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CEMM

NIMROD MODELING OF SAWTOOTH MODES USING CONTINUUM AND HOT-PARTICLE CLOSURES

Tom Jenkins — Tech-X Corporation

in collaboration with

Eric Held — Utah State University

Jake King — Tech-X Corporation

Scott Kruger — Tech-X Corporation

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Abstract

We summarize ongoing efforts to model giant sawtooth onset and growth in DIII-D shot 96043 using NIMROD. The RF heating used in this discharge gives rise to an energetic ion population that alters the sawtooth stability boundary; the conventional sawtooth cycle is replaced by longer-period `giant sawtooth' oscillations of much larger amplitude. We explore the use of both continuum kinetic and particle-in-cell closures to numerically represent the RF-induced hot-particle distribution, and investigate the role played by the form of this distribution, including a possible high-energy tail drawn out by the RF, in determining the altered mode onset threshold and subsequent nonlinear evolution. Equilibrium reconstructions from the experimental data are used to enable these detailed validation studies. Effects of other parameters on the sawtooth behavior, such as the plasma Lundquist number and hot-particle β-fraction, are also considered. Ultimately, we hope to assess the degree to which NIMROD's extended MHD model correctly simulates the observed linear onset and nonlinear behavior of the giant sawtooth, and to establish its reliability as a predictive modeling tool for these modes

Sawtooth basics

Normal sawtooth mode

-Plasma has q(0) > 1, peaked current density on axis

-Ohmic heating introduced (e.g. 80 keV neutral beam)

-Plasma near axis preferentially heated (higher J) ➔ decreased core resistivity (~ $T^{-3/2}$) ➔ further current peaking, decreased q(0)

-(1,1) internal kink instability triggered when q(0) < 1, which rearranges magnetic flux and flattens temperature profile

-Cycle repeats

DIII-D shot #96043

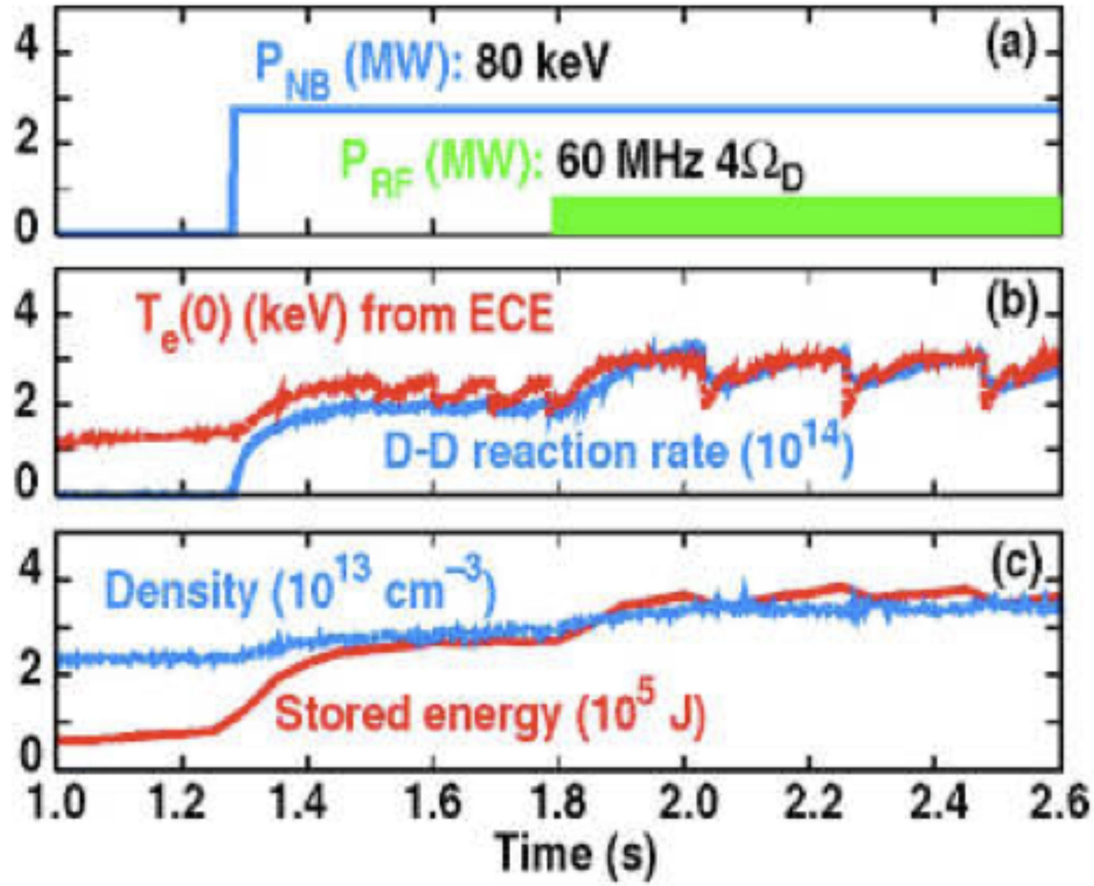


Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

Giant sawtooth basics

Giant sawtooth mode

-Energetic particle population (e.g. induced by RF heating, or fusion reactions) alters stability of internal kink mode

-Higher temperatures and stored energies achievable even with q(0) < 1

-Terminates like a normal sawtooth crash, but with larger amplitude

-Potential trigger for ELMs, NTMs, large heat transfer to vessel wall

"slow leak" description
"soft β limit"

