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Courant Institute of
Mathematical Sciences

Magneto-Fluid Dynamics Division

Abstracts of
Sherwood Theoretical Meeting

March 22-23, 1971

AEC Research and Development Report

Plasma Physics
December 1, 1971

New York University

UNCLASSIFIED

New York University
Courant Institute of Mathematical Sciences
Magneto-Fluid Dynamics Division

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Abstracts of

SHERWOOD THEORETICAL MEETING

March 22-23, 1971

U. S. Atomic Energy Commission
Contract No. AT(11-1)-3077

UNCLASSIFIED

Sherwood Theoretical Meeting

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Courant Institute of Mathematical Sciences, New York University

All sessions will be held in Room 109 Warren Weaver Hall [251 Mercer Street, 4th Street East of Washington Square] except session IV-b in Room 102.

Where known, underlined author will present paper.

Opening, 9 am, Monday, March 22

Welcome; Louis Nirenberg, Director, Courant Institute

Announcements

Session I, Chairman - C. L. Hedrick

1. W. A. Newcomb, "Stationary Equilibria of a Guiding-Center Plasma" (15 min.)

2. D. Lortz, E. Rebhan, G. Spies, "Sufficient Criteria for Magnetohydrodynamic Stability" (15 min.)

3a. R. Gajewski, "MHD Equilibrium of an Elliptical Plasma Cylinder"

b. R. Dagazian, B. Coppi, "MHD Instabilities in Confinement Configurations with Shaped Magnetic Surfaces" (20 min.)

4. N. Friedman, "Free Boundary Scylla Equilibria" (15 min.)

COFFEE

5. H. Grad, A. Kadish, D. Stevens, "Free Boundary Tokomak Equilibria with No Outer Wall" (15 min.)

6. J. Freidberg, "Stability of the Straight $\ell = 1$ Scyllac Configuration" (15 min.)

7. H. Grad, J. Marsh, H. Weitzner, "A Spectral Analysis of the Screw Pinch" (15 min.)

R1. Report from Princeton, J. Dawson, P. Rutherford (30 min.)

Session II, 2 pm, Chairman - J. Freidberg

R2. Report from Los Alamos, R. Morse (20 min.)

8. J. D. Callen, "Tokomak Kinetic Theory; Trapped-Particle Modes" (15 min.)

Session II (cont'd)

9. A. A. Ware, A. B. Macmahon, "Trapped Particle Contributions to the Heating in a Tokomak" (15 min.)

10. S. Yoshikawa, J. Schmidt, "Anomalous Viscosity as a Possible Explanation for an Anomalous Skin Effect" (15 min.)

COFFEE

11. F. L. Hinton, C. W. Horton, Jr., "Bootstrap Current-Driven Drift Instability in Tokomaks" (15 min.)

12. D. K. Bhadra, C. S. Liu, T. Ohkawa, "Neoclassical Diffusion in Axisymmetric Torus in the "Intermediate Region"" (15 min.)

13. N. K. Winsor, E. C. Bowers, M. A. Hellberg, J. M. Dawson, "Shocks and Diffusion in Low-β Toroidal Plasmas" (15 min.)

14. W. Ross, M. N. Rosenbluth, "Stabilization of Dissipative Trapped Particle Instability" (15 min.)

R3. Report from N. Y. U., H. Grad (20 min.)

Session III, 9 am, Tuesday March 23, Chairman - N. Winsor

R4. Report from M. I. T., B. Coppi (20 min.)

15. D. V. Anderson, J. Killeen, "Computation of Finite-Beta Plasma Equilibria in Minimum-B Mirror Systems" (15 min.)

16. D. Dobrott, H. Grad, "Limiter Induced Perturbations and the Small Gyro Radius Limit" (15 min.)

17a. B. D. Fried, C. F. Kennel, "Low Frequency Drift Cyclotron Instability"

b. D. W. Forslund, R. L. Morse, C. W. Nielson, "Electron Cyclotron Drift Instability" (20 min.)

COFFEE

18. E. J. Valeo, C. Oberman, W. L. Kruer, "Nonlinear Theory for Plasma Driven by a Large-Amplitude, High-Frequency Field" (15 min.)

19. C. L. Hedrick, Jr. "Suppression of the Whistler Instability by Relativistic Effects" (15 min.)

20. J. P. Freidberg, B. M. Marder, "High Frequency A. C. Electrostatic Plasma Instabilities" (15 min.)

21. P. K. Kaw, Y. C. Lee, "Dynamic Stabilization of Drift Waves by Radial and Azimuthal High-Frequency Fields" (15 min.)

R5. Report from Oak Ridge, G. Guest (20 min.)

Session IV-a, 2 pm, Chairman - D. Dobrott

R6. Report from LRL, D. Pearlstein (20 min.)

R7. Report from Texas, W. E. Drummond (20 min.)

22. H. R. Strauss, "High Pressure Tokomak Equilibrium" (15 min.)

23. J. M. Dawson, H. P. Furth, F. H. Tenney, "Thermonuclear Power Production by Nonmaxwellian Ions in a Closed Magnetic Field Configuration" (15 min.)

24. M. S. Chu, "Thermonuclear Reaction Waves at High Densities" (15 min.)

COFFEE 3:45 pm

25. J. P. Holdren, A. H. Futch, Jr., J. Killeen, A. A. Mirin R. F. Post, "Multi-Species Fokker Planck Calculations and Q Values for D-T and D-He Mirror Reactors" (15 min.)

26. J. M. Kindel, F. W. Perkins, "Will Ion Cyclotron Waves Heat Tokomak Plasmas?" (15 min.)

27a. R. Lee, R. N. Sudan, "Cross Field Injection of a Relativistic Electron Beam into a Magnetized Plasma"

b. R. V. Lovelace, R. N. Sudan, "Precession of a Strong E-Layer" (20 min.)

28. N. C. Christofilos, "E-Layer Equilibria in the Presence of a Toroidal Field" (15 min.)

Session IV-b, 2:45 pm, Room 102, Chairman - R. Gajewski

29. P. H. Sakanaka, T. C. Marshall, C. K. Chu, "Formation and Interaction of Ion Acoustic Solitary Waves in a Collisionless Warm Plasma" (12 min.)

30. P. N. Hu, "Evolution of Shock and Nonlinear Compression Waves" (12 min.)

31. E. A. Williams, C. Oberman, "Noise Broadening of Trapped-Particle Echoes" (12 min.)

32. T. M. O'Neil, J. H. Winfrey, "Nonlinear Interaction of a Small Cold Beam and a Plasma, Part II (Random Phase Approximation Revisited)" (12 min.)

COFFEE 3:45 pm

33. M. Bineau, "On The Existence of Force Free Magnetic Fields in a Simply Connected Domain" (12 min.)

34. R. E. Aamodt, "Containment in Open-Ended Systems in the Presence of Fluctuating Electric Fields" (12 min.)

Session IV-b (cont'd)

35. M. N. Rosenbluth, P. H. Rutherford, R. Hazeltine, "Effect of Field Asymmetries on Neoclassical Confinement in Tokomaks" (12 min.)

36. R. L. Dewar "A Lagrangian Theory for Nonlinear Wave-packets in a Collisionless Plasma" (12 min.)

37. F. V. Coroniti, B. D. Fried, R. B. White, "Shock Wave Structure in a Two Component Plasma" (12 min.)

Supplementary Program (by Title or in Review Papers)

C. D. Striffler, T. Kammash, "Evolution of Unstable Bernstein Modes in Astron Plasma"

R. K. Varma, C. W. Horton, Jr., "A Schrodinger Description of Non-Adiabatic Particle Loss and of Magnetic Surfaces"

J. K. Percus, G. J. Yevick, "Decay of Pair Correlations in a Classical Fluid"

U. Daybelge, "Neoclassical Electrical Conductivity of Plasmas with Trapped Particles"

G. Schmidt, "Self Consistent Field Theory of Relativistic Electron Rings"

F. Winterberg, "Magnetically Insulated Ultra-High Voltage Transformer and Controlled Fusion Research"

S. H. Schneider, C. K. Chu, "Numerical Simulation of Strong Plasma Shock Waves Produced in an Electromagnetic Shock Tube"

B. Rosen, G. Schmidt, "The Role of Trapped Particles in Plasma Instabilities"

W. B. Thompson, "Low Frequency Heating"

M. Lampe, J. B. McBride, W. M. Manheimer, J. H. Orens, R. N. Sudan, "Theory and Simulation of the Beam Cyclotron Instability"

W. L. Kruer, J. M. Dawson, "Electron Vortices and Anomalous DC Resistivity"

P. K. Kaw, R. M. Kulsrud, "Relativistic Particle Motion in Super-Intense Laser Beams"

E. A. Frieman, J. M. Greene, K. E. Weimer, "Toroidal Effects on Kink Modes in Tokomaks"

H. Okuda, B. Rosen, J. M. Dawson, "Computer Simulation of Drift-Cyclotron Instability"

J. T. Yen, "Analysis of Nonlinear Plasma Waves"

G. Bateman, "Energy Principle with Given Distributed Inductance"

T. Yeh, "Hydromagnetic Stability of Bifurcated Equilibria"

Stationary Equilibria of a Guiding-Center Plasma*

William A. Newcomb

University of California, Lawrence Radiation Laboratory

Livermore, California

We consider equilibrium states of a guiding-center plasma, assuming either $\underline{v} = 0$ (special case) or $\underline{v} \neq 0$ (general case), where $\underline{v}$ is the drift velocity $(= \underline{E} \times \underline{B}/B^2)$. Let H be the total energy. Assuming $\underline{v} = 0$, the variational condition for equilibrium may be given as $\delta H = 0$, provided that $\delta J = 0$ is imposed as a side condition, where J is the areal invariant in the two-dimensional phase plane of the particle motion in the parallel direction. This was first shown by Grad, and we generalize it now for nonvanishing $\underline{v}$. In the general case, the equilibrium condition is again $\delta H = 0$, but now with two side conditions: $\delta J = 0$ (as before), and $\delta C = 0$, where C is a modified circulation integral for the macroscopic $\underline{v}$ motion. The explicit form of C is derived from a symmetry principle: invariance under excahnge of equivalent flux lines. (In the same way, the areal invariant arises from the principle of invariance under exchange of equivalent particles, so that a unified treatment of both the side conditions may be given.)

