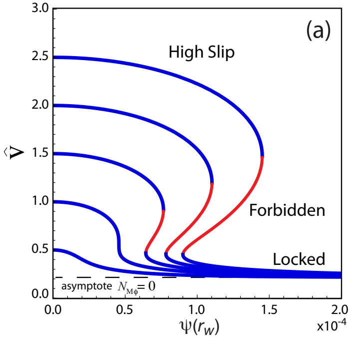


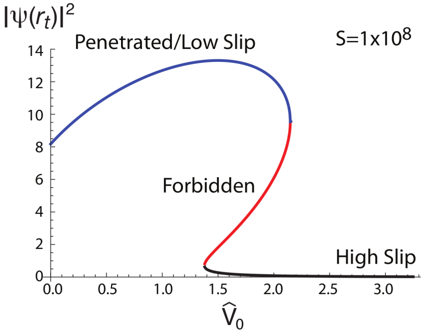
Reconnected flux $|\tilde{\psi}(r_t)|^2$ *vs* plasma rotation



- Notice one or three intersections of N_M with N_V (possibly two extra)
- v_0 controls the intercept of N_V while viscosity controls its slope.

Upward/downward bifurcations between unlocked (high-slip) and locked states are possible.





- High-slip states where plasma at $r = r_t$ rotates fast enough to shield the error field ($|\tilde{\psi}(r_t)| \ll |\tilde{\psi}(r_w)|$).
- Locked states where the plasma rotation at $r = r_t$ is slowed down ($|\tilde{\psi}(r_t)| \sim |\tilde{\psi}(r_w)|$)), but not stopped: $v(r_t) \to \omega_r/k$ as $|\psi(r_w)| \to \infty$.

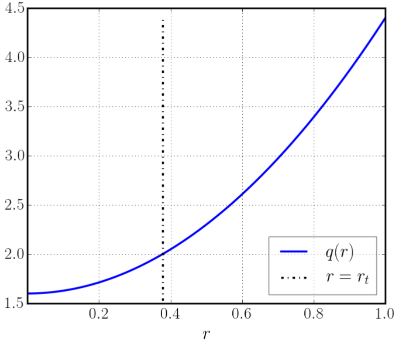
Scott effect for nonlinear drift-tearing modes

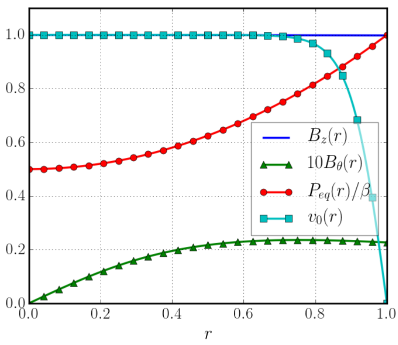
- For a large enough island width W such that $k_\parallel' W c_s \gtrsim \omega_*$, sound wave flattens pressure across island.
  - $\Rightarrow$ slows diamagnetic propagation,
  - $\Rightarrow$ allows island to grow to resistive MHD level.
  - $\Rightarrow$ weakens the finite rotation locking effect: plasma locks back to zero velocity

Question: Is there a corresponding Scott effect for the Glasser effect? We expect the Scott effect to slow down ω_r in the same fashion as ω_* and cause the plasma to lock back to <u>zero</u> velocity.

Use a periodic cylinder to simulate a large-aspect ratio ($R/r_w = 10$) torus

- A hollow pressure (quadratic) profile to mimic favorable average curvature in a torus ($D_m \to D_s < 0$).
- $1.6 \leq q(r) \leq 4.4$ with the rational surface located at $r_t = 0.38$ for $(m, n) = (2, 1)$ tearing mode.
- B_z very slightly paramagnetic.





