TEM turbulence in stellarators - its simulation and its optimisation

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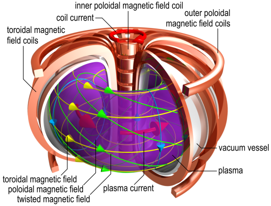
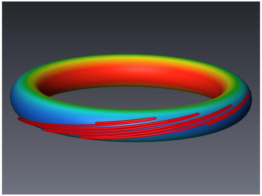
Tokamaks vs. stellarators

Tokamak

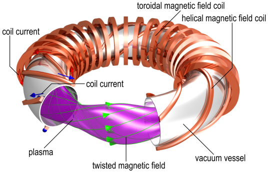


$$\frac{\mathrm{d}}{\mathrm{d}\Phi} = 0 \rightarrow p_\Phi = mrv_\Phi + qRA_\Phi = \mathrm{const}$$

trapped particles
remain on flux surface

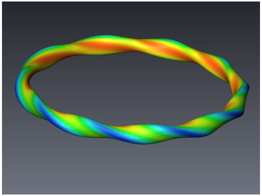


Stellarator



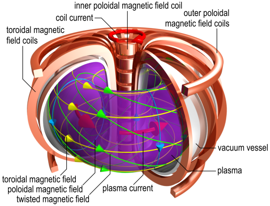
$$\frac{\mathrm{d}}{\mathrm{d}\Phi} \neq 0 \rightarrow p_\Phi \neq \mathrm{const}$$

trapped particles
drift radially outwards



Tokamaks vs. stellarators

Tokamak



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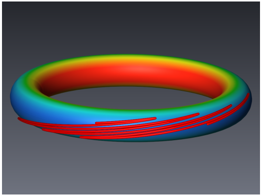
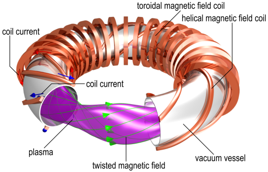
trapped particles
remain on flux surface



Stellarator



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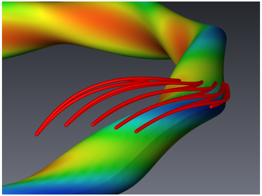
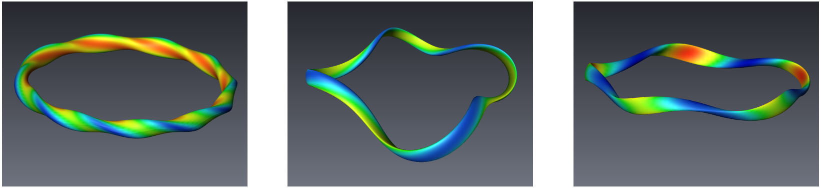
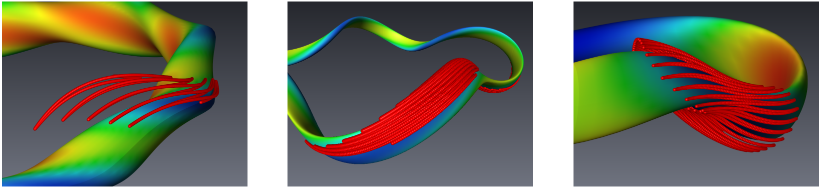


Trapped particles are radially confined in advanced stellarators

Classical stellarator e.g. Large Helical Device (LHD)	Quasi-symmetry e.g. Helically Symmetric Experiment (HSX)	Quasi-isodynamicity e.g. Wendelstein 7-X (W7-X)



Magnetic field strength $|\mathbf{B}|$ on the flux surface of $r/a = 0.5$



Trajectory of a trapped particle

Low neoclassical transport motivates turbulent transport optimisation

Neoclassical transport: already shown to be below the level of tokamaks (down to a collisionality of $\nu^* = 10^{-3}$)

Anomalous transport is expected to be the dominant transport channel for outer radii

- investigate **microinstabilities** which trigger small-scale turbulence
- Ion-temperature gradient (ITG) mode - limits ∇T
- **Trapped-electron mode (TEM)** - limits ∇n