*Work performed under the auspices of the U.S. Atomic Energy Commission.

Sufficicient Criteria for Magnetohydrodynamic Stability

D. Lortz, E. Rebhan, G. Spies

Max-Planck-Institut fur Plasmaphysik, Euratom Association

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A class of sufficient magnetohydrodynamic stability criteria has been derived for equilibria which are periodic along the magnetic field lines (i.e. having some symmetry or closed field lines). The sufficient criteria known hitherto are contained as special cases. In this class there is an optimum criterion which, in general, is less restrictive than the former sufficient criteria. However, its application requires solving an eigenvalue problem. More convenient criteria have been derived from this optimum criterion for helically symmetric systems, including the limiting cases of axial and plane symmetry. These criteria appear well suited to calculating lower bounds for a critical β.

MHD Equilibrium of an Elliptical Plasma Cylinder*

Ryszard Gajewski

Department of Physics and Research Laboratory of Electronics, M.I.T.

Cambridge, Massachusetts 02139

It has been suggested[1] that departure from the circular shape of the toroidal plasma column cross section may be advantageous to stability. One way of achieving the desired shape of the column cross section is to properly shape the conducting shell which must coincide with one of the magnetic surfaces in vacuum.

In the present study magnetic surfaces are found analytically in the vacuum region surrounding a cylindrical plasma column at MHD equilibrium. The cross section of the column is assumed an ellipse and the longitudinal current density is assumed constant over the cross section. The magnetic surfaces are deformed ellipses, elongated along the major axis (Fig. 1), and bound by a separatrix (Fig. 2). The separatrix tends to infinity when the ellipticity $\varepsilon \rightarrow 0$, and approaches the surface of the column as $\varepsilon \rightarrow 1$. The case when the longitudinal current is confined to the surface of the column is also discussed.

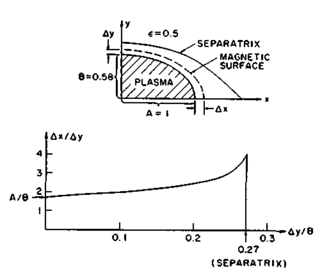

Fig. 1

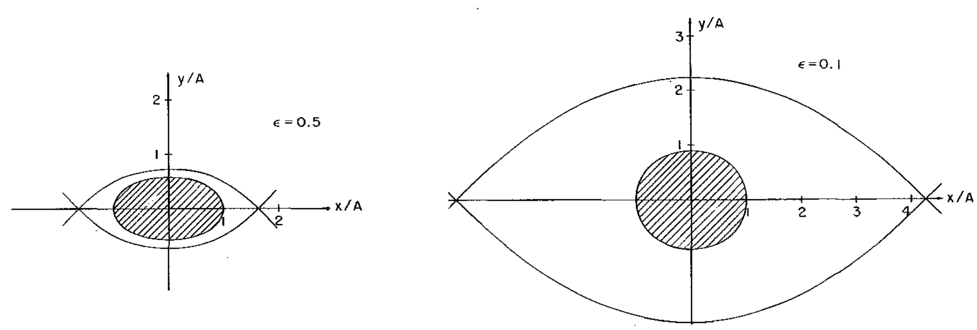

Fig. 2

*This work was supported by the U. S. Atomic Energy Commission (Contract AT(30-1)-3980).

[1] L. S. Solovev, V. D. Shafranov, E. I. Yurchenko, Nucl. Fusion, Special Supplement 1969, p. 25.

MHD Instabilities in Confinement Configurations with Shaped Magnetic Surfaces*

R. Dagazian and B. Coppi

Massachusetts Institute of Technology

The equation of motion is solved for a cylinder of plasma with an elliptically or triangularly deformed cross section immersed in a strong magnetic field. A dispersion relation is obtained for "kink" like perturbations. In particular, the stability of the plasma column under such perturbations is studied in the presence of the metallic casing which serves to shape the magnetic surfaces.

*Work supported in part by the U.S. Atomic Energy Commission, Contract AT(30-1)-3980.

Free Boundary Scylla Equilibria

N. Friedman

Cornell University

Ithaca, New York

Straight helical equilibria have been computed using a free boundary flat pressure model. Given the shape of the outer fixed conductor, the shape of the plasma is calculated. One special case is $\ell = 1$, which is pertinent to the Scylla device. In this case, the outer boundary is specified to be an eccentric circle, and this eccentricity determines the helical field perturbation. Additional parameters are the helical wavelength, the ratio of the radius of the plasma to that of the wall, and the magnitude of the longitudinal (pinch) current. The plasma shape and displacement are calculated as a function of these parameters. The eccentricity creates a mutual inductance and from this we can compute either the strength of the induced z-current if the azimuthal flux is kept fixed or the induced axial EMF if the z-current is held at zero.

Free Boundary Tokomak Equilibria with No Outer Wall

Harold Grad, Abraham Kadish, Donald Stevens

Courant Institute of Mathematical Sciences, New York University

A two dimensional idealization for perfectly conducting Tokomak equilibria has been studied. Containment is provided by the vertical field alone; there are no outer conducting walls. Except for trivial scaling, there is a one parameter family of free boundary equilibria specified by the aspect ratio or by the ratio of vertical field strength to the constant value of the field at the plasma. For small vertical field, the aspect ratio is large, and the plasma cross-section is approximately circular. With increasing vertical field, the plasma shape distorts, and the axial plasma dimension becomes much larger than the radial dimension. The results are obtained by conformal mapping and are explicit in terms of elliptic integrals. Asymptotic formulas have been obtained for large and small aspect ratio and numerical values in between. Curves are given for the variation of all plasma dimensions, temperature, pressure, density, and current as the vertical field is varied adiabatically (as in the proposed ATC experiment).

These results are preliminary to the calculation of actual toroidal free boundary equilibria without walls.

Stability of the Straight $\ell = 1$ Scyllac Configuration*

J. P. Freidberg

Los Alamos Scientific Laboratory

The stability of a straight θ pinch with a superimposed $\ell = 1$ field is studied for the case of arbitrary β, in the sharp boundary model. Previous investigations of this problem have relied on one of two expansions depending upon the relative size of two dimensionless parameters, $\varepsilon = ha \equiv$ pitch number x plasma radius and $\delta \equiv$ shift/plasma radius. In the first expansion it is assumed that $\delta << \varepsilon \lesssim 1$. For this case, leading order terms in δ and leading and first order terms in ε are kept. The plasma is found to be weakly unstable to the $m = 1$ mode. However, substituting into the toroidal equilibrium relations for Scyllac parameters results in a violation of the original ordering; that is for equilibrium it is necessary that $\delta \approx 1$, $\varepsilon \approx .1$. This led to the development of the second expansion in which it is assumed that $\varepsilon << \delta \lesssim 1$. In this expansion leading order terms in ε and leading and first order terms in δ are kept. The results of this theory also predicted weak instability although for typical parameters, the growth rate was considerably smaller than that of the first expansion. The calculation presented here is based on the second expansion which is more closely related to experimental conditions. By calculating growth rates numerically we are able to carry the expansion to leading and first order terms in ε and arbitrary δ. The main result of the calculation is that for experimental parameters the additional corrections to the second expansion appear to be very important and somewhat suprisingly predict growth rates very close to those of the first expansion except for very high β.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

A Spectral Analysis of the Screw Pinch

Harold Grad, Jeffrey Marsh and Harold Weitzner

Courant Institute of Mathematical Sciences, New York University

Previous analyses of the stability of the screw pinch equilibrium in ideal MHD theory have been done in many different ways and are the source of many qualitative concepts. In order to learn more about this simple system we analyse the stability directly from the equations of motion and determine the spectrum of oscillation and growth frequencies.

Typical of far more general topological toroidal configurations we find first that there are instabilities not observable in the usual δw formulation. While such instabilities grow algebraically with time and not exponentially they may well be of some significance. Even the ordinary stable theta pinch may exhibit such instabilities. A nonlinear study of such instabilities is also given.

For a given k and m number of the perturbation the continuous and stable and unstable point spectra are described, the role of the Suydam criterion in terms of the spectrum is given. For systems only slightly Suydam unstable growth rates associated with the Suydam modes are estimated. Various pathological cases are explored for significance. The applicibility of perturbation theory to various instabilities is explored. Possible relationships between mathematical pathology and physical significance are considered.

Tokamak Kinetic Theory; Trapped-Particle Modes*

J. D. Callen

Department of Aeronautics and Astronautics

Massachusetts Institute of Technology, Cambridge, Mass.