DIII-D shot #96043

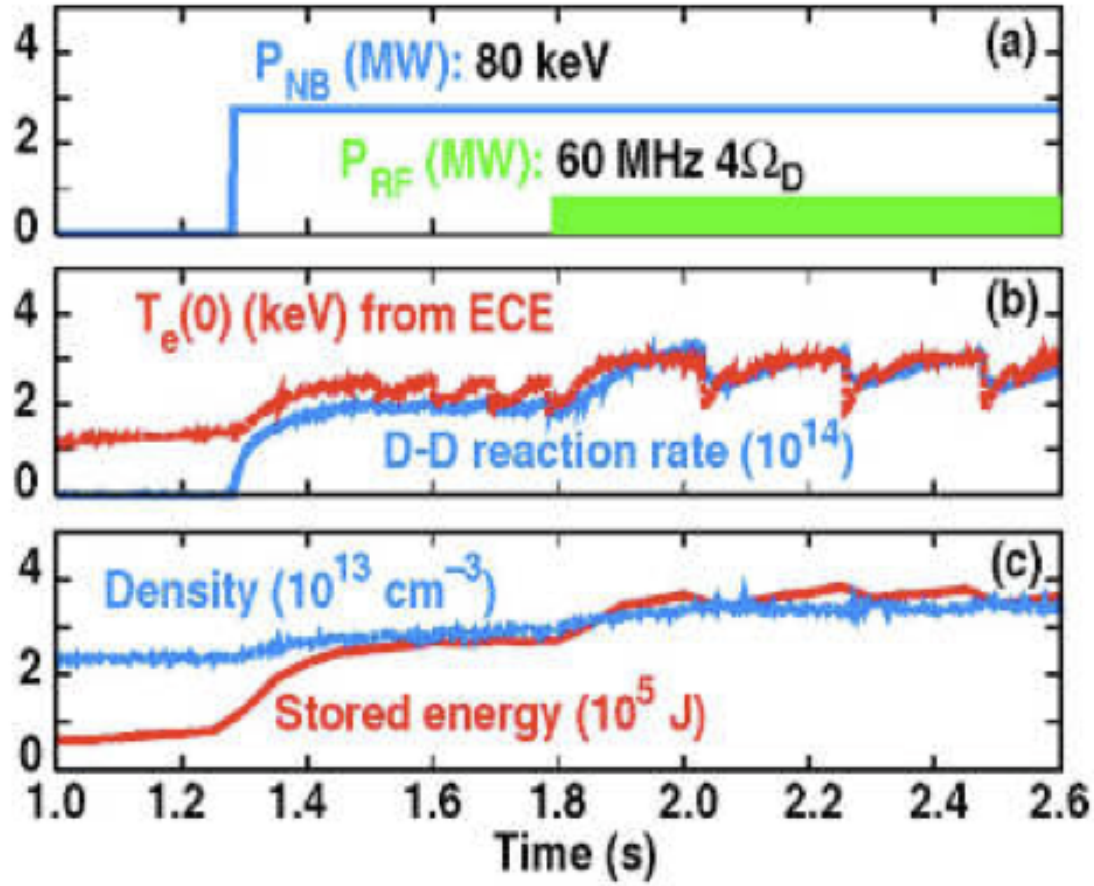


Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

History/context for this modeling effort

•The initial giant sawtooth modeling for this project was done by Dr. Dalton Schnack in 2009-2010, using NIMROD. It included linear scans of sawtooth and giant sawtooth onset in the resistive MHD and 2-fluid regimes, both with and without hot-particle stabilization (kinetic PIC). Complete stabilization of the sawtooth was not observed computationally.

•Modeling hot-particle stabilization of the giant sawtooth is a challenging numerical problem, since resistive MHD physics, 2-fluid effects, parallel closures, and hot-particle dynamics all influence the mode evolution.

•Code developments since Dalton's initial work have improved NIMROD's memory management, grid generation, and 2-fluid methods, enabling computational studies to proceed more easily and with greater accuracy.

•Tech-X researchers took up the project after Dalton passed away in late 2013.

Hot-particle sawtooth stabilization in NIMROD: computational approaches

Momentum equation has an extra term:

$$\rho\frac{\partial\vec{V}}{\partial t}+\rho(\vec{V}\cdot\nabla)\vec{V}=\vec{J}\times\vec{B}-\nabla\cdot\overleftrightarrow{P}-\nabla\cdot\overleftrightarrow{P}_{hot}$$

Continuum kinetic:

$\overleftrightarrow{P}_{hot}$ from moments of continuum solution to drift-kinetic equation (E. Held)

Kinetic PIC:

$\overleftrightarrow{P}_{hot}$ from moments of PIC distribution, evolving according to drift-kinetic equation (C. Kim, D. Schnack, T. Jenkins)

$$\overleftrightarrow{P}\rightarrow(1-\beta_{frac})\overleftrightarrow{P}$$

$\overleftrightarrow{P}_{hot}=\beta_{frac}\overleftrightarrow{P}$ comes from energetic particles, via T_i (energetic particles have low density and high temperature; n_{hot} << n but P_{hot} ~ P)

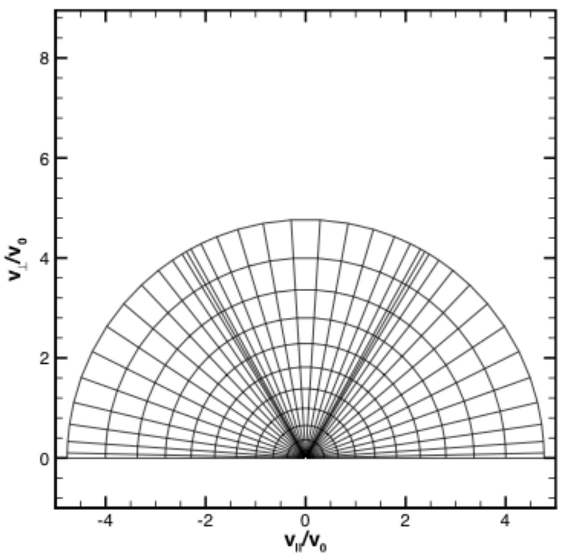
Current form of hot-particle pressure tensor contribution: slowing-down distribution

$$f_\alpha=\frac{P_0\exp(P_\zeta/\psi_n)}{\varepsilon^{3/2}+\varepsilon_c^{3/2}}$$

