Analytical and numerical study of the lower-hybrid-relevant wave equation in a toroidal plasma

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An analytical study of the two-dimensional wave equation, which describes the lower hybrid wave propagation in tokamak plasmas and general magnetic field equilibrium, has been performed by means of a multiple spatial scale approach, based on the Poisson Summation formula. This technique is strictly related with that discussed earlier in Ref. [1] and, when applied to plasma instabilities, reduces to the well-known "ballooning formalism" [2]. In applying this formalism, the starting equation for the scalar potential in the cold plasma limit reduces to two nested one-dimensional equations: the first governs the wave structure along the magnetic field lines, while the second describes the slow radial dependence of the wave envelope, for which the usual WKB asymptotic technique is applied. This mixed WKB-full-wave approach can be entirely justified on the basis of spatial scale separation in the radial direction and, for waves that have parallel group velocity faster than in the perpendicular direction (e.g. the lower hybrid wave). Choosing □ as the dual variable to $qR_0k_{||} = m + nq$ with respect to the Fourier transform (where q is the safety factor, R_0 is the major radius of the torus, $k_{||}$ is the parallel wave-vector, and m and n are, respectively, the poloidal and toroidal mode number), i.e., as a dimensionless coordinate along the magnetic line of force, a 1D ordinary differential equation (ODE) for the scalar potential in □□space is derived and numerically solved. The operator $□_k = k_r / nq□$, acting on the envelope amplitude $A(r)$ can be considered as a parameter (the normalized radial wave-vector). When solved, the 1D ODE along the field line coordinate yields a local dispersion relation connecting r with the normalized radial wave-vector , which can be solved numerically. The wave dynamics in the radial direction is determined by a system of two ordinary differential equations (ray tracing) for the position and the radial wave-vector, whose Hamiltonian function is the previously determined dispersion relation. The system of ray tracing equation describes the WKB evolution of the envelope and, for this reason, it could be referred to as envelope tracing. Preliminary results from numerical solutions of the full 2D wave equation in a pseudo-toroidal coordinate system are also presented. Here, we use a spectral approach based on the Fourier analysis of the scalar potential in the periodic variables □ and □. The starting partial differential equation (PDE) for the potential is replaced by a system of coupled ordinary differential equations along r, which can be numerically solved by standard methods. The complete solution can be finally obtained by numerical methods based on the Fast Fourier Transform (FFT). Well-known characteristic behaviors of the lower-hybrid wave propagation, such as resonance cones of a Gaussian beam in a cold plasma etc., are studied as benchmarks of the numerical solution.

A discussion of the full numerical solution when compared to the previously described analytical study is also presented.

[1] F. Zonca and L. Chen, Phys. Fluids B**5**, 3668, (1993)

[2] J. W. Connor, R. J. Hastie and J. B. Taylor, Phys. Rev. Lett. **40**, 396, (1978).