Thermal Island Destabilization

and the Greenwald Density Limit

Roscoe White, David Gates, Dylan Brennan

Plasma Physics Laboratory, Princeton University

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Introduction

• Magnetic reconnection is ubiquitous in the magnetosphere, the solar corona, and in toroidal fusion research discharges.

• In a fusion device a magnetic island saturates
at the magnetic energy minimum of the configuration.
Further modification of the current density profile in the island interior causes additional growth or contraction of the saturated island.

• An island is thermally isolated from the outside plasma and can heat or cool depending on the balance of Ohmic heating and radiation loss, changing the resistivity and the current in the island.

• A model of island destabilization due to radiation cooling of the island is constructed. An additional destabilization effect is described, and it is shown that a small imbalance of heating can lead to exponential growth of the island.

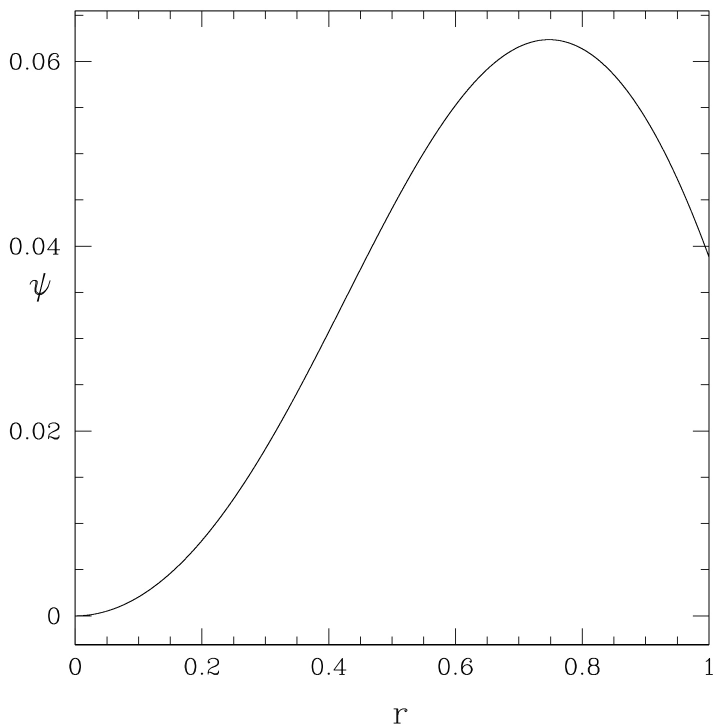
Large aspect ratio approximation, circular equilibrium.
Cylindrical geometry r, θ with a conducting wall $r = 1$.
Model profiles given by the form of the current density profile

$$j(r) = \frac{j(0)}{[(1 + (r/r_0)^{2\nu}]^{1+1/\nu}},$$

with the associated field helicity given by

$$q(r) = q(0)[(1 + (r/r_0)^{2\nu}]^{1/\nu},$$

r_0 gives the width of the current channel
$\nu = 1, 2, 4$ are peaked, rounded, and broad.
Furth, Rutherford, Selberg 1973
Disruption test case, slightly above m=2 instability threshold
$r_0 = 0.548$, $\nu = 1.53053$, $q(0) = 0.9$, $r_s = 0.7288$.