P_ζ	canonical toroidal momentum
ψ_n	normalized poloidal flux
ε_c	critical slowing-down energy

Continuum kinetic formulation (E. Held)

Write the drift-kinetic equation as



$$\frac{\partial f}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D)\cdot\left[\nabla f - \frac{1-\xi^2}{2\xi}\nabla \ln B \frac{\partial f}{\partial \xi} - \frac{s}{2}\nabla \ln T_0 \frac{\partial f}{\partial s}\right] - C(f) +$$

$$\frac{1-\xi^2}{2\xi}\left[-\xi^2\frac{\mathbf{b}}{B}\cdot\frac{\partial \mathbf{B}}{\partial t} + \frac{q}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D)\cdot\mathbf{E} + \xi^2\frac{\mathbf{E}\times\mathbf{B}}{B^2}\cdot\nabla \ln B\right]\frac{\partial f}{\partial \xi} +$$

$$\frac{s}{2}\left[-(1-\xi^2)\frac{\mathbf{b}}{B}\cdot\frac{\partial \mathbf{B}}{\partial t} + \frac{q}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D)\cdot\mathbf{E} + (1+\xi^2)\frac{\mathbf{E}\times\mathbf{B}}{B^2}\cdot\nabla \ln B\right]\frac{\partial f}{\partial s} = 0$$

formulated in pitch angle and normalized speed coordinates

$$\xi = v_{||}/v \qquad\qquad s = v/v_0$$

and with drift velocity

$$\mathbf{v}_D = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{T_0 s^2}{qB^2}\left[\left(1+\xi^2\right)\mathbf{b}\times\nabla B + 2\xi^2\mu_0\mathbf{J}_\perp + (1-\xi^2)\mu_0\mathbf{J}_{||}\right] + \frac{m v_0 s\xi}{qB^2}\mathbf{b}\times\frac{\partial \mathbf{B}}{\partial t}$$

Discretize velocity space, use NIMROD's finite-element machinery to evolve hot-particle distribution on the velocity grid

Moments of hot-particle distribution form the hot-particle pressure tensor

Kinetic PIC formulation

Drift-kinetic equation for hot ions:

$$\frac{\partial f_\alpha}{\partial t} + \vec{u} \cdot \vec{\nabla} f_\alpha + a \frac{\partial f_\alpha}{\partial v_\parallel} = C(f_\alpha)$$

Change of variables: evolve parallel particle motion instead of pitch angle and speed

$$\vec{u} = v_\parallel \hat{b} + \frac{\vec{E} \times \vec{B}}{B^2} + \frac{v_\parallel}{\Omega_\alpha} \hat{b} \times \frac{\partial \hat{b}}{\partial t} + \frac{\mu}{q_\alpha}\left(\frac{\hat{b} \times \vec{\nabla} B}{B} + \hat{b}\hat{b} \cdot \left(\vec{\nabla} \times \hat{b} \right) \right) + \frac{v_\parallel^2}{\Omega_\alpha} \left(\vec{\nabla} \times \hat{b} - \hat{b}\hat{b} \cdot \left(\vec{\nabla} \times \hat{b} \right) \right)$$

$$a = \frac{q_\alpha}{m_\alpha} \hat{b} \cdot \vec{E} - \frac{\mu \hat{b} \cdot \vec{\nabla} B}{m_\alpha}$$

collisions determine equilibrium hot-particle distribution function (slowing-down distribution)

present implementation

Standard delta-f approach:

$$f_\alpha = f_{\alpha 0} + \delta f_\alpha$$
$$\vec{u} = \vec{u}_0 + \delta \vec{u}$$
$$a = a_0 + \delta a$$

$$\frac{d}{dt}\left(\frac{\delta f_\alpha}{f_{\alpha 0}} \right) = -\frac{\delta \vec{u} \cdot \vec{\nabla} f_{\alpha 0}}{f_{\alpha 0}} - \frac{\delta a}{f_{\alpha 0}} \frac{\partial f_{\alpha 0}}{\partial v_\parallel}$$