Large configuration space in 3D: opportunity for turbulence optimisation

- What we already know from linear theory - analytically and numerically
- How these microinstabilities behave nonlinearly
- How these observations can help us to optimise stellarators for both neoclassical and turbulent transport

Outline

We can show analytically: the more particles with $\omega_{*a} \cdot \overline{\omega_{da}} < 0$ the better

- We define

$$P_e = -\mathrm{Re} \int \frac{\mathrm{d}l}{B} \int \mathrm{d}^3 v \; e \left(v_{\|} \hat{\mathbf{b}} + \mathbf{v_d} \right) \cdot \nabla \phi^* J_0 f_{e_1} \cong \mathbf{j} \cdot \mathbf{E}$$

as rate of the gyrokinetic energy transfer from the field to electrons

[Proll, Helander, Connor and Plunk, PRL 2012] and [Helander, Proll and Plunk, PoP 2013]

  - $P_e < 0$ for a destabilising influence of the kinetic electrons

We can show analytically: the more particles with $\omega_{*a} \cdot \overline{\omega_{da}} < 0$ the better

- Energy transfer rate for the electrons near marginal stability ($\gamma \to 0$)

$$P_e = \frac{\pi e^2}{T_e} \int \frac{\mathrm{d}l}{B} \int \mathrm{d}^3 v \delta(\omega - \overline{\omega}_{de}) \overline{\omega}_{de} (\overline{\omega}_{de} - \omega_{*e}^T) |\overline{J_0 \phi}|^2 f_{e0}$$

$\omega_{*e} \propto k_y \frac{\mathrm{d} \ln n_a}{\mathrm{d} r}$ - diamagnetic frequency, defined to be < 0 here

$\overline{\omega}_{de} = \overline{\mathbf{k}_\perp \cdot \mathbf{v}_{d,a}}$ - precessional drift frequency, bad curvature corresponds to < 0

- $P_e < 0$ for a destabilising influence of the kinetic electrons
- TEM rely on a resonance between the two frequencies $\omega_{*e}^T \cdot \overline{\omega_{de}} > 0$

Quasi-isodynamic configurations are stable towards TEMs

- Contours of constant $|\mathbf{B}| = |\nabla\psi \times \nabla\alpha|$ poloidally closed

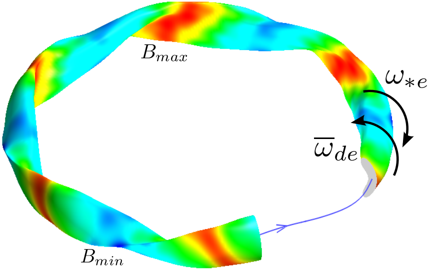
  ψ = toroidal flux, radial coordinate

  α = field line label, binormal coordinate

- bounce averaged radial drift vanishes $\overline{\mathbf{v}_d \cdot \nabla\psi} = 0$
- Action integral of the bounce motion - adiabatic invariant

  $J(\psi) = \int m v_{\|} \mathrm{d}l$

- in maximum-J-configurations with $\partial J/\partial\psi < 0$:
- favourable bounce-averaged curvature for all orbits → TEMs are stabilised



Subbotin *et al. Nucl. Fusion* **46** *2006, courtesy of Y. Turkin*

- direction of the precessional drift

$$\omega_{*a} \cdot \overline{\omega}_{da} < 0$$

$\omega_{*a} \propto k_\alpha \frac{\mathrm{d}\ln n_a}{\mathrm{d}\psi}$ - diamagnetic frequency

$\overline{\omega}_{da} \propto -k_\alpha \frac{\partial J}{\partial\psi}$ - precessional drift frequency

We can show analytically: the more particles with $\omega_{*a} \cdot \overline{\omega_{da}} < 0$ the better

- Energy transfer rate for the electrons near marginal stability ($\gamma \rightarrow 0$)