An integral equation governing kinetic modes in a collisionless, current-carrying, Tokamak-type plasma, which includes magnetic shear and finite "banana-width" effects, has been derived. The derivation is accomplished by using the magnetic flux coordinates of an axisymmetric system. Namely, we use ψ, θ, ζ in which ψ is the poloidal flux function, ζ the toroidal (axisymmetry) angle and θ a poloidal angle defined in terms of the surfaces (χ) orthogonal to the ψ surfaces in such a way that $\underline{B} = \underline{\nabla}\psi \times \underline{\nabla}(q\theta - \zeta)$ where $q = q(\psi) = 2\pi/\iota$ is the "safety factor". The particle orbits, including the gyro-, bounce and drift motions, are derived in terms of these coordinates. Integrating the perturbed Vlasov equation along these orbits for perturbations of the type $\Phi_\ell(\psi, \theta)\exp(i\ell\zeta - in\theta - i\omega t)$ and substituting the result into Poisson's equation, we obtain the integro-differential equation for $\Phi_\ell(\psi, \theta)$ governing electrostatic perturbations in an axisymmetric system. The equation is similar to those obtained for axisymmetric multipoles[1] and axisymmetric magnetic mirror systems.[2] It differs from those works in that: 1) magnetic shear is explicitly included; 2) finite gyroradius and finite "banana-width" corrections

*Research supported by the National Science Foundation (Grant GP-9557) and in part by the U. S. Atomic Energy Commission (AT(30-1)-3980).

[1]P. H. Rutherford and E. A. Frieman, Phys. Flu. 11, 569 (1968); J. B. Taylor and R. J. Hastie, Plasma Phys. 10, 479 (1968).

[2]C. W. Horton, J. D. Callen and M. N. Rosenbluth (to be published).

in the ψ motion are kept; and 3) the magnetic field is not a vacuum one, but rather one which allows for a plasma current.

We have used this new integral equation to investigate the trapped-particle interchange modes[3] ($\omega << \omega_{bi}$, the ion bounce frequency). In particular, we study the radial (or ψ) dependence of the unstable modes and the effects of shear, removing the previous assumptions that $\frac{\omega_{bi}}{\omega} << |\ell q - n| << 1$ (close, but not on a mode-rational surface). The ψ, θ eigenfunction problem is solved iteratively by first solving the θ (integral) equation and then solving the ψ (differential) equation. Generally speaking, the unstable modes are found to follow the magnetic field structure in such a way that they are always as flute-like as possible. High ℓ number modes are found to be stabilized by the finite banana-width effects.

[3] B. B. Kadomtsev and O. P. Pogutse, Soviet Phys. - JETP 24, 1172 (1967).

Trapped Particle Contributions to the Heating in a Tokamak

A. A. Ware and A. B. Macmahon
University of Texas at Austin

Using the transport equations of Rosenbluth and Hinton, the steady state solutions have been obtained for n, T_e, T_i and B_θ allowing for ohmic heating and the various trapped particle effects. Assuming the aspect ratio is fixed and utilizing a dimensionless form for the equations, the single parameter which determines the character of the solutions is $a\omega_{pi}/c$, where a is the minor radius and ω_{pi} the ion plasma frequency; this parameter controls the degree of thermal coupling between electrons and ions. The properties of these solutions will be discussed.

In the Tokamak experiments T-3 and ST, consideration of the time constants involved indicate that their pulse lengths are too short for such a steady state to be attained. Compression and heating continue throughout the pulse. Initially, ohmic heating will be the dominant heating process, but once β_θ becomes comparable with unity the dominant process will be compressional heating caused by the trapped particle pinch effect. Approximate calculations show that a sharp spatial peak in temperature can result, as observed in ST. The plasma in T-3 does not reach this stage because of the longer time scale and the fact that the driving force for the compression (the external electric field) is reversed.

Anomalous Viscosity as a Possible Explanation for an Anomalous Skin Effect

S. Yoshikawa and J. Schmidt

Plasma Physics Laboratory, Princeton University, Princeton, New Jersey

Anomalous viscosity of electrons across the magnetic field may be responsible for the observed anomalous skin effect in tokamaks. Here, a model is proposed to determine the anomalous viscosity. This may have relevance in collisionless shocks across the magnetic field. In the model, electrons which drift parallel to B, have a velocity shear in the perpendicular direction (say x). The electrons excite waves which have the group velocity finite in the x direction. Then the waves which get the momentum from the electron drift propagate to the region where the drift velocity of electrons is smaller than the phase velocity of waves parallel to B. Then the momentum is transferred to those slow electrons. That is, waves act as carriers to transfer the momentum between spatially-separated electrons. In practice, the mean free path of the wave perpendicular to B, ℓ, is estimated. The momentum transfer rate between electrons and waves, τ^{-1}, also is calculated from quasilinear theory. The anomalous (kinematic) viscosity is then estimated as $\ell^2\tau^{-1}$. The amplitude of the wave is left as a parameter to be determined by full nonlinear theory and/or by experiments. This work was supported by U.S. Atomic Energy Commission Contract AT(30-1)-1238.

Bootstrap Current-Driven Drift Instability in Tokamaks

F. L. Hinton and C. W. Horton, Jr.
University of Texas at Austin

The stability of collisionless drift waves is examined for Tokamak geometry. The equilibrium distribution functions are those given by neoclassical transport theory in the banana regime. These distribution functions are known to give a "bootstrap" current, carried by trapped particles, due to the spatially non-uniform distribution of bananas. This current is shown to be the dominant destabilizing effect for drift waves ($\omega \approx \omega_*$) with m-numbers in the range $1 \ll m \lesssim B_\phi/B_\theta$, where m is the quantum number of the mode variation that short way around the torus. If the electron to ion temperature ratio satisfies $\sqrt{T_e/T_i} > r_n/a_{i\theta}$, where r_n is the density scale length and $a_{i\theta}$ is the average ion Larmor radius in the poloidal magnetic field, then the destabilizing effect of the bootstrap current dominates that of ion inertia for all m. Furthermore, the ion acoustic wave ($\omega \cong k_\parallel c_s$) is also driven unstable by the bootstrap current in this case.

A quasilinear analysis of the trapped particle diffusion resulting from these instabilities shows that, for a given wave energy spectrum, the particle flux (averaged over a magnetic surface) is proportional to $1/B_\theta^2$.

Neoclassical Diffusion in Axisymmetric Torus
in the "Intermediate Region"

Dilip K. Bhadra, Chuan S. Liu, and Tihiro Ohkawa
Gulf General Atomic Company
San Diego, California

Higher-order collisional effects on the neoclassical diffusion in the "intermediate" region of a toroidal plasma are studied. Since the major contribution to the diffusion is from the low-$v_{\|}$ particles, the Fokker-Planck equation is solved for an asymptotic solution with small parameter $(\nu_c/\nu_t)^{1/2}$ (ν_c, ν_t being the collision and transit frequency respectively). The resulting diffusion coefficient is of the form $D = D_o[1 + \alpha(\nu_c/\nu_t)^{1/2} - \beta(\nu_c/\nu_t) \ldots]$ where $D_o = (\pi/2)^{1/2}(2\pi\rho_e/lR)(\rho_e^2\Omega_e)$ is the result previously obtained by Galeev and Sagdeev. The coefficient α is small and vanishes in the absence of a radial electric field. The coefficient β is positive definite, of the order of unity. Thus the major higher-order collisional effects tend to reduce the diffusion coefficient from D_o. This decrease is due to the reduction of the random walk frequency. The half-power of (ν_c/ν_t) is inherent to this kind of boundary-layer type problem.

Work supported by the U. S. Atomic Energy Commission, Contract No. AT(04-3)-167, Project Agreement No. 38.

Shocks and Diffusion in Low-β Toroidal Plasmas*

N. K. Winsor, E. C. Bowers,+ M. A. Hellberg,** and J. M. Dawson

Plasma Physics Laboratory, Princeton University, Princeton, N.J. 08540

The low-β fluid equations have been examined for axisymmetric solutions which are stationary ($\partial/\partial t = 0$) up to the order at which resistive diffusion causes the macroscopic parameters to change. With an ordering of the resistivity motivated by experimental parameters, the stationary solutions display poloidal rotation and a density profile with a discontinuity curve(shock) extending radially from the magnetic axis to the wall. When viscosity is added to the equations, both shocked solutions and unshocked solutions are possible, depending on the relative magnitude of the resistivity and viscosity. Numerical simulation studies agree with these analytic solutions. Cases with more viscosity and less resistivity reveal no shock, a material flux approximately classical (Pfirsch-Schluter) and a negative potential on axis as in ST tokamak experiments. When resistivity is lowered, a density shock yields fluxes more than an order of magnitude greater than classical and the axis is positive, as in Model C Stellarator measurements. Movies of the density contours for these cases illustrate the physical processes involved.

+Present address: Imperial College, London, England

**On leave from University of Natal. The author gratefully acknowledges receipt of an Overseas Study Grant from the South African Atomic Energy Board.

*This work was supported by U.S. Atomic Energy Commission Contract AT(30-1)-1238; Use was made of computer facilities supported in part by National Science Foundation Grant NSF-GP 579.

Stabilization of Dissipative Trapped Particle Instability

David W. Ross and Marshall N. Rosenbluth

The Institute for Advanced Study, Princeton

The calculation of the stability of the trapped particle drift mode, which is driven by electron collisions, is amended to include untrapped ion Landau damping in addition to the trapped ion collisional damping considered previously.[1] It is found that the Landau term is stabilizing if $dT/dr = 0$,[2] but that it changes sign if $d \ln T/d \ln r > 2/3$. In the latter case the numerical results predict that ion collisions are insufficient to stabilize the mode under typical operating conditions of several proposed Tokamaks.

