Stability Study of the Hall-MHD Model

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Because of a growing interest in plasmas with mass flow and its effects on various aspects of fusion studies such as turbulence, reconnection and equilibrium states, it becomes important to investigate the effects of two-fluid plasma model. We base our theory on Hall magnetohydrodynamics(HMHD), which encompasses single fluid ideal magnetohydrodynamics(MHD) with the addition of the Hall term. We have initiated a thorough study of linear stability of HMHD plasmas by analyzing, in particular, the effect that the Hall term bears on the stability of equilibria with no flow, which are also equilibrium states of classical MHD.

The linearized equations are cast in a Hamiltonian form, similar to the classical form for MHD plasmas with flow [1]. However, this requires the introduction of two displacement vectors, ξ_i and ξ_e. The Hamiltonian form enables easy comparison with MHD results. For example, subject to a mild additional restriction, such as incompressibility, we show that an MHD stable plasma is also HMHD stable. We develop general stability criteria as well as a bound on the complex eigenfrequencies of the system and their location in the complex plane. The bounds indicate that the phenomenon of overstability is the common case.

We examine with particular attention the theory of ballooning modes. From the ballooning expansion, we derive an equation that involves four displacement components, two each for ξ_i and ξ_e. We find that if ideal MHD is stable with regard to ballooning modes, then HMHD is also stable. We then concentrate on the special case of a cylindrical screw pinch. Stability involves five parameters, rather than just s and α, the shear and pressure gradient, respectively. This case is analytically tractable and leads to a fourth order algebraic equation for the eigenmodes. We determine that if ideal MHD is unstable with regard to ballooning modes, then HMHD is also unstable and the growth rate of the instability is actually increased by the Hall term. We are in the process of exploring the existence of a stability criterium for this special case. Also, for the general case we plan on studying the stability properties of HMHD ballooning modes by solving numerically the ballooning equations.

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References

[1] E. Friedman and M.Rotenberg Rev. Mod. Phys. **32**, 898 (1960).

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