

# Computation of singular currents at rational surfaces in non-axisymmetric MHD equilibria

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**Abstract.** Ideal MHD predicts the existence of singular current densities forming at rational surfaces in three-dimensional equilibria with nested flux surfaces. These current singularities consist of a Pfirsch-Schlüter  $1/x$  current that arises *around* rational surfaces as a result of finite pressure gradient and a Dirac  $\delta$ -function current that develops *at* rational surfaces and presumably prevents the formation of islands that would otherwise develop in a non-ideal plasma. These currents play a crucial role in describing (1) the plasma response to non-axisymmetric boundary perturbations, (2) the equilibrium and stability of non-axisymmetric, toroidally confined plasmas, and (3) the triggering of reconnection phenomena such as tokamak sawteeth. While analytical formulations have been developed to describe such currents in simplified geometries, a numerical proof of their existence has been hampered by the assumption of smooth functions made in conventional MHD equilibrium models such as VMEC. Recently, a theory based on a generalized energy principle, referred to as *multiregion, relaxed MHD* (MRxMHD), was developed and incorporates the possibility of non-smooth solutions to the MHD equilibrium problem. Using SPEC, a nonlinear implementation of MRxMHD, we provide the first numerical proof of their mutual existence and a novel theoretical guideline for the numerical computation of three-dimensional ideal MHD equilibria with singular currents [1]. Examples of such kind of equilibria are shown for both slab and cylindrical geometries, and the numerical results are thoroughly verified against analytical predictions from linearly perturbed ideal MHD equilibria. Based on these results, we present the hypothesis that non-axisymmetric ideal MHD equilibria only exist for discontinuous rotational transform profiles.

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