weight equation

Moments of hot-particle PIC distribution form the hot-particle pressure tensor

Sawtooth stability

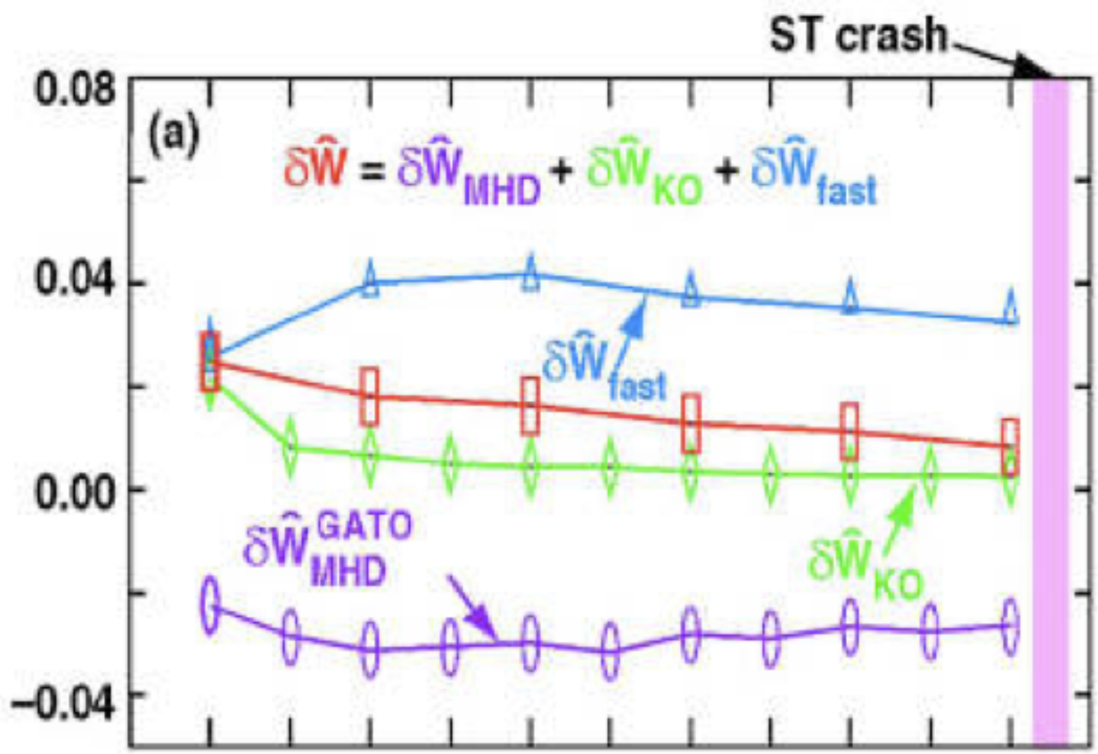


Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

Hot-particle effects

Bulk pressure tensor effects

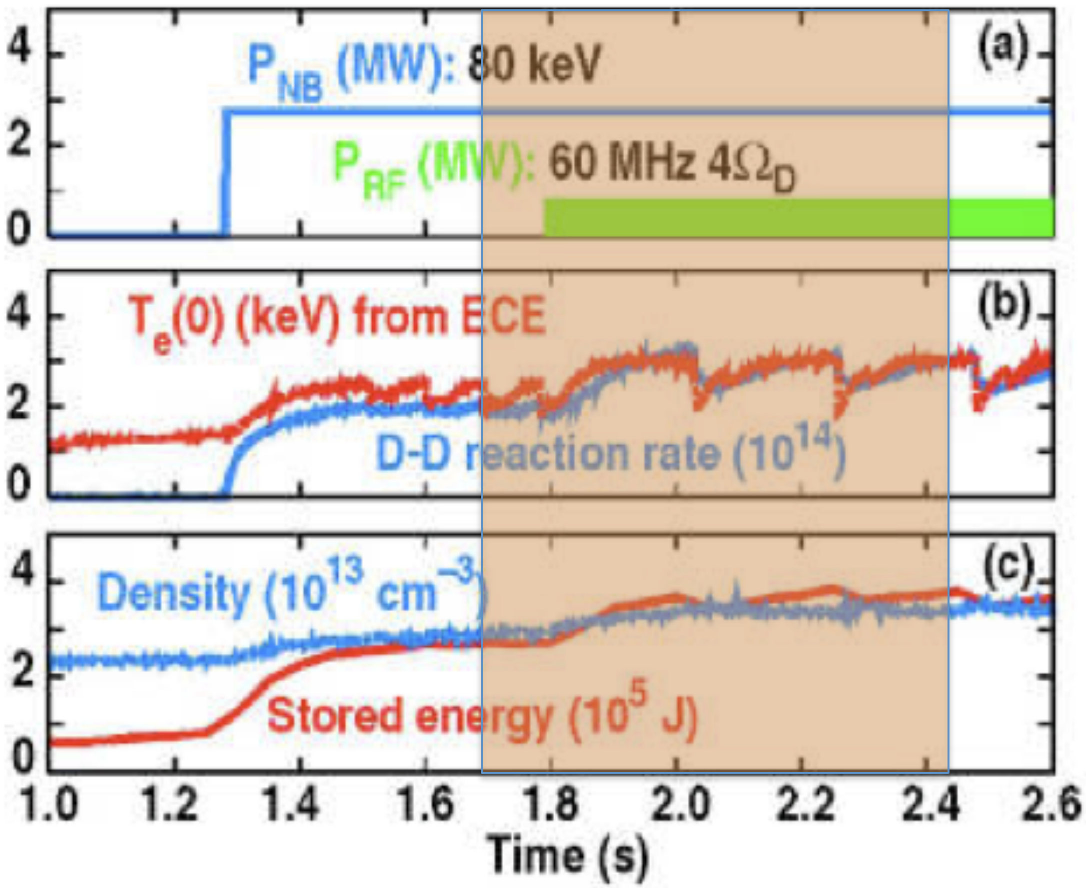
Ideal MHD effects

Total stability parameter

- Does ideal MHD + hot-particle kinetics explain everything?
- Role of two-fluid effects?

NIMROD reads the DIII-D equilibrium files

DIII-D shot #96043



•40 EFIT files (provided by Alan Turnbull) from within the orange box, covering both conventional and giant sawtooth cycles.

Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

Hot particles can also modify the equilibrium

Conventional Grad-Shafranov:

$$\vec{\nabla}\times\vec{B}=\mu_0\vec{J}_p$$

$$\vec{J}_p\times\vec{B}=\vec{\nabla}p_p$$

$$\vec{\nabla}\cdot\vec{B}=0$$

+axisymmetry: yield

$$\Delta^*\psi=-\mu_0R^2p_p'-FF'$$

$$F=F(\psi)=RB_\theta(R,Z)$$

$$p_p=p_p(\psi)$$

$$\vec{B}=\vec{\nabla}\psi\times\vec{\nabla}\theta+RB_\theta(R,Z)\vec{\nabla}\theta$$