Visco-resistive MHD is implemented with the NIMROD multi-fluid framework

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = \nabla \cdot D_n \nabla n, \tag{5}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \overleftrightarrow{\mathbf{\Pi}}, \tag{6}$$

$$\partial \mathbf{B}/\partial t = -\nabla \times \mathbf{E} + \kappa_{\nabla \cdot \mathbf{B}} \nabla \nabla \cdot \mathbf{B}, \tag{7}$$

$$n \left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = -(\Gamma - 1) \frac{p}{2} \nabla \cdot \mathbf{v} - \nabla \cdot \boldsymbol{q}, \tag{8}$$

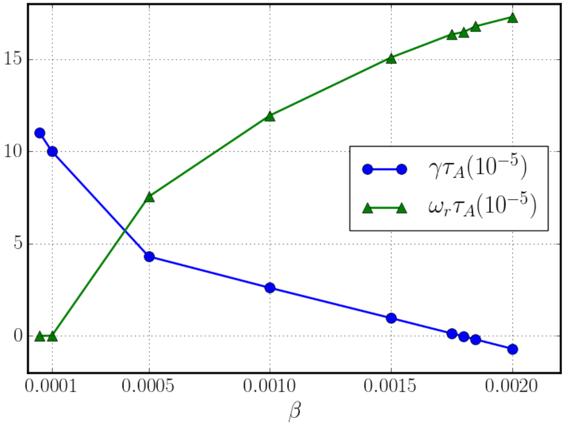
where $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$, $\rho = m_i n$, $\nabla \times \mathbf{B} = \mathbf{J}$, $p = n(T_i + T_e) = 2nT$, $\mathbf{q} = -\kappa_{\|} \nabla_{\|} T - \kappa_{\perp} \nabla_{\perp} T$ and $\overleftrightarrow{\mathbf{\Pi}} = \mu \nabla \mathbf{v}$ or $\mu \left[\nabla \mathbf{v} + (\nabla \mathbf{v})^T - \frac{2}{3} \nabla \cdot \mathbf{v} \right]$

- Dimensionless: $r \rightarrow r/r_w$, $\mathbf{B} \rightarrow \mathbf{B}/B_0$, and $t \rightarrow t/\tau_A$. $v_A = r_w/\tau_A$
- Constant and uniform diffusivities but anisotropic heat conduction.
- The equilibrium is kept static in time: introduces a momentum source.

Typical simulation parameters

- Aspect ratio, $R/r_w = 10$, $r_w = 1$ with $1.6 \leq q(r) \leq 4.4$
- $\beta \leq 0.002$
- Initial $B_z(r = 0) \equiv B_0 = 1$ such that $v_A = 10^7$ m/s for a chosen ρ.
- Lundquist number $S = \tau_R/\tau_A = 10^5$; Prandtl number: $Pr = 0.1$
- $(m, n) = (2, 1)$ error field with $10^{-7} \leq \epsilon \leq 10^{-3}(?)$
- Equilibrium (axial) flow: $v_0 \leq 50$ km/s $(= 0.0046 v_A)$.
- Glasser phase speed: $\omega_r/k = 19.2$ km/s
- $D_n = 0.05 < \kappa_\perp = 0.1 < \mu = 10.9 < \eta = 109 < \kappa_{\nabla \cdot \mathbf{B}} = 10^4 < \kappa_\| = 10^5$ with $\kappa_\|/\kappa_\perp = 10^6$
- Coarse poloidal resolution: $m = 0, 1, 2$. Convergence checked up to 11 modes for a few cases.
- Time asymptotic state for nonlinear simulations takes several τ_R

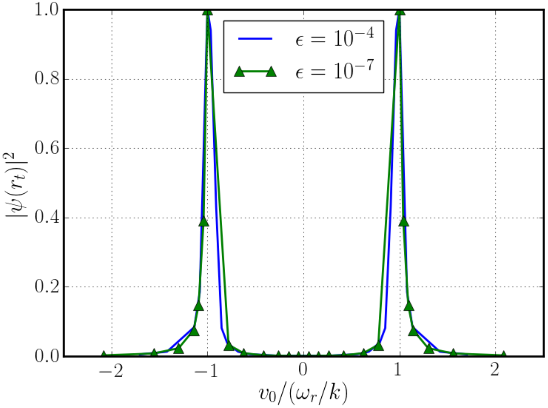
Linear simulations show real frequency and stabilization of the $(2,1)$ tearing mode as β increases $(v_0 = 0)$



- Glasser oscillations: $\omega_r > 0$ (green) for $\beta > 10^{-4}$.
- The mode becomes damped: $\gamma < 0$ (blue) for $\beta \geq 0.0018$.