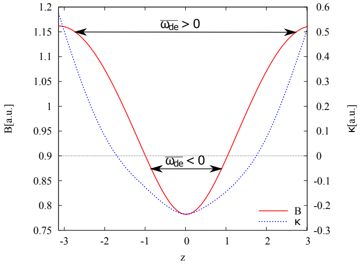
$$P_e = \frac{\pi e^2}{T_e} \int \frac{\mathrm{d}l}{B} \int \mathrm{d}^3 v \delta(\omega - \overline{\omega}_{de}) \overline{\omega}_{de} (\overline{\omega}_{de} - \omega_{*e}^T) |\overline{J_0 \phi}|^2 f_{e0}$$

ω_{*e} - diamagnetic frequency, defined to be < 0 here
$\overline{\omega}_{de}$ - precessional drift frequency, bad curvature corresponds to < 0

  - $P_e < 0$ for a destabilising influence of the kinetic electrons
  - TEM rely on a resonance between the two frequencies $\omega_{*e}^T \cdot \overline{\omega_{de}} > 0$

P_e more negative the higher the fraction of trapped particles with "bad average curvature" $\overline{\omega_{de}} < 0$

- $\overline{\omega_{de}}(\lambda) \propto \int_{z_1}^{z_2} \frac{\kappa(1-\lambda B(z)/2)}{\sqrt{1-\lambda B(z)}} \mathrm{d}z$ with local curvature κ, pitch angle λ and bounce points z_i
- if particle trapped in region of bad local curvature $\kappa \rightarrow \overline{\omega_{de}} < 0$
- worst average curvature if B and κ are in phase



Outline

Numerical Simulations

- start with realistic stellarator equilibrium created with

VMEC

[Hirshman and Whitson, Phys. Fluids 26 (1983)]

- create flux tube geometry usable by GENE with

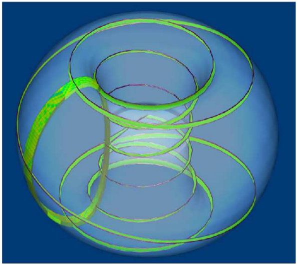
GIST

[Xanthopoulos, Cooper, Jenko, Turkin, Runov and Geiger, PoP 16 (2009)]

- perform linear electrostatic collisionless flux tube simulations with

GENE

[Jenko, Dorland, Kotschenreuther and Rogers, PoP 7 (2000)]



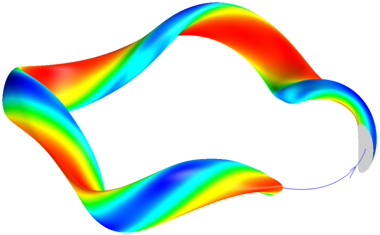
Example for a flux tube

M. Barnes, PhD thesis 2008

Simulated geometries: HSX and W7-X

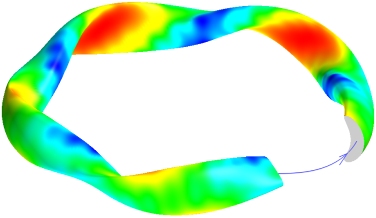
Magnetic field strength B, red $= B_{max}$, blue $= B_{min}$.

HSX



- quasi-helically symmetric stellarator
- aspect ratio: $A = 8$

W7-X



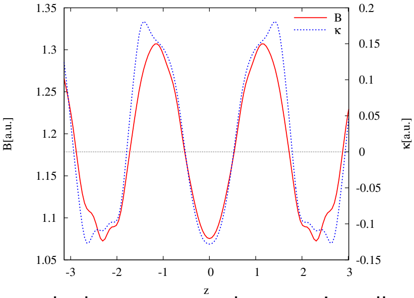
- approaching quasi-isodynamicity
- aspect ratio: $A = 10$
- trapped particles in the almost straight sections

WENDELSTEIN 7-X

Simulated geometries: HSX and W7-X

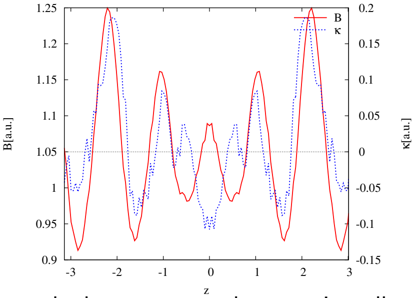
Magnetic field strength B and curvature κ along a magnetic field line.
$z = 0$ in the outboard midplane of the bean plane.