Modified Grad-Shafranov [see E. V. Belova et al., *Phys. Plasmas* **10**, 3240 (2003)]:

$$\vec{\nabla}\times\vec{B}=\mu_0(\vec{J}_p+\vec{J}_h)$$

$$\vec{J}_p\times\vec{B}=\vec{\nabla}p_p$$

$$\vec{J}_h\times\vec{B}=\vec{\nabla}p_h$$

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\cdot\vec{J}_h=0$$

+axisymmetry: yield

$$\Delta^*\psi=-\mu_0R^2p_p'-HH'-\mu_0GH'+\mu_0RJ_{h\theta}(R,Z)$$

$$H=H(\psi)=RB_\theta(R,Z)-G(R,Z)$$

$$p_p=p_p(\psi)$$

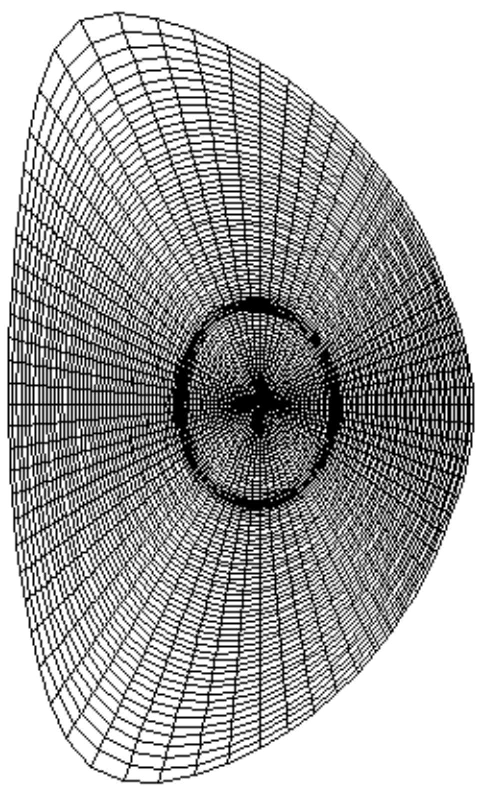
$$\vec{J}_h=\vec{\nabla}G\times\vec{\nabla}\theta+RJ_h(R,Z)\vec{\nabla}\theta$$

$$\vec{B}=\vec{\nabla}\psi\times\vec{\nabla}\theta+RB_\theta(R,Z)\vec{\nabla}\theta$$

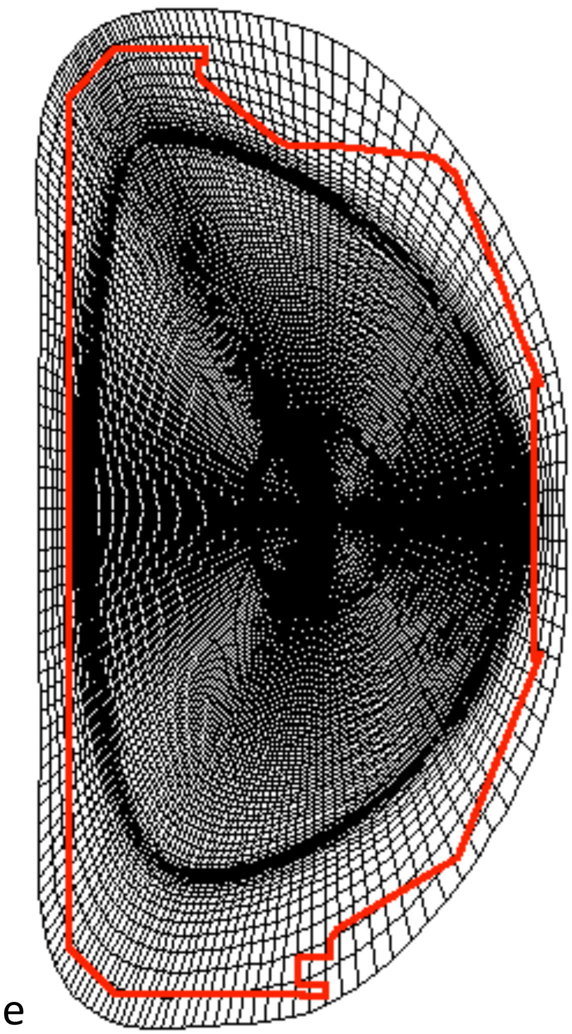
We are exploring whether a more general equilibrium solve is needed.