Linear simulations show maximum reconnected flux $|\tilde{\psi}(r_t)|$ when plasma rotates at the phase speed $v_0 = \pm\omega_r/k$

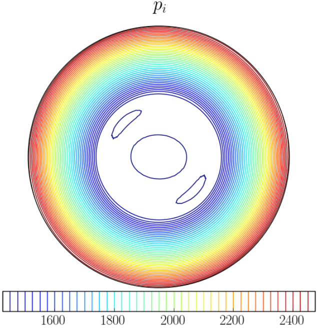
- $v \to -v$ symmetry.
- Values normalized to their respective maxima for each trace.



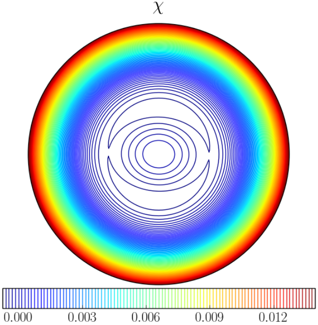
Weakening of the Glasser effect observed in nonlinear simulations for error field $\epsilon \gtrsim 10^{-5}$

- ϵ provides the means to control the island width W because $W \propto |\tilde{\psi}(r_t)|^{1/2} \propto |\tilde{\psi}(r_w)|^{1/2}$ and $\epsilon = im\tilde{\psi}(r_w)/r_w$:
- Flattening of axisymmetric pressure and an $m = 2$ island are observed.

pressure profile ($\epsilon = 10^{-4}$)



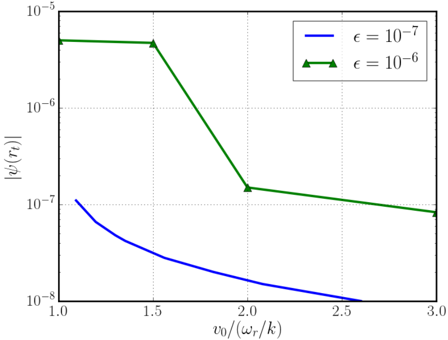
Helical flux χ contours ($\epsilon = 10^{-4}$)



Scott parameter: $k'_{||} W c_s/\omega_r = 1.6$ for $\epsilon = 10^{-4}$

Nonlinear simulations to look for bifurcation/locking to finite frequency

- Order parameters: $\psi(r_t)$ and $v(r_t)$
- Control parameters: $\psi(r_w)$ (or ϵ) and v_0
- Simulations at very small error fields show NO locking, because
  - $N_v >>> N_M$ or
  - v_0 is still too small (à la bifurcation diagram of Slide 8)



Summary

- Periodic cylinder with $R/r_w = 10$ with
  - a hollow pressure profile to model average good curvature
  - an equilibrium unstable to $(2,1)$ tearing mode at $r = 0.38$
- $\gamma \to \gamma \pm i\omega_r$ with $\gamma \leq 0$ at $\beta \geq 0.0018$ due to the Glasser effect.
- Resonant response to a static error field observed for $v_0 = \pm\omega_r/k$.
  - A pair of complex conjugate roots lead to $v \to -v$ symmetry.
  - No such symmetry for ω_*.
- **Quasilinear theory:** Plasma locks to finite velocity under the influence of EM and viscous torques.
- **Scott effect:** when the island width W is large, the pressure is flattened across the island and ω_* slowed down.
  - $\Rightarrow$ Plasma locks back to zero velocity
  - $\Rightarrow$ **Q:** Does the Scott effect also slow down ω_r and lock to 0 velocity?
- Flattening of the pressure is observed for $\epsilon \gtrsim 10^{-5}$.
- Still looking for the locked state.