Chandrasekhar equilibria of compact toroids with Alfvénic flows

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Abstract

The Chandrasekhar equilibria are a class of stationary ideal magnetohydrodynamic equilibria stabilized by magnetic-field-aligned Alfvénic flows against fixed-boundary ideal MHD modes with a solenoid-free perturbation in the displacement vector.[1] The basic idea of the Chandrasekhar equilibrium is to have the inertia of a field-aligned Alfvénic plasma flow balance the magnetic curvature force plus the parallel gradient of magnetic pressure,

$$\rho \mathbf{U} \cdot \nabla \mathbf{U} = \frac{1}{\mu_0} \mathbf{B} \cdot \nabla \mathbf{B}. \tag{1}$$

The overall force balance is then reduced to

$$\Pi \equiv p + B^2/2\mu_0 = constant, \tag{2}$$

for a steady state plasma. The stabilization effect of a field-line-aligned Alfvénic flow is not limited to ideal modes. For example, in the Chandrasekhar equilibrium of a Harris sheet, the Alfvénic flow is found to stabilize the resistive tearing modes.[2] Unlike the Grad-Shafranov equilibrium of a static plasma where two free functions (of magnetic flux), usually pressure and plasma current, are required to specify an equilibrium, the specification of a Chandrasekhar equilibrium admits total freedom in magnetic field design. For any specified $\mathbf{B}$, the plasma flow is solved from equation (1) and the corresponding plasma pressure is found from equation (2). Analytic Chandrasekhar equilibrium of FRC and spheromak can be elegantly solved[3] with Chandrasekhar-Kendall modes[4] from a field constraint of the form

$$\sum_n c_n \left(\nabla\times\right)^n \left(\mu_0 \mathbf{J}\right) = \lambda \mathbf{B}$$

where c_ns $(n = 0, 1, 2, \cdots)$ are constant coefficients and $(\nabla\times)^n \equiv \overbrace{\nabla\times\nabla\times\cdots\nabla\times}^{n}$. Favorable confinement property of nested closed flux surfaces and the unusual ideal magneto-hydrodynamic stability of such compact toroids are of interest for both magnetic trapping of high energy electrons in astrophysics and confinement of high temperature plasmas in laboratory. The similarity and important distinctions between Chandrasekhar compact toroid equilibria[3] and those predicted by relaxation theories[5] will be clarified. This work was supported by DoE OFES.

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