Some equilibrium solutions are better than others

growth rate = $3.13 \times 10^4\ s^{-1}$

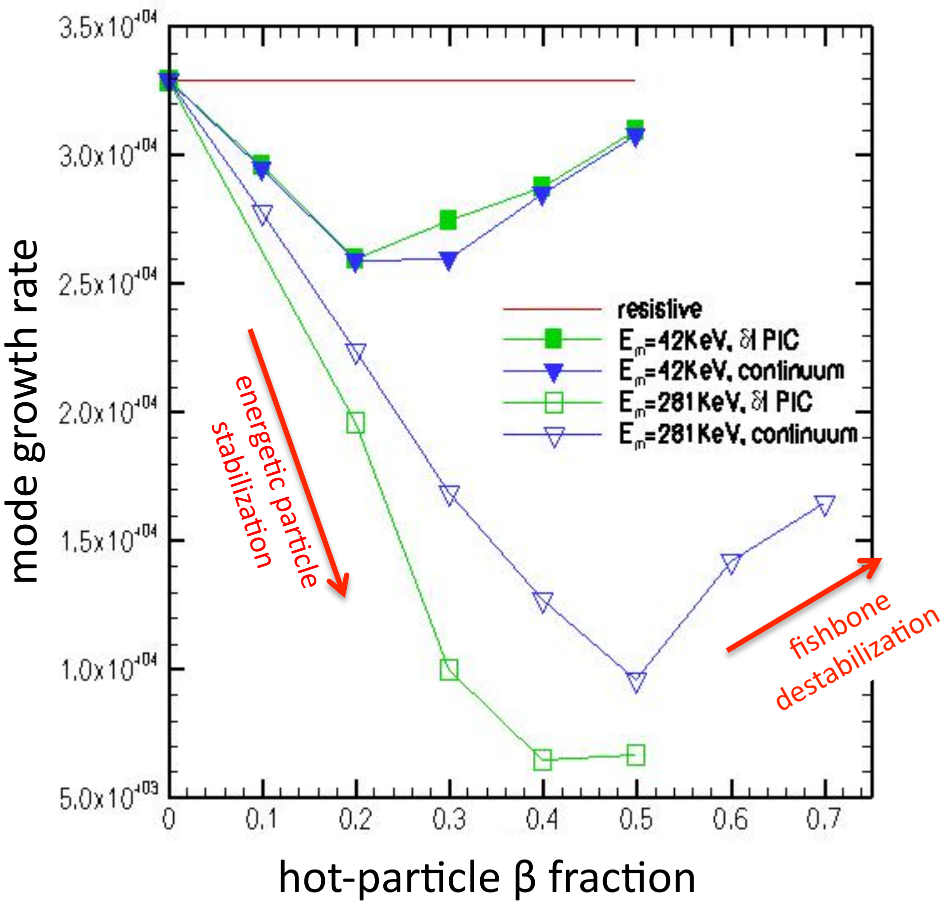


growth rate = $3.28 \times 10^4\ s^{-1}$



- Conducting wall is stabilizing
- Inaccuracy in Grad-Shafranov solve near separatrix can introduce significant current variation (relative to EFIT equilibrium), modifying growth rates and particle dynamics. New GS solver (fgnimeq; see also J. King poster) handles the transition across the separatrix.

Continuum and kinetic PIC benchmarks



E_m = peak energy of slowing-down distribution function

Related issues

•Energetic particle kink stabilization, fishbone destabilization as β fraction increased

•Stabilization (3rd adiabatic invariant – toroidal precession of energetic trapped particles modifies MHD) requires

$\omega_{pd}/\gamma_R >> 1$

(growth slow compared to precession) but for these cases,

$\omega_{pd}/\gamma_R = 1.5$ (42 keV)

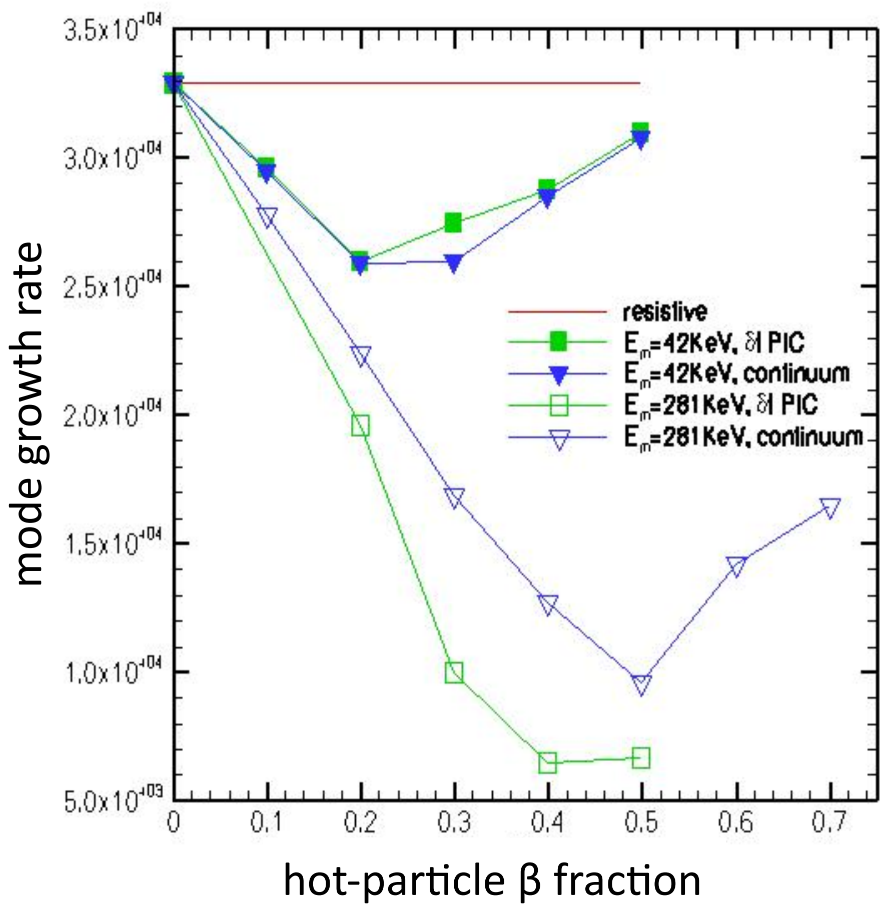
$\omega_{pd}/\gamma_R = 10$ (281 keV)