[1] David W. Ross, Marshall N. Rosenbluth, and D. P. Kostomarov, Bull. Am. Phys. Soc. 15, 1400 (1970).

[2] R. Z. Sagdeev and A. A. Galeev, Sov. Phys.-JETP 24, 1172(1967).

Computation of Finite-Beta Plasma Equilibria

In Minimum-B Mirror Systems*

David V. Anderson and John Killeen

University of California, Lawrence Radiation Laboratory

Plasma equilibria have been computed for various containment schemes which share the common feature of effective two-dimensional symmetry. We present a method for computing the equilibria in the general three-dimensional minimum-B mirror system, and give numerical results for finite-beta equilibria in the Baseball II magnetic configuration. We are also applying this code to the 2X II experiment and to the design of the mirror fusion experiment (MFX). For the case of open field lines the hydromagnetic equilibria equations with tensor pressure reduce to the equation $\nabla^2\theta = 1/\nu(\nabla\theta\ \nabla\nu)$ where $\vec{B} = 1/\nu(\nabla\theta)$ and $\nu = (p_{||} - p_{\perp})/B^2 - 1$. We can write θ as $\theta_c + \theta_p$, where θ_c is the potential function of the vacuum magnetic field, i.e., $\vec{B}_c = -\nabla\theta_c$, and θ_p is the potential function of the magnetic field due to the plasma. We solve the equation for $\theta_p(r,\theta,z)$ on a cylindrical domain by finite difference methods. An implicit iterative algorithm similar to the ADI scheme is used to solve the finite difference form of the equation. We assume that the distortion of the original vacuum field by the plasma is negligible at sufficient distance from the plasma region. This distance determines r_{max} and z_{max} of the cylindrical domain and on this outer boundary $\theta_p = 0$. By employing a sparse mesh in the exterior region we can extend the boundary surfaces to verify the effect of this condition. The location of the coils poses no problem-the boundary surface can enclose the coils. We use the pressure functions $p_\perp(B)$ and $p_{||}(B)$ given by Taylor and obtain solutions corresponding to a sequence of betas until stability is violated.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

Limiter Induced Perturbations and the Small Gyro Radius Limit

Donald Dobrott and Harold Grad

Courant Institute of Mathematical Sciences, New York University

Although Tokomaks are designed to have approximately symmetric fields, the limiter (and fringing electric fields) makes the geometry strongly asymmetric. One effect is to preferentially scrape off circulating as compared to trapped particles. More significantly, the limiter partially blocks the flow of toroidal current. The simplest non-dissipative macroscopic model which allows current and flow to depart from magnetic flux surfaces is one with Hall effect. We examine the fully non-linear finite β theory of this model in axially symmetry. The system of equations is eighth order: twice elliptic, for subsonic flow (distinct boundary conditions for magnetic and fluid flow stream functions), plus four real characteristics (flux lines and streamlines, each counted twice). The limit of small Hall effect (small gyro-radius) is very singular and includes unusual boundary layer effects.

Low Frequency Drift Cyclotron Instability

Burton D. Fried and Charles F. Kennel
University of California, Los Angeles

We consider the instability associated with ions streaming across a magnetic field at sufficiently high velocity, V_D, that the ion cyclotron radius exceeds the plasma dimensions.

Forslund, et al[1] have called attention to the possible importance of this instability for plasma heating, and have presented solutions of the linear dispersion relation and also the results of particle simulation calculations of the nonlinear behavior for the case of purely perpendicular propagation of the unstable waves involved, the frequencies being of order electron cyclotron frequency (Ω_e). Recent attempts to observe this instability in laboratory plasmas were carried out by J. M. Sellen at TRW Systems and by P. Barrett and R. Taylor at UCLA; in both cases, it was found that the dominant instability appears to be of order $V_D/r_{ce} = (V_D/a_e)\ \Omega_e << \Omega_e$, due to the fact that, as in all laboratory configurations, k_{11} is not exactly zero. (Typical minimum values of $k_{11}r_{ce}$ are .01 to .05). We show that the instability in this case is essentially an ion acoustic wave, the dispersion relation being modified in certain essential ways by the magnetic field. The fastest growing waves have $k_\perp r_{ce}$ of order 1 and $\gamma = \mathrm{Im}\omega$ of order $(\Omega_e/\Omega_i)^{1/2}$.

If T_e/T_i is large, then the behavior is governed entirely by the two dimensionless parameters, $Q = (V_D/a_e)\ (\sqrt{2}/k_z r_{ce})$ and $A = (\Omega_e/\omega_{pe})^2$, where $a_e = \sqrt{2T_e/m}$. As T_e/T_i decreases, the growth rate is diminished by ion Landau damping. Illustrative curves showing the dependence on Q and A and comparison with the experimental results will be presented.

[1] Forslund, Morse and Nielson, Phys. Rev. Lett. 25 1266 (1970)

Electron Cyclotron Drift Instability*

D. W. Forslund, R. L. Morse and C. W. Nielson

Further studies of the electron cyclotron drift instability[1] indicate that it is this instability and not an essentially magnetic field free ion acoustic instability which is responsible for anomalous diffusion (or resistivity) in high density containment experiments and in collisionless shocks. This instability is much less sensitive to T_e/T_i than the ion acoustic instability and in fact persists when $T_i > T_e$ and $V_d << V_e$ (V_d and V_e are the drift and electron thermal velocities respectively). A rough instability criterion is $V_d/V_e > \omega_c/\omega_p$ and $V_d \gtrsim V_i$ although some instability exists for smaller drifts. Detailed ω vs k root diagrams will be shown as well as numerical simulations of the resulting electron cyclotron turbulence. In particular, anomalous diffusion will be clearly demonstrated. The implications for high β controlled fusion devices will be discussed.

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*This work performed under the auspices of the U. S. Atomic Energy Commission.

[1]D. W. Forslund, R. L. Morse and C. W. Nielson, Phys. Rev. Letters, 25, 1270 (1970).

Nonlinear Theory for Plasma Driven by a Large-Amplitude, High-Frequency Field*

E. J. Valeo, C. Oberman, and W. L. Kruer

By means of an asymptotic multiple-time-scale technique, we systematically develop equations capable of describing the nonlinear evolution of a parametrically driven homogeneous plasma. The small expansion parameter is the electron-to-ion mass ratio. A general expression for the nonlinear (in both the pump and fluctuation intensities), high-frequency energy absorption rate is derived as a function of the fluctuation amplitudes which may be computed nonlinearly within the framework of the theory. The linearized form of these equations is compared with the results of others. A novel consequence of the linearized theory is the prediction of a finite phase relation between the self-consistent high-frequency waves and the pump wave, which necessitates a modification of the random-phase approximation of usual weak turbulence theory. Upon consideration of the evolution of the electron distribution function, but with neglect of the competitive processes of mode coupling, a saturation value for the wave amplitudes is calculated and is seen to be in reasonable agreement with results from a numerically simulated plasma. A subsidiary wave vector expansion of the equations is carried out and the important processes of electron-by-ion wave scattering and electron-to-ion wave decay are shown to be the dominant nonlinear mode coupling processes. Upon transformation to normal mode (Bloch function) form, the equations for the electric potential are shown to describe a regime where growth and oscillation are of competitive size. The results of the analysis of the effects of these important mode-coupling processes on saturation will be given.

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Suppression of the Whistler Instability by Relativistic Effects*

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The anisotropy driven whistler instability has been predicted to be particularly virulent for ECH plasmas. Indeed, it has been observed in a number of experiments. In this paper it is shown that relativistic effects can reduce the growth rate to a negligible level as a consequence of the relativistic spread in the cyclotron frequency, which causes resonant particles in the tail of the distribution function to contribute to damping rather than growth.

For several distribution functions it is found that, for fixed anisotropy, the growth rate decreases dramatically with temperature (in many cases the growth rate becomes negligible for temperatures which are small compared to the rest energy of an electron). In general, distribution functions with higher tails show a more pronounced decrease in growth rate. These results are in keeping with the experimental observations of N. Lazar that two-frequency heating simultaneously increases the tail of the distribution function and suppresses the whistler instability. The results are also consistent with the observations of R. Dandl (no whistler instability is observed and the distribution function always has a rather high tail).

*Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

High Frequency A. C. Electrostatic Plasma Instabilities*

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The instabilities induced in a plasma under the influence of an electric field oscillating at near the plasma frequency are studied. By numerically solving the integral equation which describes the linearized behavior of the plasma we are able to formulate a rather detailed and comprehensive picture of the stability properties for a wide range of parameters.

*This work performed under the auspices of the U. S. Atomic Energy Commission

Dynamic Stabilization of Drift Waves by Radial and Azimuthal High-Frequency Fields*

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We have investigated the stabilization of density gradient driven low-frequency drift instability ($\omega << \Omega_i$; $k_\perp a_e, k_\perp a_i << 1$; a_e and a_i refer to the electron and ion Larmor radii and the rest of symbols have usual meanings) and the drift cyclotron instability ($\omega \sim \Omega_i$, $k_\perp a_i \gtrsim 1$) by radial or azimuthal high-frequency fields with a frequency of the order of upper hybrid resonant frequency. This scheme of stabilization is much more efficient than the conventional method[1] of applying axial high-frequency fields because the drift waves, which essentially propagate at right angles to the magnetic field, couple more strongly to fields oriented in that direction. Furthermore, one can also stabilize the more dangerous flute-like drift cyclotron waves (with $k_{||} = 0$) by this scheme in contrast to the conventional method using axial high-frequency fields only.

