Paleoclassical Electron Heat Transport In Toroidal Plasmas

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An important new mechanism for radial electron heat transport has recently been identified [1,2]. It results from a combination of parallel free-streaming and the "paleocalassical" Coulomb collisional processes of parallel electron heat conduction and magnetic field diffusion in low collisionality toroidal plasmas. The key new physical point is that as magnetic field lines diffuse radially (with $D_\eta \equiv \eta/\mu_0 = \nu_e\, c^2/\omega_p^2$), they carry electron heat equilibrated over a long distance $2L$ along magnetic field lines. The parallel half-length L is the minimum of the electron collision length λ_e and a maximum effective field line length which is the minimum of the half-length of a low order rational field line ($2\pi R_0 q\, n^\circ$) or an electromagnetic-skin-depth- and magnetic-shear-determined length $\ell_{\max} \equiv \pi R_0 q\, n_{\max}$ in which $n_{\max} \equiv (\pi\delta_e q')^{-1/2}$ is a maximum toroidal Fourier harmonic number, $\delta_e \equiv c/\omega_p$ is the electromagnetic (em) skin depth and $q' \equiv dq/dr$. Since L is much longer than the poloidal periodicity half-length $\pi R_0 q$, the radial electron heat diffusivity is a multiple $M \equiv L/\pi R_0 q >> 1$ (typically $M \gtrsim 10$) times the magnetic field line diffusivity: $\chi_e^{\rm pc} \sim M\, \nu_e\, c^2/\omega_p^2$.

The evolution equation for the magnetic flux ψ is obtained from a combination of Faraday's and Ampere's laws using an Ohm's law that includes electron inertia:

$$\left(1 - \delta_e^2 \nabla^2\right) \frac{d\psi}{dt} = \frac{\eta_\|}{\mu_0} \nabla^2 \psi - \frac{\partial \Psi}{\partial t}, \quad \text{with } \delta_e \equiv \frac{c}{\omega_p} \text{ (em skin depth) and } \frac{\partial \Psi}{\partial t} \equiv E_z^A. \tag{1}$$

Here, E_z^A is the inductive toroidal electric field, which, in resistive equilibrium, balances the $\eta_\|$-induced magnetic field diffusion. When $\delta_e^2 \nabla^2 << 1$ (i.e., for $x > \delta_e$) ψ and hence the magnetic field lines diffuse with a diffusion coefficient $D_\eta \equiv \eta_\|/\mu_0 \simeq \nu_e \delta_e^2 \sim (\Delta x)^2/\Delta t$, which implies diffusive steps $\Delta x = \delta_e$ in an electron collision time $\Delta t = 1/\nu_e$. For $x < \delta_e$ ($\ell_\| > \ell_{\max}$) the solution of (1) for ψ becomes spatially evanescent.

Radial transport of electron heat "contained" on diffusing magnetic field lines is calculated using a Chapman-Enskog approach [3]. In the paleoclassical model magnetic field diffusion occurs on the same time scale as parallel free-streaming and heat conduction of electrons. This effect is taken into account by transforming from a stationary coordinate system to one moving with the magnetic flux function ψ: $\partial/\partial t|_{\mathbf{x}} = \partial/\partial t|_\psi + \partial\psi/\partial t|_{\mathbf{x}} \partial/\partial\psi$, with $\partial\psi/\partial t|_{\mathbf{x}}$ from (1). Then, the kinetic equation for the kinetic distortion F away from a dynamic Maxwellian f_M has an additional $-\partial\psi/\partial t|_{\mathbf{x}} \partial f_M/\partial\psi$ "source" or drive term. To lowest order F and T_e are nearly constant along magnetic field lines [2] — electrons and their heat are frozen to the magnetic field lines and hence flux sufaces which, according to (1) with $\delta_e^2 \nabla^2 << 1$, are diffusing.

The divergence of the conductive parallel electron heat flow induced by $\partial\psi/\partial t|_{\mathbf{x}} \neq 0$ in the presence of a radial electron temperature gradient is [1,2] $\nabla_\| q_{\|e} \simeq -[n_e/(2A_{01})] \partial\psi/\partial t|_{\mathbf{x}} dT_e/d\psi$ at all points along a magnetic field line whose effective length is $2L$. Integrating it from $-L$ to L and dividing by the field line poloidal periodicity length $2\pi R_0 q$ (due to the inverse ballooning transform [4] in axisymmetric toroidal geometry), the effective electron temperature evolution equation $(3/2) n_e dT_e/dt = -\langle \nabla_\| q_\| \rangle + \cdots$ becomes

$$\frac{3}{2}\frac{dT_e}{dt} = M\left(\frac{\eta_\|}{\mu_0}\nabla^2 T_e - E_z^A \frac{dT_e}{d\psi}\right) + (\chi_e^{\rm nc} + \chi_e^{\rm anom})\nabla^2 T_e + \frac{Q_e}{n_e} \quad \Longrightarrow \quad \chi_e^{\rm pc} \equiv M\frac{\eta_\|}{\mu_0} \sim \left(\frac{L}{\pi R_0 q}\right)\nu_e \frac{c^2}{\omega_p^2}. \tag{2}$$

This is a diffusion equation for the electron temperature T_e with a paleoclassical diffusion coefficient $\chi_e^{\rm pc}$; it relaxes a factor of M faster than ψ does. Usually $dT_e/d\psi < 0$ and the second term on the right in (2) is positive; it apparently causes the often-observed [5] electron "heat pinch."

The paleoclassical model of radial electron transport is in reasonable accord [1,2] with many features of "anomalous" electron heat transport, including: 1) Magnitude (e.g., $\chi_e^{\rm pc} \sim 2$ m^2/s for TFTR ohmic plasmas at $r/a \simeq 0.5$), 2) $\chi_e^{\rm pc}$ increases with r, 3) Alcator confinement scaling ($\propto n_e$) and magnitude, 4) Profile resiliency and larger transient than equilibrium transport (because the heat pinch partially cancels $\chi_e^{\rm pc}$ diffusion), and 6) Reduced transport and internal transport barriers at low order rational surfaces [6].

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