HSX



- bad curvature and magnetic well overlap
- $\omega_{*e} \cdot \overline{\omega_{de}} > 0$ for a large fraction of trapped particles

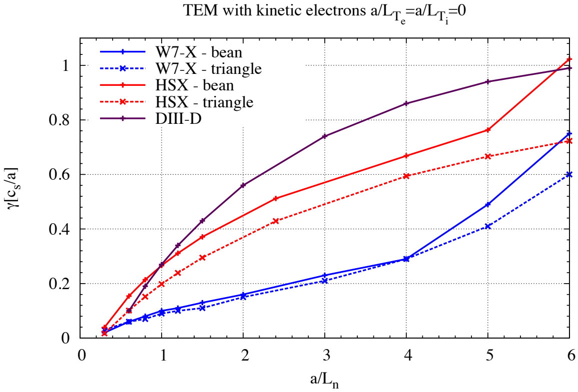
W7-X



- bad curvature and magnetic well separated at center of the flux tube
- $\omega_{*e} \cdot \overline{\omega_{de}} > 0$ for a smaller fraction of trapped particles

W7-X has fewer particles with $\omega_{*e} \cdot \overline{\omega_{de}} > 0$ and lower TEM growth rates

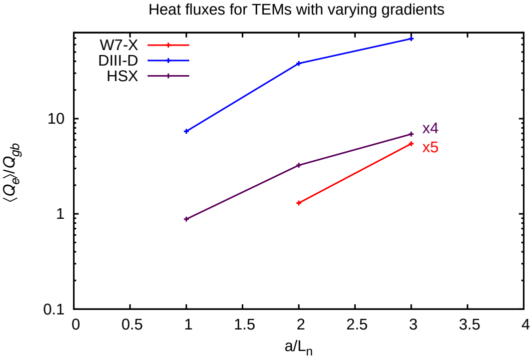
- W7-X has lower linear growth rates
- The critical gradient in both stellarators is lower than in a typical tokamak



Highest TEM growth rates γ at density gradients a/L_n with $a =$ minor radius including DIII-D data for comparison

The enhanced stability of W7-X prevails also nonlinearly

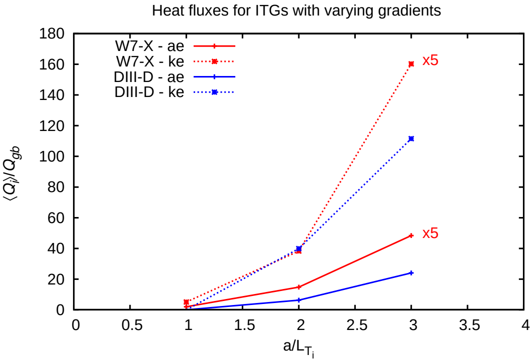
- W7-X has lower TEM heat flux than a typical tokamak



TEMs with pure density gradient in DIII-D, W7-X and HSX.
The factor accounts for the difference in aspect ratio.

The enhanced stability of W7-X prevails also nonlinearly

- W7-X has lower TEM heat flux than a typical tokamak



Ion temperature gradient modes with pure ion temperature gradient, with adiabatic (ae) and kinetic (ke) electrons in the DIII-D tokamak and W7-X. The factor accounts for the difference in aspect ratio.