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[1] Ya. B. Fainberg and V. D. Shapiro, Sov. Phys. JETP **25**, 189 (1967); M. Okamoto, T. Amano, and K. Kitao, J. Phys. Soc. Japan **29**, 1041(1970).

*This work was supported by U. S. Atomic Energy Commission Contract AT(30-1)-1238.

High Pressure Tokamak Equilibrium

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Assuming an ideal MHD equilibrium, we find there is no limit to the maximum pressure p_{max} obtainable in a Tokamak of given magnetic field strength B, aspect ratio ε^{-1}, and rotational transform ℓ. There is a limit to the total energy a Tokamak can contain; this may be expressed as $\bar{p} < \bar{p}_o = \alpha\varepsilon(\ell/2(\ell/2\pi)^2 B^2/8\pi$ where $\bar{p}$ is the pressure averaged over the plasma volume and α is a number of order unity. The pressure and rotational transform have their maximum values on the magnetic axis. As $\bar{p}$ and p_{max} increase with ℓ, ε, B fixed, the distance δ from the magnetic axis to the outer edge of the plasma decreases. When $\delta < .7r$, where r is the plasma radius, the toroidal current begins to reverse on the inner edge of the plasma. For $\delta < .6r$, $\bar{p}$ has reached its limiting value $\bar{p}_o$, and $\delta = (3\bar{p}_o/p_{max})r$. The axis moves to the edge in the limit $p_{max} \rightarrow \infty$. In general, there are two classes of high $\bar{p}$ Tokamak equilibria, "deep" solutions with arbitrarily large p_{max} and "shallow" ones with p_{max} limited to about $2\bar{p}_o$. The magnetic axis of shallow solutions does not displace very much as $\bar{p}$ and p_{max} tend to their limiting values. In deriving all these results, $p(\psi)$ and $I^2(\psi)$ have been chosen quadratic in ψ, where p is the pressure, I is the poloidal current flux, and ψ is the poloidal magnetic flux. (For $p(\psi)$ linear in ψ, there are only shallow solutions).

Thermonuclear Power Production by Nonmaxwellian Ions in a Closed Magnetic Field Configuration*

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High-energy neutral beam injection into the tokamak configuration can be used to implement the ancient idea of a target-plasma fusion reactor. In a cold-triton plasma with $T_e = 5$ keV, a 180-keV deuteron on the average 2.4-folds its energy while thermalizing. (This factor includes initial kinetic energy, fusion release, and standard blanket reactions.) Assuming moderate efficiencies of conversion from thermal to electrical power, net power can be produced. Optimization calculations have been carried out and will be presented. The main advantage of the scheme is to lower the appropriate Lawson-type requiremnets significantly relative to the one-component reactor case. The stability problem will also be discussed: it does not appear to be severe, if the energetic-ion component is fairly isotropic.

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Thermonuclear Reaction Waves at High Densities*

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Phenomena are studied of the ignition and the development of plane thermonuclear reaction waves propagating through solid density deuterium-tritium plasmas. The state of the plasma is described by a two temperature gas and optically thin. The equations governing the change of density, velocity, electron temperature, ion temperature and degree of reaction are derived from the conservation laws and the laws of reaction kinetics. The physics are contained within a set of five coupled non-linear partial differential equations. Terms in the equations take explicit account of thermonuclear reactions and heating, conduction, electron-ion equilibration, bremsstrahlung cooling and other fluid dynamic effects. Qualitative behavior of the physical variables is discussed. Solutions of the equations are obtained by employing finite difference techniques. Due to the properties of the plasma and the form of the reaction cross section, substantial physical differences exist between the fusion driven reaction waves and detonation waves in chemically reactive gases. The ignition energy threshold is obtained for simple geometries and the above threshold case is studied. Throughout the development, the wave structure changes from pure conduction, to plasma-schock-like behavior, and finally, it is dominated by thermonuclear heating and bremstrahlung cooling effects. The final steady wave structure is presented. The thermonuclear energy yield from small D-T pellets has also been studied.

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Multi-Species Fokker Planck Calculations and Q Values For D-T and D-He^3 Mirror Reactors*

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The feasibility of the mirror machine as a fusion reactor depends in large part on the achievable values of Q, the ratio of thermonuclear power to injected power in steady state. Under the assumption that the end losses are due primarily to weak Coulomb collisions, Q can be determined from a solution of the coupled, time-dependent, Fokker-Planck equations for the species present. The computer code used in the present work obtains distributions $f_i(v,t)$, for an arbitrary number of species; the mirror machine is approximated by a magnetic square-well, and the ϕ and θ dependences of the distributions in velocity space are removed by assuming azimuthal symmetry and by separation of variables, respectively. Improvements over previous, related work are: explicit treatment of 3-component systems such as D-T-e and D-He^3-e; refined calculation of reaction rates, $\langle\sigma_{ab}v\rangle$, integrating explicitly over numerically determined f_i for both reactants; and more rigorous treatment of the Coulomb logarithm, $\ln\Lambda$. Results to be presented emphasize Q values for the D-He^3 fuel cycle, with injection energies in the ranges, 200 keV $\leq E_D \leq$ 800 keV, 800 keV $\leq E_{He} \leq$ 2000 keV. Values of Q_{DT} versus injection energy, their sensitivity to the means of calculating $\langle\sigma v\rangle$, and new results concerning the mass dependence of individual $n\tau$'s will also be discussed.

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Will Ion Cyclotron Waves Heat Tokamak Plasmas?

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The object of this work is to determine the conditions under which ion cyclotron waves can efficietnly heat ions in the central regions of tokamak plasmas, and what consequences (such as electron heating) ensue if the ion cyclotron waves are not launched in the optimum manner. Our model of a tokamak is a plasma slab of varying density perpendicular to a confining magnetic field with a varying magnitude but constant direction. As a result, ω and $k_{\parallel}$ remain fixed as a wave propagates and the dispersion relation determines the (complex) wave vector k_x. Typical results are that ion cyclotron waves should be launched with $k_{\parallel} \approx [\omega_{pi}^2 \; \Omega_c/c^2 v_{thi}]^{1/3} \approx 0.7 \text{ cm}^{-1}$ and $\omega \approx 0.95\Omega_c$ where Ω_c is the ion cyclotron frequency at the center of the plasma. If $k_{\parallel}$ is too large, the ion cyclotron waves are strongly evanescent and do not reach the central plasma. For values of $k_{\parallel}$ somewhat too small, the ion cyclotron wave converts into the slow Alfven wave which principally heats electrons. For very small values of $k_{\parallel}$, there is a mode conversion into an electrostatic wave at the plasma surface with possible surface heating.

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Cross Field Injection of a Relativistic Electron Beam into a Magnetized Plasma

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In the Astron experiment and in a similar experiment at Cornell University[1], relativistic electrons are injected into a magnetic mirror to form a layer of circulating electrons. When the beam of electrons is injected into a preionized plasma it is magnetically neutralized by a return current which is induced in the plasma by the injection process. We have computed the magnitude and decay time of this return current on the assumption that the plasma is cold and the beam is rigid. We find that such a return will flow mostly within the beam cross-section when $a\omega_p(1+\omega_c^2/\omega_p^2)^{-1/2}/C >> 1$, where a is the beam radius, ω_p is the plasma frequency, and ω_c is the cold electron cyclotron frequency. The return current flows for a time of the order of ω_{ci}^{-1} $[\mu^{1/2}a\omega_p/v_o\gamma]$, where $\mu = m_e/m_i$, v_o is the beam velocity, $\gamma = (1-v_o^2/c^2)^{-1/2}$, and ω_{ci} is the cold ion cyclotron frequency.

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[1] H. H. Fleischmann, H. Davitian, R. E. Kribel, and J. A. Nation, Bull. Am. Phy. Soc. 15, 1459.

Precession of a Strong E-Layer

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Observations of weak relativistic E-layers in the Astron indicate that the motion is rigid in accord with the "super particle" model of Furth (1) in which electrons of the layer are assumed to precess as a rigid body with a frequency equal to the precession frequency of a single electron in the external fields. By perturbation of the Vlasov equation for an E-layer of arbitrary strength we investigate the affect the self-magnetic field has on the precessional motion. In terms of the parameter $\xi \equiv \omega_B^2/2\Omega_c^2$ (where ω_B is the relativistic electron plasma frequency and Ω_c the cyclotron frequency) we find that the precession frequency is reduced from that for a single electron by a factor $(1+\xi)$. In the weak E-layer limit, $\xi << 1$, the precession frequency agrees with that of Furth, but for a strong E-layer with loading factor $\zeta \sim 1$, ξ approaches $r/2a$, where r is the layer radius and a its thickness. Non-rigid motion of the layer appears in our analysis as a damping of the precession in the absence of external forces on the layer; the damping is negligible and the motion is rigid for large enough self-fields, $\xi >> \Delta p/p$ where $\Delta p/p$ is the fractional spread in canonical momentum. Stability of the precession mode depends on the sign of the mode energy, the positive energy case being stable under the influence of external dissipative forces; the sign of the mode energy is shown to be opposite to that of the external field index independent of the layer strength. For a weak E-layer with a fraction f of its charge unneutralized, the precession is stable only for $(f\zeta + n) < 0$, where n is the field index.

[1] H. P. Furth, Phys. Fluids 8, 2020 (1965).

toroidal field effects e.g. trapped particle instabilities. The relativistic proton E-layer currents substitute for the plasma current required to create the shear thus allowing a continuous operation in contrast to the pulsed operation of the Tokamak. At the same time the E-layer particles will heat the plasma to fusion temperature.

