Hall-MHD axisymmetric equilibria with flow

G. N. Throumoulopoulos*, H. Tasso†

* University of Ioannina, Association Euratom - Hellenic Republic, Section of Theoretical Physics, GR 451 40 Ioannina, Greece

† Max-Planck-Institut für Plasmaphysik, Euratom Association, D-85787 Garching, Germany

Abstract

The equilibrium of an axisymmetric magnetized plasma is investigated within the framework of the Hall-MHD model with the convective flow term included in the momentum equation for either barotropic equations of state, $P = P(\rho)$ (P and ρ are the plasma pressure and density, respectively), or incompressible flows. For barotropic plasmas the electron diamagnetic term is also kept in Ohm's law. Unlike in the single fluid MHD model, e.g. [1,2], the magnetic surfaces depart from the ion-flow surfaces which coincide with the surfaces of generalized vorticity, $\mathbf{\Omega} = \mathbf{B} + \lambda \nabla \times \mathbf{v}$ ($\mathbf{B}$, $\mathbf{v}$ and λ are the magnetic field, the velocity, and a dimensional constant associated with the Hall term, respectively). The equilibrium is governed by a PDE for the poloidal magnetic flux function ψ coupled with a PDE for the poloidal ion-flow flux function G, two algebraic ψ-integrals and two algebraic G-integrals. For incompressible flows the density becomes constant on the ion-flow surfaces. Several particular cases are examined, i.e."force-free" ($\mathbf{j} \parallel \mathbf{B}$, where $\mathbf{j}$ is the current density), magnetic field aligned-, toroidal-, poloidal-, Beltrami-, and Trkal-flows. Unlike in the single fluid MHD model [1], steady states with incompressible poloidal flows are possible; they are isodynamic-like with pressure varying on magnetic surfaces. Exact solutions can be constructed in the aforementioned particular cases. From the solutions obtained, however, only those with constant density can satisfy boundary conditions pertinent to magnetically confined plasmas; the others exhibit hollow pressure profiles. Also, it is proved that if the flows are parallel to $\mathbf{B}$ they should be incompressible and for isothermal magnetic surfaces they become isodynamic [$B = B(\psi)$]. The latter results indicate that in many cases the Hall term leads to a restriction of the possible equilibria. However, new equilibria can also appear due to the Hall term, e.g. incompressible equilibria with purely poloidal flow.

[1] H. Tasso, G. N. Throumoulopoulos, Phys. Plasmas **8**, 2378 (1998).

[2] G. N. Throumoulopoulos, G. Poulipoulis, G. Pantis, H. Tasso, Phys. Lett. A **317**, 463 (2003).