Single helicity analysis

Equilibrium helical flux a combination of toroidal ψ_t and poloidal ψ_p fluxes

$$\psi_0(r) = \psi_t - (n/m)\psi_p/2$$

$$\psi_0'(r_s) = 0, \qquad q(r_s) = m/n$$

Perturbed helical flux

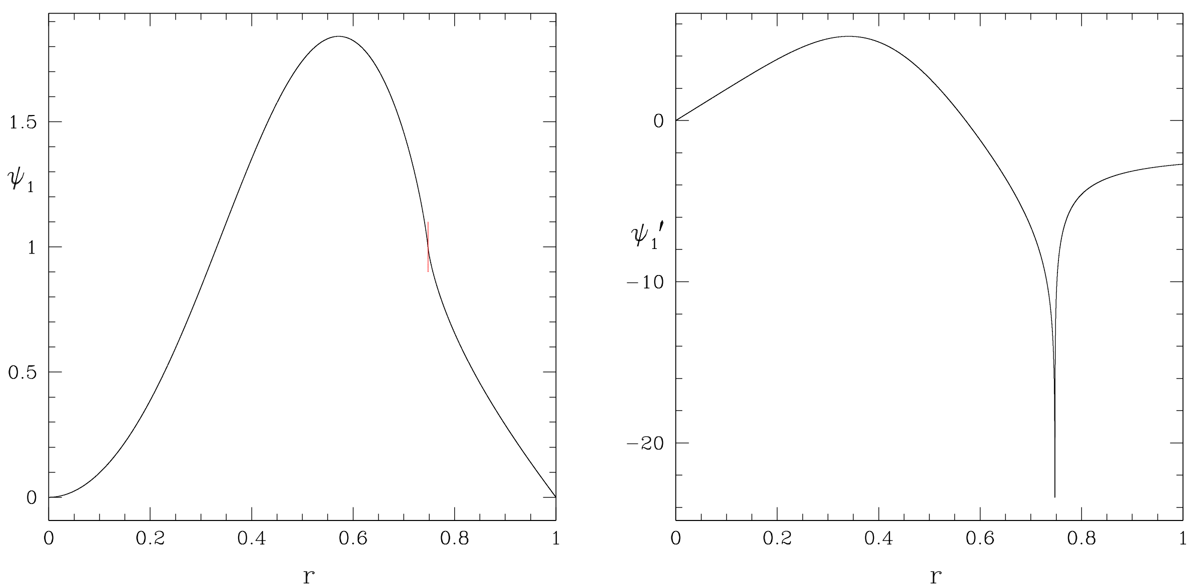
$$\psi(r) = \psi_0(r) + \psi_1(r) cos(m\theta)$$

$$\nabla_\perp^2 \psi_1 = \frac{dj}{d\psi_0}\psi_1 + \delta j_1.$$

Perturbed flux function and derivative, singular at r_s $q(r_s) = m/n$
Local analysis $r = r_s + x$

$$\psi_1 = \psi_1(0)[1 - Ax + kxln(-x)], \quad x < 0$$
$$\psi_1 = \psi_1(0)[1 - Bx + kxln(x)], \quad x > 0.$$

linear $\Delta' = A - B$



Time evolution for islands larger than the tearing layer
modified Rutherford equation

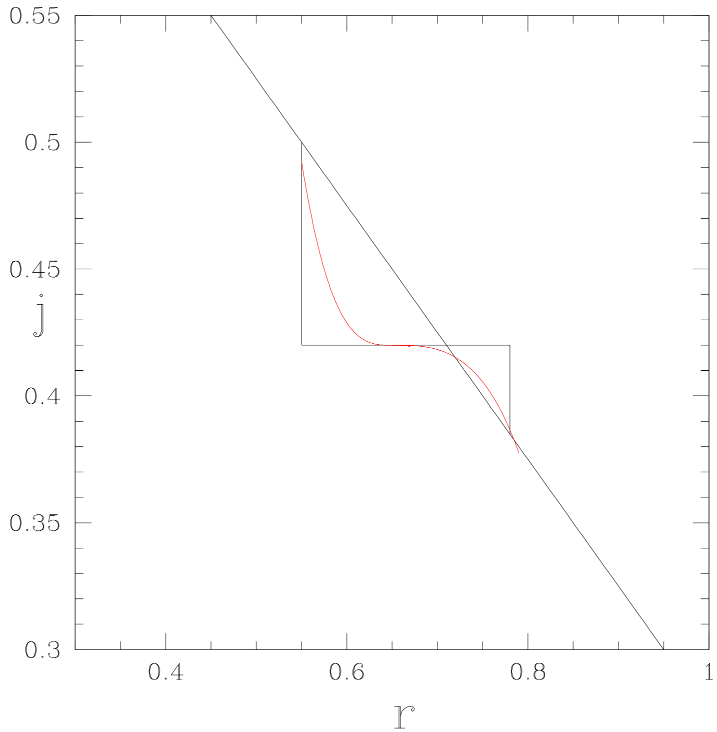
$$\frac{d\psi_1}{dt} = \frac{\eta(r_s)\Delta'(w)\psi_1}{w} - \eta(r_s)\delta j_1$$