E-Layer Equilibria in the Presence of a Toroidal Field*

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The addition of toroidal field to improve the plasma β in plasma confined in the Astron's E-layer was suggested several years ago.[1] It was only recently however that the effect of the toroidal field in the E-layer equilibrium has been investigated. The investigation so far is limited to a simplified model of the E-layer represented by a straight beam of circular cross section while the toroidal field is parallel to the beam axis. There are two classes of solutions depending whether the E-layer is paramagnetic or diamagnetic to the toroidal field. The first class of solution was discovered by S. Yoshikawa.[2] In this solution the relativistic electrons follow paths almost parallel to the magnetic field thus minimizing synchrotron radiation losses. The solution given in Reference 2 is accurate for large rates of v, where $v = (I/17{,}000\ \gamma)$, I is the beam current and γ is the electron energy expressed in rest energy coils. A more general solution valid for any value of (v) will be presented.

In the second class of solution the E-layer is diamagnetic to the externally applied toroidal field. In this way the stability properties of the E-layer's closed magnetic well are combined with Tokamak's shear to provide an improved plasma confinement hence higher β. In addition the E-layer currents generate a depression or a minimum-B in the toroidal field, thus eliminating all the

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]T. K. Fowler, et al., Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, 1968 (IAEA, Vienna, 1, 981 (1969).

[2]S. Yoshikawa, Physical Review Letters 26, 295 (1971).

Formation and Interaction of Ion-Acoustic Solitary Waves in a Collisionless Warm Plasma*

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The formation and interaction of solitary waves in a warm-ion, hot electron plasma have been studied with the Vlasov equation for ions and Boltzmann distribution for electrons, with the purpose of determining those properties hitherto known only for cold ions and the K-dV equation. A parallel study has also been made using the fluid equations for ions with a scalar pressure added.

First, by using the warm ion fluid equation, with isothermal electrons given by Boltzmann distribution, we have shown that for the same amplitude, the width of the solitary wave is smaller than that obtained from the Korteweg-de Vries equation while the Mach number is slightly larger. The limiting value of the amplitude is considerably smaller than that for the case of the cold ions, for the electron-ion temperature ratio $T = 30$, the limiting amplitude is only 1/3 of that for the case of $T = \infty$.

Second, with ions described by one-dimensional Vlasov equation and electrons by the Boltzmann distribution, we have shown that an arbitrary propagating pulse in a plasma always decays into solitary waves. We have studied the interactions of two solitary waves, a large wave catching up with a smaller one. We have found that: (1) if the initial amplitude ratio of the two waves is not too large, the two waves exchange their amplitudes and bounce apart; (2) if the initial amplitude is large, larger wave first absorbs the smaller, and later reemits it in the back, and the initial amplitudes are conserved. These properties agree qualitatively with the Korteweg-de Vries results, but the solution of the Vlasov equation gives a more complete picture of the physics of the interaction.

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Evolution of Shock and Nonlinear Compression Waves*

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The initial value and boundary value problem of weak nonlinear plasma waves in the presence of a perpendicular magnetic field is studied, based on a system of moment equations of Boltzmann equations for electrons and ions. This system, rigorously obtained by proper scalings in time and space with respect to the wave strength, describes a fully-ionized plasma with all possible dissipative mechanisms including electron and ion viscosity, coupled electron and ion heat conduction, thermal diffusion and resistivity, as well as all the distinct Hall effects. In all the plasma parameter ranges, it is found that the evolution of weak nonlinear waves is governed by the Burgers equation, subject to given initial and boundary conditions.

The formation of a shock and the evolution of a compression pulse are studied in detail. The transit time and distance for a fully developed shock are obtained. In some parameter ranges, the transit time is smaller than the electron collision time while the transit distance is smaller than the mean free path.

The time scale for the evolution of a compression pulse depends strongly on its initial width. A shock-like wavefront will shortly appear when the initial width is large as compared with the thickness of a steady shock. (In some parameter ranges, this shock thickness is smaller than the mean free path.) Eventually, however, a compression pulse becomes self-similar, with its amplitude decaying as $1/\sqrt{t}$ and its width spreading as $\sqrt{t}$.

The inclusion of the electron inertia effect leads to the Korteweg-de Vries-Burgers equation which covers a wide range of wave profiles.

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Noise Broadening of Trapped-Particle Echoes*

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It has recently been proposed by Liu and Wong that the behavior of magnetically trapped particles can conveniently be studied by investigating echoes associated with them. We extend the analysis of Liu and Wong to include the effect of interparticle collisions or weak turbulence. It is well known that echo phenomena in general are sensitive to noise, because of their dependence on the preservation of fine structure in the distribution function.

Information about the diffusion of magnetically trapped particles is of particular interest due to their apparent role in the stability and transport of toroidal plasmas. We show that the effect of noise on the echo pulses is to broaden them, provided the effective collision time is long compared with a typical bounce time in the well. If this condition does not hold, the echo is destroyed. To calculate the trapped-particle distribution responsible for the echoes, we require the propagator for a particle trapped in a magnetic well. This we compute and then evaluate the propagator for the case of tokamak geometry, and finally specialize to deeply trapped particles. We then use the previous results to solve the echo problem and discuss the results. There are interesting magnetospheric applications as well.

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Nonlinear Interaction of a Small Cold Beam and a Plasma, Part II (Random Phase Approximation Revisited)*

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In a previous paper[1], a single wave model is used to describe the initial nonlinear interaction of a small cold beam and a plasma. This description is based on the observation that after several e-foldings the band width of the growing spectrum of waves is so narrow that the electrons interact with a very nearly pure sinusoidal field. However, this solution is only locally valid, since the small remaining spectral width produces a slow spatial modulation in the phase and amplitude of the sinusoidal wave. This spatial modulation is random, since the phases of the waves within the small spatial width are random.

In the present paper a nonlocal solution is constructed by giving the locally valid sinusoidal solution a slow spatial modulation in phase and amplitude. Spatial Fourier transforms over this nonlocal solution are evaluated using the theory of random noise. This procedure does not involve the usual assumptions associated with the random phase approximation, since here the dynamics is worked out completely, before any averaging is done.

[1] "Nonlinear Interaction of a Small Cold Beam and a Plasma", T. M. O'Neil, J. H. Winfrey and J. H. Malmberg, to be published in Phys. Fluids.

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On the Existence of Force Free Magnetic Fields in a Simply Connected Domain

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A force free magnetic field $\vec{B}(\vec{x})$ carries a constant current density $\sigma(\vec{x})$ along each magnetic line. This fact suggest a natural boundary value problem for a force free field in a simply connected domain D with boundary ∂D. Let S be some part of ∂D and D(B,S) be the part of D connected with S by a line of the field $\vec{B}$. The problem is to find a field $\vec{B}(\vec{x})$ such that $\nabla \cdot \vec{B} = 0$ in D, $\vec{B} \cdot \nabla \sigma = 0$ and $\nabla \times \vec{B} = \beta \sigma \vec{B}$ in D(B,S), $\nabla \times \vec{B} = 0$ in D - D(B,S) with the boundary data of B_n on ∂D and σ on S. (That is $\vec{n} \cdot \vec{B} = B_n$ given where $\vec{n}$ is the outer normal of ∂D); β is a parameter that measures the distortion from a vacuum field $\vec{B}_o$, solution of $\nabla \cdot \vec{B}_o = 0$, $\nabla \times \vec{B}_o = 0$, $\vec{n} \cdot \vec{B}_o = B_n$ on ∂D. It is assumed that $\beta \ll 1$. With $\vec{j}(\vec{x}) = \sigma(\vec{x}) B(\vec{x})$ in D(B,S) and $\vec{j}(\vec{x}) = 0$ in D - D(B,S) the solution has the representation $\vec{B} = \vec{B}_o + \beta \vec{B}_1$, $\vec{B}_1 = \frac{1}{4\pi} \int \nabla \frac{1}{r} \times \vec{j}' \, d\tau' + \frac{1}{4\pi} \int \nabla \frac{1}{r} \times \vec{\gamma}' \, ds'$ (f' means $f(\vec{x}')$ and $\vec{x}'$ is the variable of integration) where $\vec{\gamma}$ is the solution of the integral equation on ∂D $\vec{\gamma} + \frac{1}{2\pi} \int \vec{n} \times (\nabla \frac{1}{r} \times \vec{\gamma}') dS' = - \frac{\vec{n}}{2\pi} \times \int \nabla \frac{1}{r} \times \vec{j}' \, d\tau'$. This form was given by H. Villat to the solution of the vortex problem of Poincare-Steklov (in terms of $\vec{j}$ and $\vec{B}$ this problem amounts to solve the same equations for $\vec{j}(\vec{x})$ given in D and $B_n = 0$ on ∂D)[1]. This representation allows to show by expansion the existence of a solution when the field $\vec{B}_o$ is regular (does not vanish in D) for small enough β. If B_n has uniformely λ-holder continuous first derivatives and σ has uniformely bounded derivatives on ∂D then $\vec{B}$ has uniformely ν-holder continuous first derivatives in D ($\nu < \lambda$).

[1] R. Kress-Arch-Rational Mech. anal. <u>30</u> (1968) 381-400.

