

## An Upper Bound on the Dissipation Rate of Hartmann Flow

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### Abstract

The dissipation rate of Hartmann flow is studied using dissipative, incompressible MHD. The ‘energy stability’ method is used to derive explicit expressions for nonlinear stability regions in parameter space, which are compared with numerically derived results. An upper bound on the total energy dissipation rate, equivalently the drag coefficient, is obtained by using the ‘background method’ (Doering & Constantin, *Phys. Rev. Lett.* **69**, 1648–1651 (1992).) For small Reynolds numbers the bound is proportional to the laminar dissipation rate. For Reynolds numbers large enough for turbulence, the dissipation is shown to be bounded by a function of the magnetic Prandtl number. The function is linearly increasing if the Prandtl number is larger than one and constant if it is smaller than one in agreement with the bound derived using the same method for magnetic sheared MHD flows in (Pétrélis et al. Sherwood 2003).