Hamiltonian description of ITG dynamics and coherent structures

F. L. Waelbroeck, P. J. Morrison
Institute for Fusion Studies, University of Texas

The Hamiltonian formalism constitutes an effective framework for investigating the dynamics of fluid models.[1] In particular, it provides techniques for finding conserved quantities, obtaining first integrals of the equilibrium equations, and constructing variational principles describing the stability of nonlinear coherent structures. It can also be used to guide the derivation of fluid closures by specifying the subset of higher order terms that need to be retained in order to preserve desired conservation properties. More recently, the Hamiltonian formalism has been used by Krommes and Kolesnikov to derive equations governing the generation of zonal flow and long wavelength coherent structures under the effect of stochastic forcing by short wavelength modes.

A particularly appealing feature of the Hamiltonian formalism is that it readily provides first integrals of the equations governing the structure and propagation of modons, or solitary waves. Modons usually consist of isolated or paired vortex tubes propagating in the direction perpendicular to both the magnetic field and the equilibrium density gradient. They have been shown to have remarkable resilience. Their properties are generally consistent with the observation in simulations and experiments of patterns of flow or density perturbations that enjoy a lifetime substantially exceeding the correlation time for the turbulence. Such patterns, called coherent structures, are thought to play an important role in turbulent transport. In particular, they give rise to intermittency and non-Gaussian statistics, and they determine the asymptotic behavior of the turbulent spectra. They have been clearly identified in observations of edge turbulence, where their effect on the erosion of plasma-facing components is a source of concern. In the confinement region, coherent structures are responsible for avalanches and have been observed as radially extended coherent signals in the electron cyclotron emission.

Here we construct a Hamiltonian formulation of the equations governing the dynamics of the ion temperature gradient (ITG) instability with finite ion Larmor radius (FLR). We use as a starting point the model of Kim, Horton and Hamaguchi (henceforth KHH) describing electrostatic turbulence driven by the gradient of the ion temperature in slab geometry.[2] We present Hamiltonian formulations for two different versions of the basic ITG model. The first corresponds to a traditional, fully adiabatic response for the electrons and the second to a more accurate parallel adiabatic response where the electron density is insensitive to perturbations that are constant on a flux surface. We derive a complete family of Casimir invariants for our models and use these invariants to construct a variational principle describing the equilibrium and stability of propagating modons. Our solution extends previous descriptions of coherent structures for ITG models by retaining both FLR and parallel flow effects.

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References

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