With Δ' approximately given using the unperturbed eigenfunction

$$\Delta'(w) = \frac{\psi_1'(r_r) - \psi_1'(r_l)}{\psi_1(r_s)}, \qquad w = r_r - r_l$$

The effect of the current perturbation gives a modification of Δ' by the addition of $\Delta'_{\delta j}$ with

$$\Delta'_{\delta j}(w) = -w\frac{< \delta j_1 >}{\psi_1(r_s)},$$



Island asymmetry $A = (r_r - r_x)/(r_x - r_l) - 1$ destabilizing effect producing a δj.

$$\Delta'_A(w) = f_F \frac{\int [j(r_x) - j(r)] cos(m\theta) d\theta dr}{\psi_1(r_s)},$$

f_F Fitzpatrick, island partially flattened.

Secondly, flattening not valid for small width, due to perpendicular heat diffusion Multiply $\Delta'_{\delta j}$, Δ'_A by $w^2/(w^2 + w_F^2)$,

$w_F = \sqrt{8}(\kappa_\perp/\kappa_\parallel)^{1/4}(Rr_s/ns)^{1/2}$,

For typical fusion parameters

$\kappa_\perp/\kappa_\parallel \sim 10^{-9}$

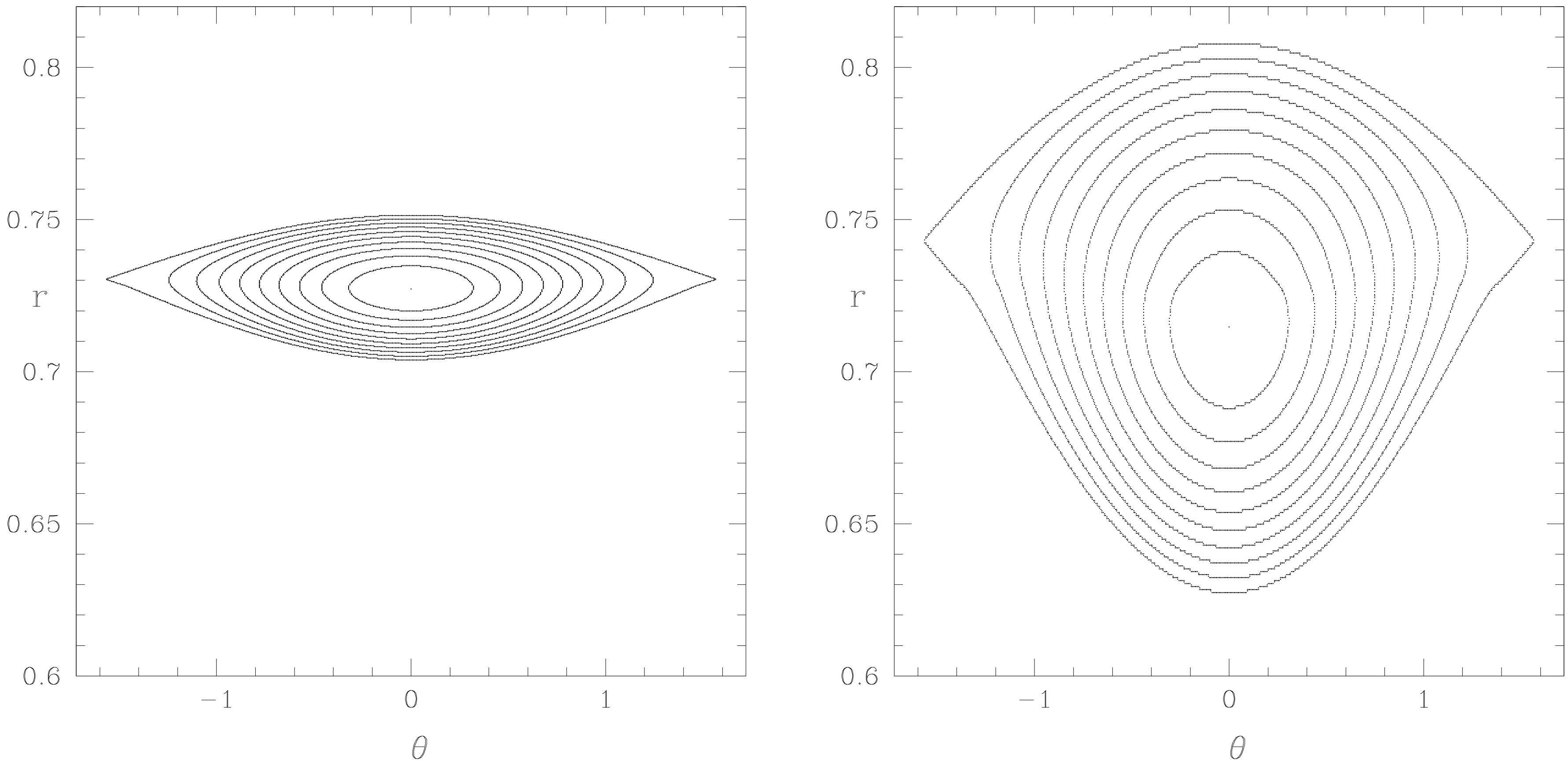
aspect ratio $R = 5$ $w_F \sim 0.02$.