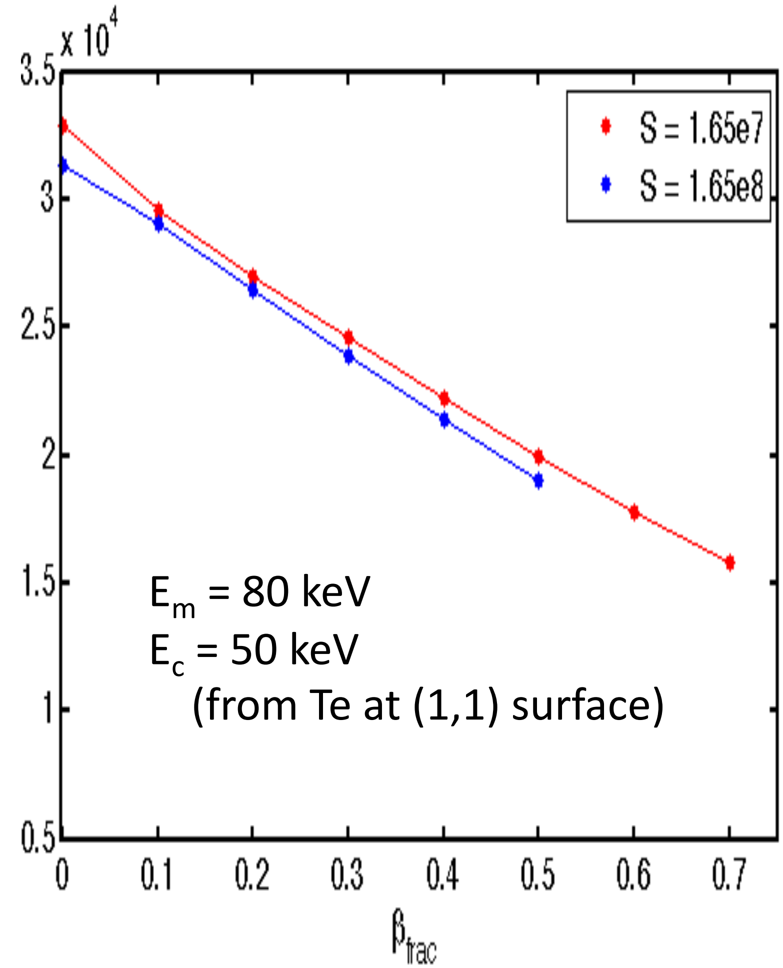
Can we run at high enough energies and Lundquist numbers to achieve full stabilization? (particle population in phase space)

How much does the form of the hot-particle distribution function matter?

PIC approach is more expensive computationally

Recent simulation results





• Energetic particle stabilization, but no fishbone effects, in new results

Hot-particle stabilization of sawtooth modes at high Lundquist number

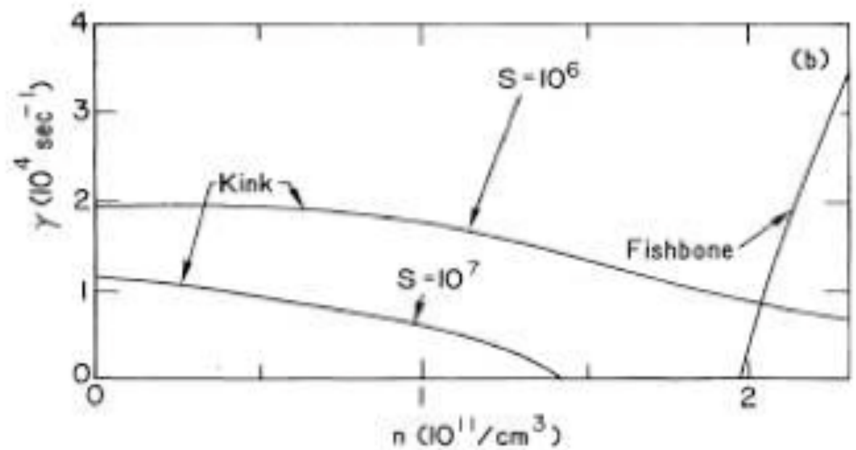


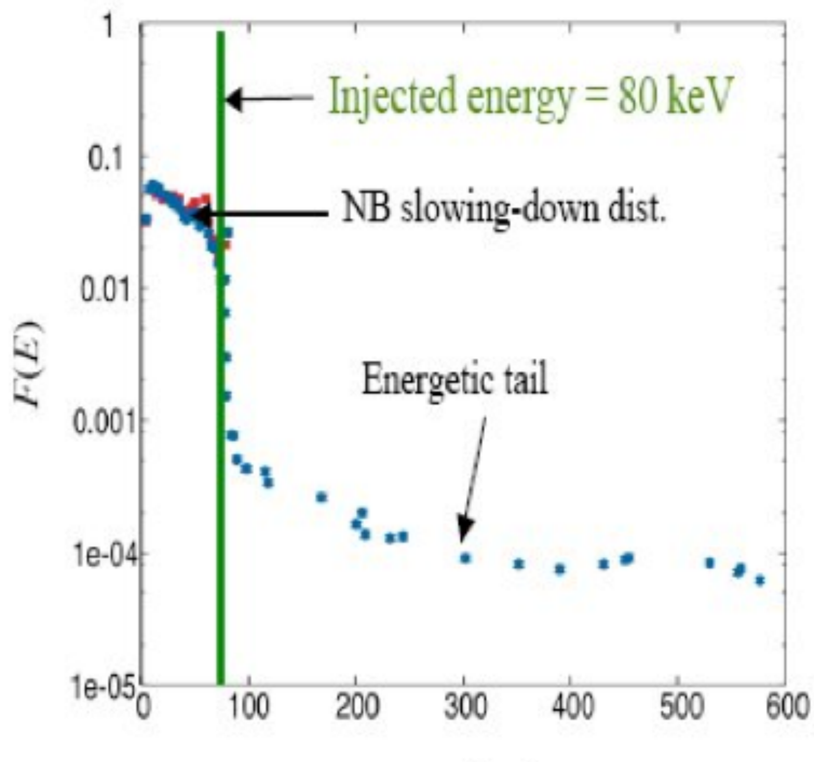
Figure from R. B. White et al., PRL **62**, 539 (1989).

