Conducting Hasegawa-Wakatani model

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The Hasegawa-Wakatani (HW) model [1] and its extensions [2-4] are considered to be the minimal models that describe electrostatic drift-waves and the turbulence-zonal flow interaction. In this work, a new conducting HW (CHW) model is proposed that more accurately represents the drift-reduced Braginskii model for the geometry of a magnetically confined plasma. In the HW models, the two key normalized parameters are inverse of characteristic length of the density gradient, κ, that controls the strength of the drift-wave instability drive, and the adiabaticity parameter, α, that controls the dissipation rate due to electrical conduction. The usual 2D HW models consider a geometry that allows the parallel wavenumber, $k_\parallel$, to be decoupled from the perpendicular wavevector, $k_\perp$. They then consider constant α, which implies a prescribed constant $k_\parallel$. For a tokamak or stellarator, the magnetic field has a dominant toroidal component and a smaller poloidal component, so one can still approximate the perpendicular plane as the poloidal plane. However, if one assumes either toroidal or helical symmetry, then the poloidal wavenumber generates both a finite perpendicular component and a finite parallel component, so that the parallel and perpendicular dynamics are coupled together. Now that there are multiple $k_\parallel$, one must use a more accurate form for dissipation: electrical conduction generates parallel diffusion, $\alpha \propto \sigma_\parallel k_\parallel^2$, where $\sigma_\parallel$ is the parallel conductivity.

Simulations of the new CHM model are performed and compared to the original and modified HW models using both finite difference [5] and conservative finite element [6] formulations. The CHW model is able to recover the turbulence bifurcation found in the modified HW model [3] because it also eliminates zonal damping for $k_\parallel = 0$. In the adiabatic limit ($\alpha \gg 1$), strong zonal flows are generated, turbulence is suppressed, and separate populations of drift-waves and zonal flows can be clearly identified. In the hydrodynamic limit ($\alpha \ll 1$), this model produces a mixed isotropic turbulence (similar to the original 2D HW model [1]) along with low $k_\perp$ convective cells (similar to the 3D HW model [2]) at the saturation stage, indicating a robust nonlinear energy transfer process from intermediate $k_\parallel$ to low $k_\parallel$. Moreover, unlike the extended HW model [4] which shows turbulence suppression with both good and bad curvature; when the CHW model is applied to curved magnetic fields by adding the interchange drive, a linear correlation between the turbulence dynamics and the curvature is found. Just as one would expect: in the bad curvature case, turbulence enhances and the dominant mode moves toward to a lower k; while in the good curvature case, turbulence is suppressed. Another advantage of simulating the CHW model is a significant relaxation of the Courant-Friedrichs-Lewy (CFL) condition because the $k \to 0$ singularity in the dispersion relation is eliminated. Therefore, a larger time-step compared to the original/modified HW models can be used to accelerate the simulation – which is especially important when the simulation domain is large compared to the gyroradius.

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This work was supported by LLNL PLS LDRD 20-ERD-038 and was performed for US DOE by LLNL under DE-AC52-07NA27344. LLNL-ABS-847056

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