The effective value of Δ' due to perturbed j and flattening can be found analytically using local approximations. Including the Fitzpatrick factor for small islands we have

$$\Delta'_{\delta j}(w) = 16 \frac{< \delta j_1 >}{\psi_0''} \frac{w}{w^2 + w_F^2}, \qquad \Delta'_A(w) = \frac{2 j'(r_x)}{\pi \psi_0''} \frac{w^2}{w^2 + w_F^2} f_A,$$

where f_A takes account of asymmetry A and degree of island flattening, given by $f_A = A f_F$. Both ψ_0'', $j'(r_x)$ negative

Determination of f_A requires analysis with thermal transport
Qualitative results are independent of f_A for values between 0.5 and 1



Magnetic islands including the effect of current flattening. The small island has amplitude $\alpha = 10^{-4}$, and width $w = .05$, $A = .26$. The amplitude of the large island is $\alpha = 1.5 \times 10^{-3}$, with width $w = 0.18$, and asymmetry $A = .78$. Profile parameters are those of the disruption test case with $f_F = 1$.

Suttrop et al, Asdex U, 2/1 island, Nucl Fus 37.1, p119 (1997)

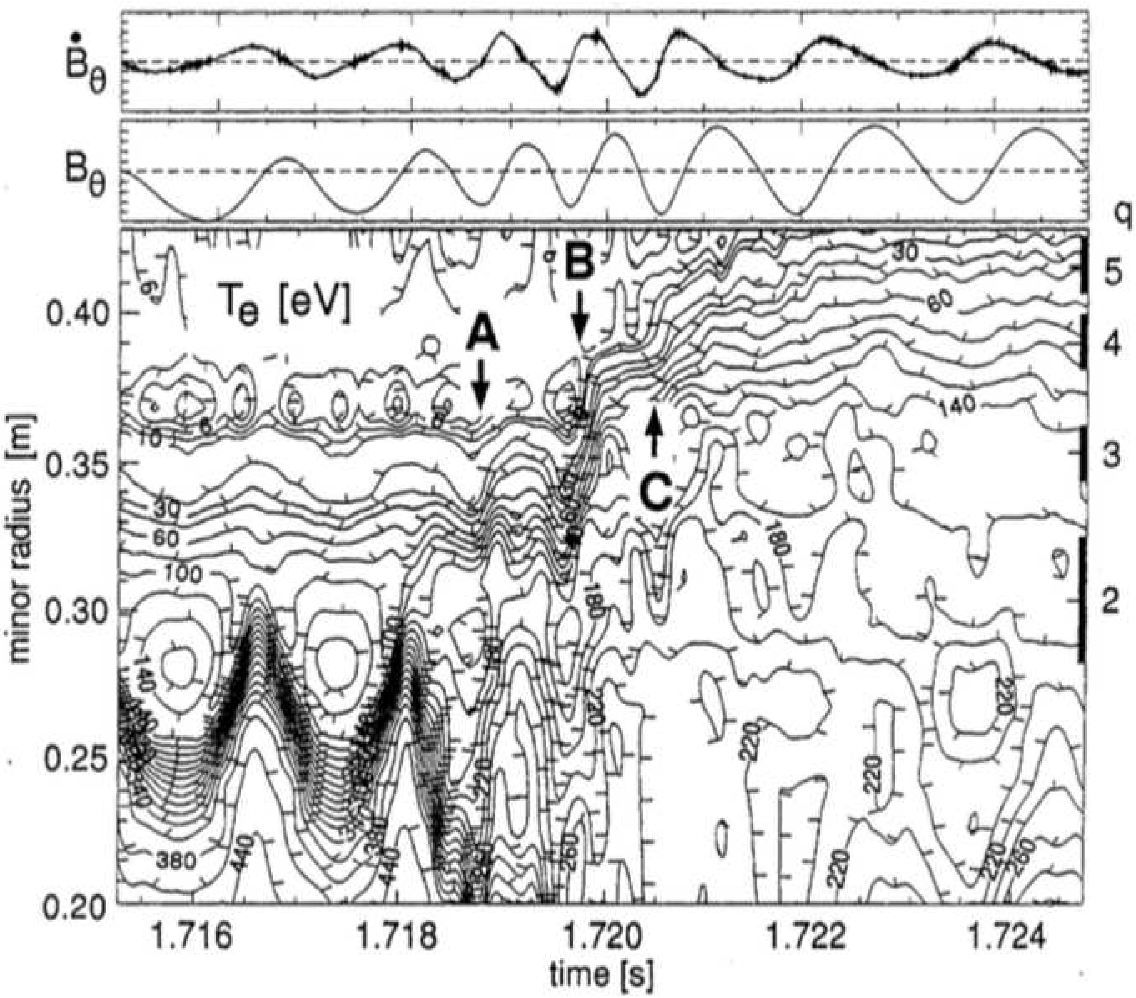
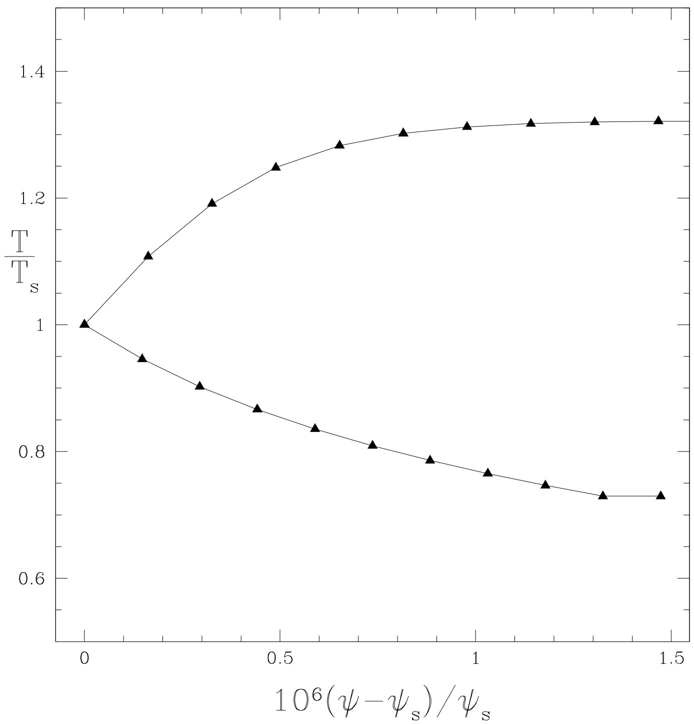