Containment in Open-Ended Systems in the Presence of Fluctuating Electric Fields*

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The lack of constancy of the energy and magnetic moment due to fluctuating electric fields need not drastically effect the containment properties of mirror traps. If the amplitude of the fluctuating electric fields are small enough so that a particle E x B drifts only a fraction of a wavelength in a cyclotron period, then in addition to this drift the main effect of the electric fields is to slowly modulate the Larmor motion. This slow modulation can be described by the equations of motion averaged over a Larmor cycle. In the important case that the fields have (a) frequency close to a harmonic of the gyro frequency (b) long wavelength along the external magnetic field and (c) short wavelength across it, these time averaged equations are shown to have Hamiltonian form and are periodic. Hence, an adiabatic invariant for the perpendicular motion exists, this invariant reducing to the usual magnetic moment in the limit when the fluctuating electric fields are zero. Motion along the lines of force, for the averaged equations, implies a second constant of the motion which corresponds to the time averaged energy in the frame of reference rotating with the cyclotron frequency. Using these two constraints, single particle ion motion in a mirror system is examined and it is shown that small amplitude waves induce large magnetic moment excursions. This implies that unstable cyclotron waves which depend sensitively on the recurrent phase relations of the simple Larmor cycle will be stabilized at low amplitudes. However, these constraints also suggest that when the fluctuating fields vanish well inside the magnetic mirror field maximum, particle containment in the presence of quasi-stationary waves is affected only as an ion transits a "beach region" where the frequency of the waves is extremely close to an integral multiple of the ion cyclotron frequency. This is a result of the fact that only in these resonance regions is the particle motion "irreversible." Such a local process minimizes the overall effect of the large magnetic moment excursions and greatly reduces expected particle loss rates.

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Effect of Field Asymmetries on Neoclassical Confinement in Tokamaks*

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Experiments are being carried out on the Princeton ST tokamak on the influence on confinement of local increases in toroidal field.[1] We have considered the effect of such a symmetry on neoclassical transport theory. The usual axisymmetric field is used but a local increase in field δB is simulated by imposing specular reflection at $\phi = 0$ for particles with $\varepsilon - \mu B_0 < \mu \delta B$; here ε and μ are the energy and magnetic moment, and $B = B_0[1 - (r/R)\cos\theta]$. In contrast to the axisymmetric problem it is found necessary to solve for a curious boundary layer between trapped and untrapped particles: this solution is developed by the Wiener-Hopf method. In the low collision frequency regime both the radial diffusion and the transport due to the driving E-field[2] (trapped particle pinch effect) are strongly modified. Surprisingly, in the case of small rotational transform and $\delta B/B \gg r/R$, the latter becomes a particle flux of order $F \simeq (nE/B)(r/R)(B/\delta B)^{3/4}(v_T/\nu R)^{1/2}$, being therefore of increased magnitude at low collision frequency, and radially <u>outward</u> in direction.

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A Lagrangian Theory for Nonlinear Wavepackets in a Collisionless Plasma*

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A rather general method for treating the nonlinear interaction between a wavepacket and a collisionless plasma is presented. The method is applicable to a wide class of waves, including laser pulses, but the specific wave investigated is the electron plasma wave. The method consists in representing the potential and particle disturbances in terms of slowly-varying (in space and time) complex amplitudes for the fundamental and harmonics, substituting these into the Low Langrangian for the plasma, averaging, and simplifying by elimination of all amplitudes but that of the fundamental of the wave potential. The resultant Lagrangian gives rise to a nonlinear Schrodinger equation for the wave potential. The nonlinearity arises both from the purely nonlinear frequency shift and from quasilinear modification of the linear dispersion relation. The latter effect includes nonlinear Landau damping, which is shown to lead to instability of the wave envelope, thus suggesting that particles moving with the group velocity can lead to break up and dissipation of nonlinear wavepackets. The matrix element for nonlinear Landau damping extracted from this theory is shown to agree in the appropriate limit, with that of previous theories.

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Shock Wave Structure in a Two Component Plasma

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Nonlinear fluid theory equations have been used in many instances to discuss the structure of the leading edge of a shock wave, or in the absence of dissipation, a solitary pulse.[1] Recently, the use of a light ion tracer as a diagnostic has been reported[2] and offers the possibility of a detailed examination of the mechanisms involved in collisionless shock structure. In this paper we investigate the effect of the addition of a light ion contaminant on the previous treatments of shock structure referred to in (1).

The nonlinear coupled fluid and kinetic equations are solved, the light ions treated kinetically and the heavy ions as a fluid, and the resulting shock structure discussed as a function of T_e/T_i, mass ratio, and contamination. The two-fluid limit of cold light ions is also obtained. The analysis leads to a new critical Mach number, M_e as a function of the percentage light ion contamination, the ratio T_e/T_i, and the mass ratio of the two ion species. It describes the transition between light ion reflection by the shock front and the penetration of the front by the light ions. Significant changes in shock amplitude and leading edge profile occur at this critical mach number, particularly at large values of T_e/T_i. Shocks of greatly differing amplitudes (as much as a factor of 10 for an A-He plasma) may be found for the same values of light ion contamination and mach number by varying T_e/T_i.

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Evolution of Unstable Bernstein Modes in Astron Plasma*

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Using the linearized Vlasov equations for the Astron system we investigate the stability of the Bernstein[1] modes of the background plasma as a result of interaction with the relativistic E-Layer. We find that this interaction occurs at those cyclotron harmonics of the beam which lie in a range above each harmonic of the background electron cyclotron frequency. For the Bernstein modes between the first and second electron harmonics the interaction occurs in the frequency range between the electron cyclotron frequency and the upper hybrid frequency. Considering the system to evolve from zero plasma density the critical density necessary for the onset of an instability at a fixed beam harmonic is calculated. The growth rate is also calculated as a function of the density and it is shown that at each beam harmonic the growth rate decreases rapidly with increasing background density. These results agree favorably with recent experimental observations in Astron[2].

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A Schrödinger Description of Non-Adiabatic Particle Loss and of Magnetic Surfaces

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A model is obtained which describes the non-adiabatic loss of particles from magnetic mirror traps through Schrodinger-like equations

$$\frac{-i\bar{\mu}}{n}\frac{\partial\psi(n)}{\partial t} = -\frac{1}{2m}\left(\frac{\bar{\mu}}{n}\right)^2\frac{\partial^2\psi(n)}{\partial x^2} + (\bar{\mu}\Omega)\ \psi(n),\quad n = 1,2,3,\ldots$$

with the probability density G(x,t) being given by

$$G(x,t) = \sum_n \psi^*(n)\psi(n) \ ;$$

here the role of n is played by the first action invariant $\bar{\mu}$. The process of non-adiabatic loss is thus found to be analogous to the tunnelling of potential barriers in quantum mechanics, the potential $(\bar{\mu}\ \Omega)$ here being the potential for the adiabatic motion. The predictions of this model for the life times compare very well with experimental results. An application of these methods for the study of the structure of magnetic surfaces is indicated.

Decay of Pair Correlations in a Classical Fluid

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The spatial Fourier components of the pair correlation function of a classical fluid oscillate in time over a considerable range of wave vector and time, lending credence to an analysis in terms of appropriate normal modes. The decay of these modes can be described heuristically by a phase mixing of the corresponding oscillators, and results in an anomalous behavior for plasmas, from the point of view of simple Vlasov theory. To approach the problem systematically, it is convenient to focus attention upon the particle density and upon a suitably defined microscopic velocity potential. A straightforward singular perturbation theory about the independent modes can then be carried out to ellicit the long time behavior. However, the equal time values of the velocity potential correlations are required as input data. It is pointed out that the assumption of universal Gaussian correlation both reproduces the leading order perturbation theory results and allows direct computation of the relevant equal time correlations.

Neoclassical Electrical Conductivity of Plasmas with Trapped Particles

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Plasmas in strong, spatially inhomogeneous magnetic fields have a lower electrical conductivity than plasmas in homogeneous fields. For such plasmas a solution method for the kinetic equation with Fokker-Planck operators is given. The non-local electrical conductivity is calculated and compared with previous results.

*Visiting Scientist

Self Consistent Field Theory of Relativistic Electron Rings

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Self consistent compression of a thin relativistic electron ring by an azimuthally symmetric confining field is considered. The relativistic particle Hamiltonian is expressed in terms of an effective potential in two dimensions, a function of the constant canonical angular momentum and the vector and scalar potentials which are determined self consistently.

For a neutralized beam the self consistent ring radius is found to be smaller than the low β value (contrary to naive intuition), while for a pure electron ring the radius expands. The particle energy always decreases due to collective effects. The variation of the ring cross section during compression is estimated via an adabatic invariant associated with the oscillating particle in the effective potential well. Curves describing the variation of these quantities in terms of beam and field parameters have been obtained numerically. Implications to possible fusion devices are discussed.

Magnetically Insulated Ultra-High Voltage Transformer and Controlled Fusion Research

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If the magnetic field insulation principle[1] is applied to a transformer very high voltages seem to be attainable[2]. With the aid of such a transformer continuous intense streams of relativistic electron beams could be extracted from the high voltage terminal by the field emission process. These electron streams could be efficiently used for heating a plasma confined in a magnetic field, especially for solving the energy injection problem in open ended systems. In this case the thermonuclear fuel injection would have to be simultaneously accomplished, which could be done most expediently by low-energy large-mass injection with a catapult, propelled by a superconducting magnetic travelling wave macron accelerator[3].

The electron streams produced with such a transformer could be also used to charge up the proposed superconducting levitated ring capacitor[1], making possible the production of intense relativistic electron beams with energies exceeding anything so far achieved and thus raising the prospect of thermonuclear micro-bomb ignition[1,4].