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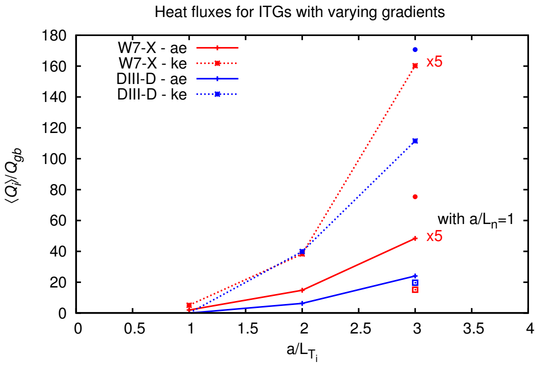
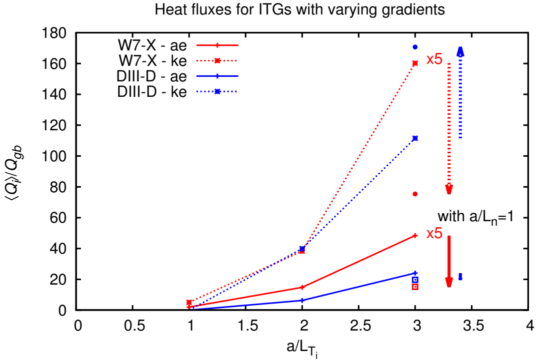
- W7-X has lower TEM heat flux than a typical tokamak



Ion temperature gradient modes with pure ion temperature gradient, with adiabatic (ae) and kinetic (ke) electrons in the DIII-D tokamak and W7-X.
The factor accounts for the difference in aspect ratio.

The enhanced stability of W7-X prevails also nonlinearly

- W7-X has lower TEM heat flux than a typical tokamak
- As soon as there is a density gradient present, W7-X has lower heat fluxes than DIII-D



Ion temperature gradient modes with pure ion temperature gradient, with adiabatic (ae) and kinetic (ke) electrons in the DIII-D tokamak and W7-X.
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Outline

Motivation

Analytical stability analysis

Numerical Results

TEM optimisation

Conclusions and Outlook

The proxy Q allows a quick estimate of the TEM stability of a configuration

- Proxy function as a measure for TEM activity, based on geometry only and thus easy to compute (a lot faster than simulation)
- Idea: reduce energy transfer rate → minimise average bad curvature
- Proxy Q for the heat flux: average bad curvature, minimise this for each flux tube

$$Q = -\int_{1/B_{\min}}^{1/B_{\max}} \overline{\omega}_d(\lambda)\mathrm{d}\lambda$$

$$\overline{\omega}_d(\lambda) = \int_{-\ell_0}^{+\ell_0} H\left(\frac{1}{\lambda} - B(\ell)\right) \omega_d(\lambda, \ell) \frac{\mathrm{d}\ell}{\sqrt{1-\lambda B(\ell)}}$$

- STELLOPT: optimisation of 3D equilibria created by VMEC via proxy functions

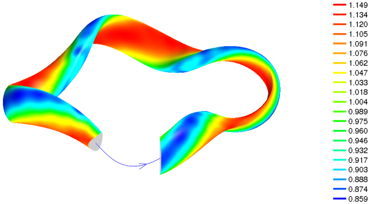
  [D.A. Spong et al 2001 Nucl. Fusion 41 711]

- minimise Q for different flux tubes on different flux surfaces

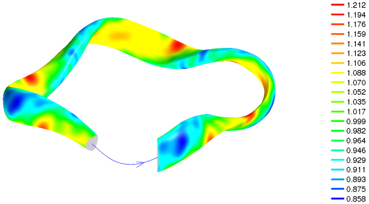
A first TEM optimisation of HSX has been achieved

- TEM-proxy has been reduced significantly, but only by relaxing the requirement of helical symmetry
- The neoclassical transport has increased slightly ($\epsilon_{eff} = 0.45\% \rightarrow 2.5\%$)

HSX (initial)



HSX (TEM-optimised)



Magnetic field strength

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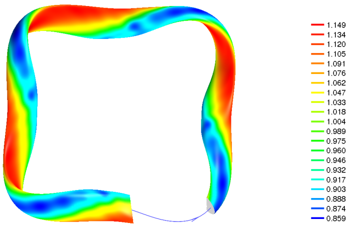