FIG. 4. A minor disruption ($q_{95} = 6$) leads to partial flattening of the T_e profile and reheating around resonant surfaces, first (A) at $q = 2$, then (B) at $q = 3$ and finally (C) around $q = 4$. No mode locking is observed. Mirnov oscillations (B_θ measured near the ECE antenna position) appear accelerated during this sequence.



Balance of Ohmic heating and radiation

$$\partial_t E = \nabla \cdot (\kappa \nabla T) + H(T) - R(T)$$

Steady state temperature profile in the island. $\kappa_{\parallel} >> \kappa_{\perp}$, $T = T(\psi)$.

Average on the flux surfaces

In the island $\nabla^2 \psi \simeq \psi_0''(r_s)$ which is order one,

and $(\nabla \psi)^2 \leq (\psi_0''(r_s) w)^2/4 << 1$, giving

$$0 = \kappa \psi_0''(r_s) \frac{dT}{d\psi} + H(T) - R(T)$$

Solve for $T(\psi)$ in the island

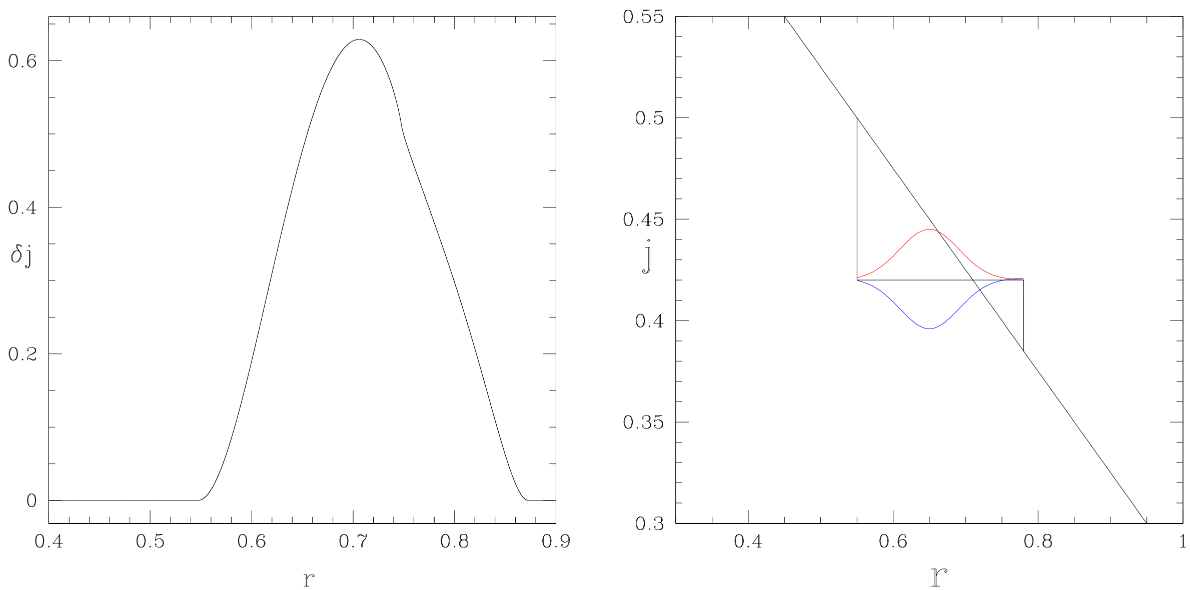
Spitzer resistivity produces a current perturbation from the temperature perturbation