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Numerical Simulation of Strong Plasma Shock Waves Produced In An Electromagnetic Shock Tube*

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A set of two-fluid Navier-Stokes equations with classical physical transport coefficients is used to compute the evolution and structure of collisional plasma shock waves in an electromagnetic shock tube. Shock speeds up to ~ 200 cm/μ sec and shocked plasma temperatures of the order of kilovolts are studied. A strong transverse bias magnetic field is employed, which significantly alters the size and shape of the shock profiles, when compared to the zero bias field case. The wave structure is different from that of a two-fluid gas dynamic shock without transverse magnetic field. Especially significant is the effect of the small ion Larmor radius in reducing the gas dynamic shock thickness by at least an order of magnitude in the transverse shock case, which permits collisional shocks to have thicknesses much smaller than the post-shock mean free path. These collisional shock waves produce very hot ions, the ion temperature increasing with the shock speed and the formation time and distances also increasing substantially with shock speed. Generally the results compare favorably with electromagnetic shock tube experimental data.

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The Role of Trapped Particles in Plasma Instabilities*

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We report on a number of Numerical experiments dealing with the role that trapped particles (TP) play in the saturation, damping and generation of plasma instabilities. It is found that returning the TP to the main body of the distribution eliminates amplitude oscillations and side band growth for both Landau damping and beam plasma instability. A redistribution of TP over the trapping region of phase space leads to significant changes in damping rates and saturation levels, and can also alter the phase of the amplitude oscillation.

*Work supported in part by the AEC and NSF.

Low Frequency Heating*

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Several problems relevant to the low frequency heating of devices, such as doublet, having high magnetic shear will be considered.

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Theory and Simulation of the Beam Cyclotron Instability

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The relative drift of electrons and ions across a magnetic field gives rise to electron cyclotron instabilities.[1-4] We present the results of computer experiments for such modes and an analysis which explains the salient features of the observations. Exponential quasilinear heating follows the onset of instability. An electrostatic energy in the wave fields builds up, the electron cyclotron resonances are broadened by turbulent scattering.[5] At a level of turbulence $\sum_k |E_k|^2/4\pi N_o T_e(0) \sim (\Omega_{ce}^2/\omega_{pe}^2)[T_e(t)/T_e(0)]^{1/2}(\Omega_{ce}/kV_{drift})$, where $T_e(t)$ is the electron temperature at time t, the effective collision frequency ν_e becomes comparable to Ω_{ce}, and a transition is made from the beam cyclotron instability to the ion acoustic instability. The system then continues to evolve exponentially, but now at the slower acoustic rate, until the fluctuating fields become strong enough to trap the ions and stabilize the instability. The final temperatures of the electrons and ions are given, e.g., $T_e \underset{\sim}{<} (5/16)^2 MV_{drift}^2$, and the important consequences for perpendicular magnetosonic shocks of the above behavior are discussed.

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*On leave of absence from Cornell University.

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Electron Vortices and Anomalous DC Resistivity*

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In computer simulation we have found that a bifurcated electron velocity distribution (two-symmetrical electron beams) results in a substantial transfer of energy to the ions. The energy transfer is a very weak function of the electron ion mass ratio. In simulations employing realistic electron-ion mass ratios we have observed a transfer to the ions of up to 15% of the total electron beam energy. A simple theory explains these results in terms of electron vortices in phase space created by the strong electron-electron instability. These vortices give rise to a number of other interesting phenomena, including an anomalous dc resistivity. When we apply a small dc field to this system, we observe an anomalous resistance for the lifetime of the vortices. The current for a time hangs up at a value much less than that determined by the electron thermal velocity (this value was found in previous simulations of plasmas driven by dc fields).[1] We present a simple picture of how vortices can effectively cause this behavior.

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Relativistic Particle Motion in Super-Intense Laser Beams*

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Recently, it has been proposed that cosmic rays can be accelerated to high energies in the intense low-frequency electromagnetic waves radiated by pulsars. The condition for relativistic acceleration is $\omega_H > \omega$ where $\omega_H = eB/mc$ is the cyclotron frequency in magnetic field B of the wave and ω is the wave frequency. The same conditions can be achieved in the laboratory by focusing laser beams with powers of order 10^{10} watts, or more, down to a few wave lengths.

We have examined charged particle motions in focussed laser pulses. The particle motion can be separated into a guiding center motion and a figure eight gyration analogous in many ways to particle motion in a magnetic field. Using three results, we discuss conditions under which effective acceleration of MeV electrons can be achieved in the laboratory. Such an acceleration may have possible application to Astron.

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Toroidal Effects on Kink Modes in Tokamaks*

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Considerations of future proposed tokamaks point to the desirability of achieving smaller aspect ratios R/a and lower values of the safety factor q. Previous investigations of tokamak kink stability[1] have been carried out in cylindrical geometry, thus neglecting the magnetic well effects of toroidal geometry and the possible coupling of modes of differing m numbers. The present calculation is based on the tokamak equilibrium expansion of Greene, Johnson, and Weimer,[2] which has been carried out to third order a/R. To go beyond the cylindrical stability theory requires an expansion to fourth order in a/R as in the results of Ware and Haas[3] who examined tokamak flute stability. The present calculations indicate the relation between β_θ, q, a/R, and various higher order equilibrium properties for pressure profiles which differ in higher order from parabolic. The results are obtained by assuming the second order neutrality requirements are satisfied and then finding corrections to these criteria.

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Computer Simulation of Drift-Cyclotron Instability*

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A study of the drift-cyclotron instability and the associated plasma diffusion by computer simulation has been started. The model used is electrostatic, two and a half dimensional, two species with uniform external magnetic field. The code is optimized and with 16,500 particles of each species on a 32 × 32 grid takes 0.4 8 seconds to complete one time step. The plasma has an initial density gradient in the x direction with the guiding center distribution ~ $\exp(-\kappa x)$ and is uniform in y direction. The velocity distribution of the ions and electrons are Maxwellian with the same temperature. This differs from the recent results by Birdsall et al,[1] who examined the drift cyclotron instability for monoenergetic ions and cold electrons. When $k_{\parallel} = 0$, an instability arises due to the coupling between negative energy ion cyclotron harmonics and the electron drift wave with a maximum growth rate of $\gamma/\omega_{ci} \approx 0.5$ for $(\omega_{pi}/\omega_{ci})^2 = 25$, of $\kappa\rho \approx 1$, and $m_i/m_e = 4$. All unstable modes grow with growth rates close to the predicted value. The most unstable mode reaches a maximum amplitude of $e\Phi/kT \approx 40$ at $\omega_{ci}t \approx 30$, followed by oscillations of its amplitude. With the development of large waves, electrons and then ions diffuse and spread out in the x direction. However, the large-amplitude waves in the y direction cause a large density modulation in that direction producing an electric field which in turn causes electrons to drift in x direction and to some extent restores the original density profile.

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Analysis of Nonlinear Plasma Waves

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The exact solution to the Vlasov equation can be expressed in terms of an exponential operator, operating on the initial velocity distribution. If we take small time-steps, this exponential operator can be approximated by a non-linear polynomial operator. The latter is used to obtain analytically at subsequent times, the velocity distribution, the electric field, the non-linear Landau damping or amplification, and the diffusion of the plasma. The plasma will occupy a finite one-dimensional region with assigned boundary conditions; the motion of the ions will be neglected.

Starting with a single wave number and initial velocity distribution containing a cubic power small disturbance, the analysis readily produces all wave numbers. By expressing non-linear exponential functions in terms of infinite series of Bessel and circular functions, we can analyze exactly the non-linear mode-mode coupling among all the wave numbers. The boundary conditions will be brought in exactly. The change in shape and width of the disturbance will be demonstrated. Diffusion and nonlinear damping or amplification is studied.

Energy Principle with Given Distributed Inductance

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Holding the shape of a set of nested flux surfaces fixed, we study the variation of the ideal MHD potential energy with respect to rearrangements of the flux and pressure profiles. The constraint on the shape of the flux surfaces is incorporated by writing $\int dr \frac{1}{2} B^2$ as an integral over a bilinear form of flux and the inverse of the differential inductance matrix. The inductance matrix is held fixed while the flux profile is varied. A number of thermodynamic models are used in the variation of the pressure profile; each model gives a necessary stability condition. For the models considered, and for a diagonal inductance matrix, the state of minimum energy is a constant-pressure sharp boundary plasma with only longitudinal current. An azimuthal current increases stability with respect to perturbations in the flux profile; smoothing the pressure profile can decrease stability.

Hydromagnetic Stability of Bifurcated Equilibria

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Helically symmetric hydromagnetic equilibria are considered in a cylindrical domain. These non-unique solutions may be regarded as bifurcated from a straight pinch. Such a bifurcated equilibrium is generally unstable unless the bifurcation point occurs at the margin of stability, in other words, at a higher order confluence of bifurcating equilibria. In the latter case, we show that the bifurcating equilibrium is stable when the sharp plasma vacuum interface has a long helical wavelength.

First we expand the equilibrium solution in a small parameter, the amplitude of the corrugation of the plasma-vacuum interface. Then we identify this class of helically symmetric equilibria as bifurcated equilibria and discuss a necessary condition for the bifurcated equilibrium to be stable. Finally using the energy principle for hydromagnetic stability we study the stability of the bifurcated equilibria.

It is an open question whether the new class of bifurcated equilibria is stable when the amplitude of the corrugation is large and when the helical wavelength is not large.

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