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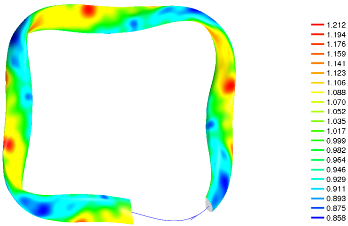

Magnetic field strength

WENDELSTEIN 7-X

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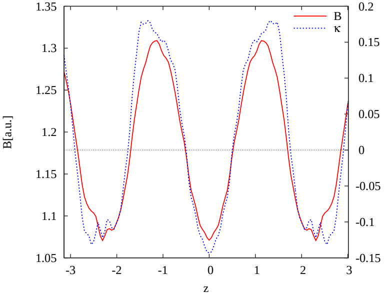
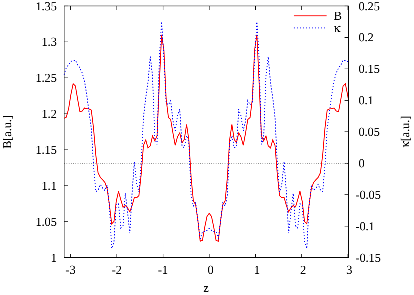


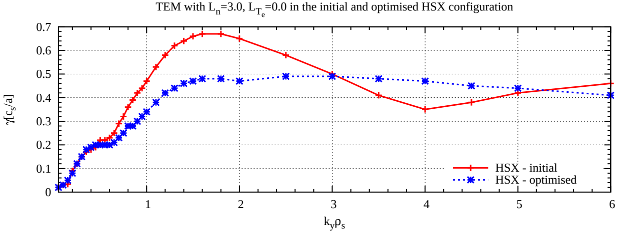
HSX (TEM-optimised)

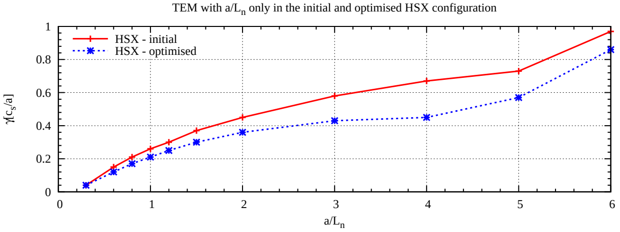


Magnetic field strength B and curvature κ along a magnetic field line. $z = 0$ in the outboard midplane of the bean plane.

The proof-of-principle optimisation was successful

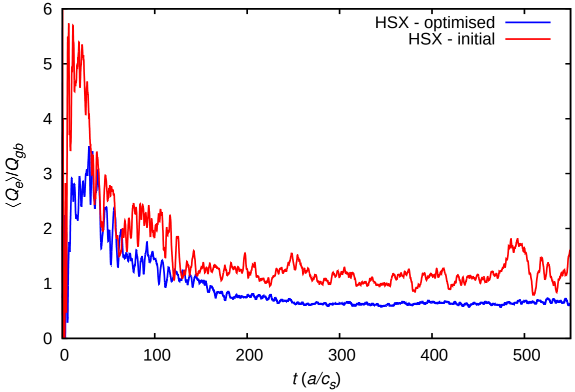
A reduction of growth rates is achieved over a large range of wave vectors and gradients.





The proof-of-principle optimisation was successful

The heat flux (here at $a/L_n = 3, a/L_{T_e} = 0$) was reduced significantly.



Here: optimisation without paying attention to the coil set (not experimentally feasible just now)

Conclusions and Outlook

Conclusions:

TEM stability:

- analytically: kinetic electrons are stabilising if $\omega_{*e} \cdot \overline{\omega_{de}} < 0$.
- numerically: W7-X, where more particles have $\omega_{*e} \cdot \overline{\omega_{de}} < 0$, has lower TEM growth rates and TEM heat flux compared with HSX. ITGs are also more stable if there is a finite density gradient.

TEM optimisation:

- developed proxy functions for use in STELLOPT
- proof-of-principle optimisation of HSX towards lower TEM activity successful, though new equilibrium not experimentally realisable

Outlook

- validate model for TEM optimisation experimentally (on HSX)
- explore optimisation space