Heating gives a positive stabilizing δj

Cooling gives a negative destabilizing δj

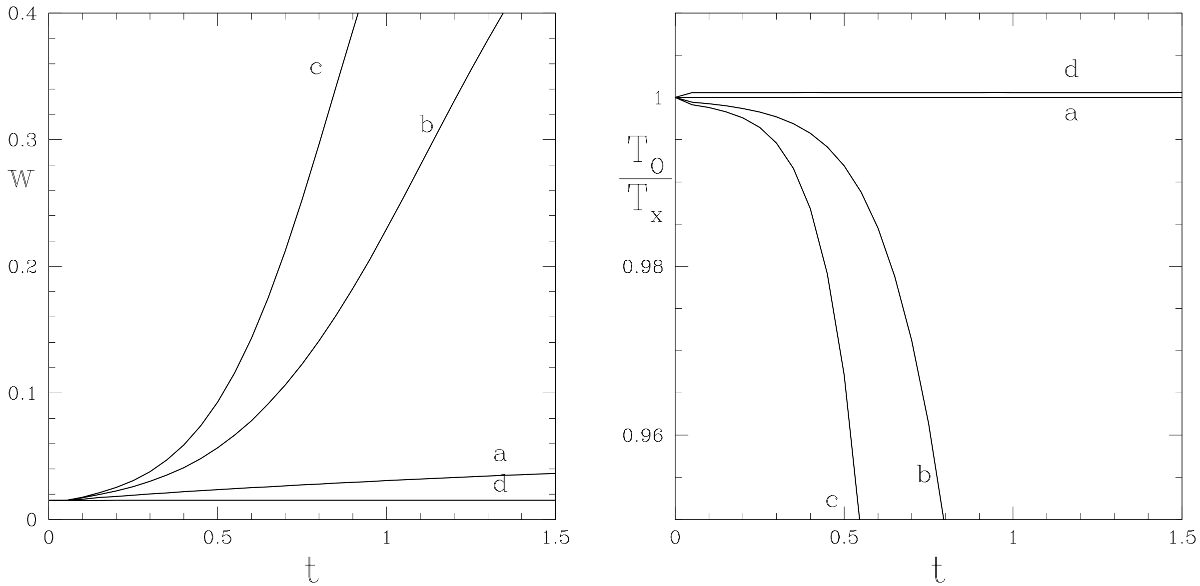
Flattening is always destabilizing

$$j_I(\psi) = j_s \frac{T^{3/2}(\psi) - T_s^{3/2}}{T_s^{3/2}},$$



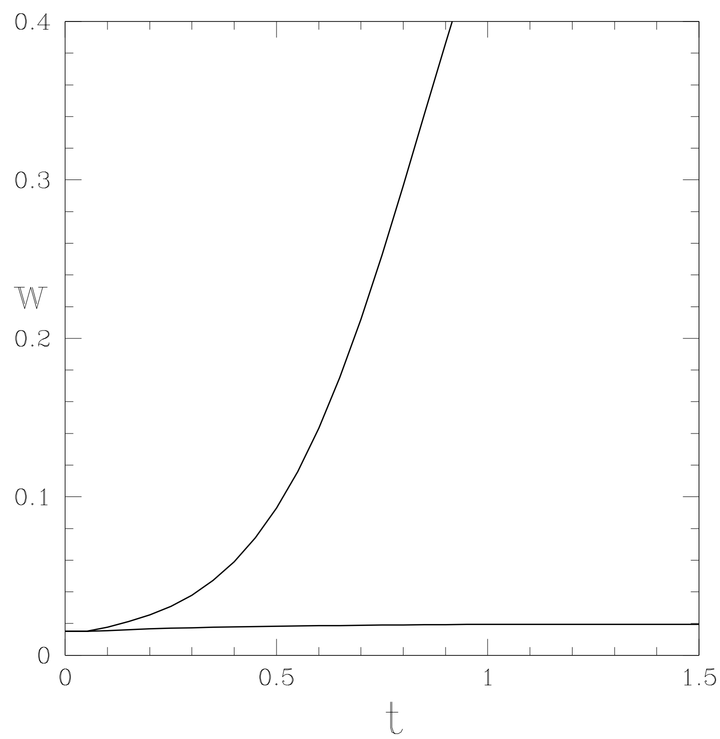
Time evolution

$$\frac{dw}{dt} = r_s^2[\Delta'(w) + \Delta'_{\delta j}(w) + \Delta'_A(w)]$$

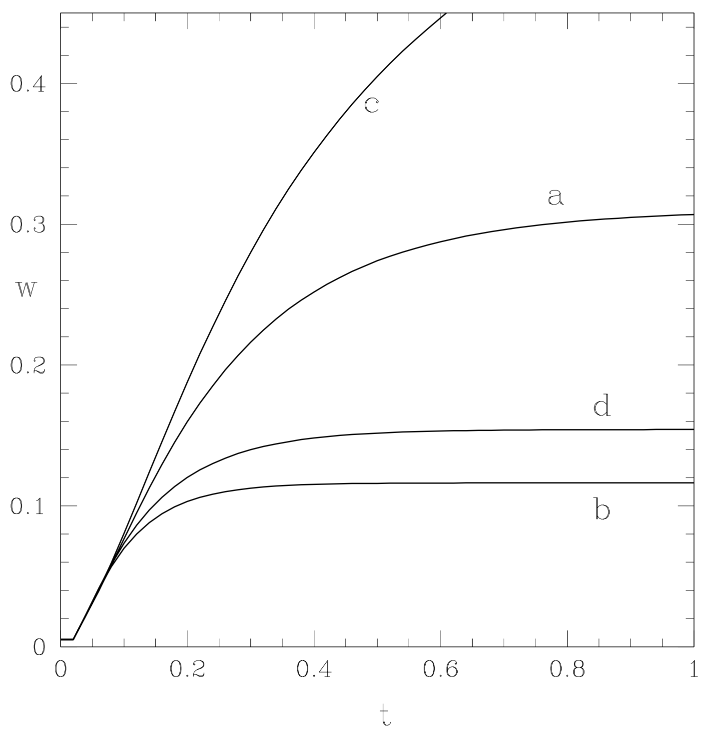


Growth of an island with a fixed temperature gradient $f_F = 1$.
a) radiation and heating balanced, b) and c) radiation dominated,
d) heating dominated. At $t = 0.2$, $T_O/T_x =$ a) 0, b) -.002, c) -.003.
In (c) temperature differential at t=0.5 with $w = 0.1$ was 3 percent.
In (d) the final central island temperature differential was .001.

Island evolution with and without the effect of asymmetric flattening. The radiation term is the same for each case. Without the effect of asymmetric flattening exponential growth is absent, and the saturation width is much smaller than the case with flattening and no cooling.



Island dynamics for any equilibrium can be approximated using the cylindrical analysis by reading data for the helical flux, q profile, and current profile.
DEBS must avoid $q = 1$ so equilibrium is more unstable



Island evolution using data from DEBS.
a) flattening, no cooling,
b) no cooling and no flattening,
c) flattening with cooling,
d) cooling no flattening.
Without asymmetric flattening exponential growth is absent, saturation width is much smaller than the case with flattening and no cooling, only slightly larger than the case without flattening or cooling.

• The $m = 2$ island has long been a candidate for the onset of significant loss of plasma to the wall and violent disruption

•The relative thermal isolation of a magnetic island and the effects of Ohmic heating and radiation can lead to rapid growth or mild contraction of a saturated island due to a tearing mode.

•Exponential growth occurs with island cooling of one or two percent. Explosive growth is absent without the asymmetric flattening naturally occuring in toroidal geometry.

•Coupled with models for Ohmic heating and radiation, this mechanism is a candidate for accounting for the Greenwald density limit.

•Currently under way:
Simulations with DEBS and NIMROD - Cooling gives exponential growth, Dylan Brennan, Dave Gates, Poster P3.014
Radiation models with impurities - Louis Delgado-Aparicio, Dave Gates
Insertion of radiation models into cylindrical code
Qian Teng et al Poster P2.019