Accounting of Magnetic, Cross, and Kinetic Helicities in Nonlinear Two-Fluid Relaxation Simulations*

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The magnetic helicity, a well-known topological quantity that measures the knottedness of magnetic field lines, is conserved in the ideal limit for single-fluid MHD. Two-fluid models, which include Hall physics in the generalized Ohm's law, allow for separation of electron and ion flows on the order of the ion skin depth. Within the Hall-MHD model, the hybrid helicity, a weighted sum of the magnetic, cross, and kinetic helicity, is invariant in the ideal limit with cold ions [Turner, IEEE Transactions on Plasma Science, PS-14, No. 6. 1986]. Extended-MHD incorporates first-order ion finite-L'armor-radius effects through the Braginskii gyroviscous stress tensor. We show that the hybrid helicity of Hall-MHD is not conserved with warm ions.

The evolution of the magnetic helicity, cross helicity, and kinetic helicity is analyzed in numerical simulations of a driven-damped plasma pinch. The initially unstable paramagnetic pinch equilibrium undergoes a nonlinear relaxation event which brings the plasma to its characteristic reversed field state. The evolution of the helicities throughout the event are reported and a reconstruction of the coupling terms is performed.

The magnetic helicity evolution couples to the cross helicity through the electron pressure gradient parallel to the magnetic field, but this is found to be insignificant in the simulations. The cross helicity is in turn coupled to the kinetic helicity through the projections of both the Hall term and the electron pressure term onto the fluid vorticity. The coupling here is stronger, but there is competition from the gyroviscous stress term in both the cross and kinetic helicity evolution. The magnetic energy drops by ~2% during the relaxation event while the magnetic helicity is more robustly conserved and changes only by ~0.5% with a modest Lundquist number of S=20,000. The hybrid helicity is well-conserved for simulations with cold ions but is not when warm ion effects are included.

Poster Submission