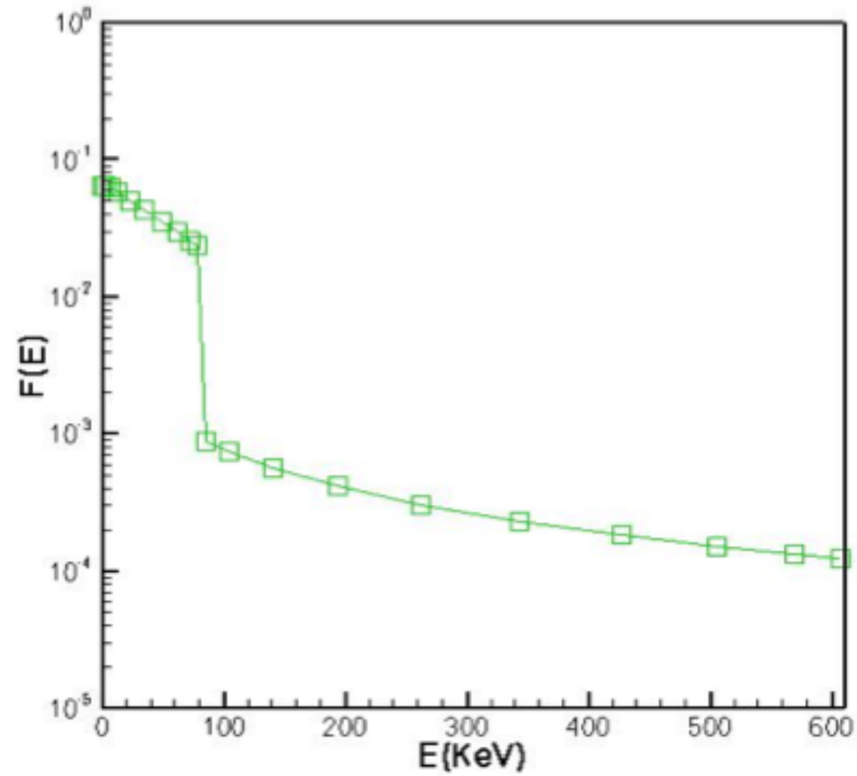
•For a fixed hot-particle density, higher Lundquist number is stabilizing
•Fishbone destabilization is expected at higher densities

Continuum closures – hot-particle RF tail

- High-energy tail and/or thermal ions needed for stabilization (only partial stabilization achieved from slowing-down distribution)

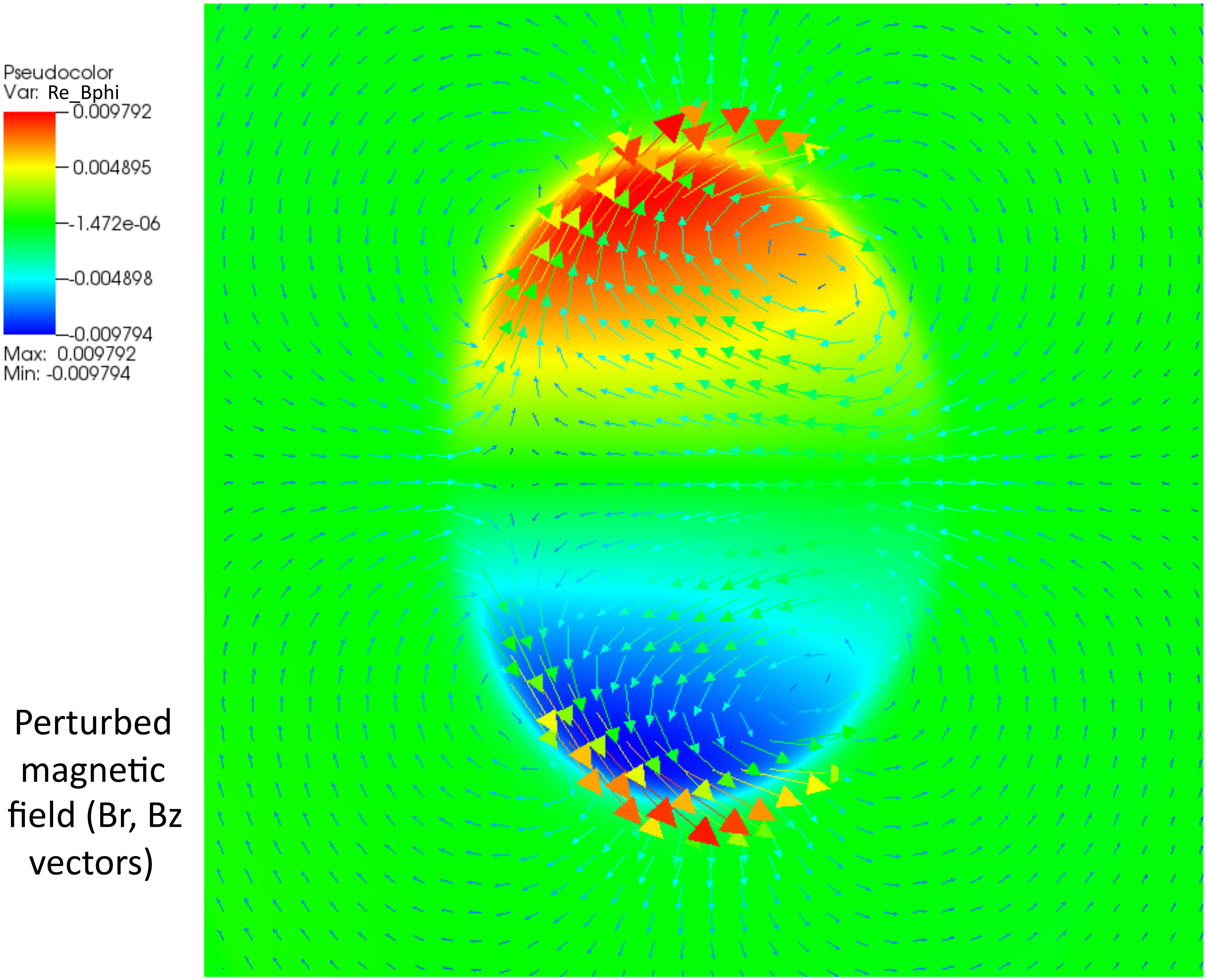
- Continuum closure developments include high-energy tail model – further testing needed to assess efficacy



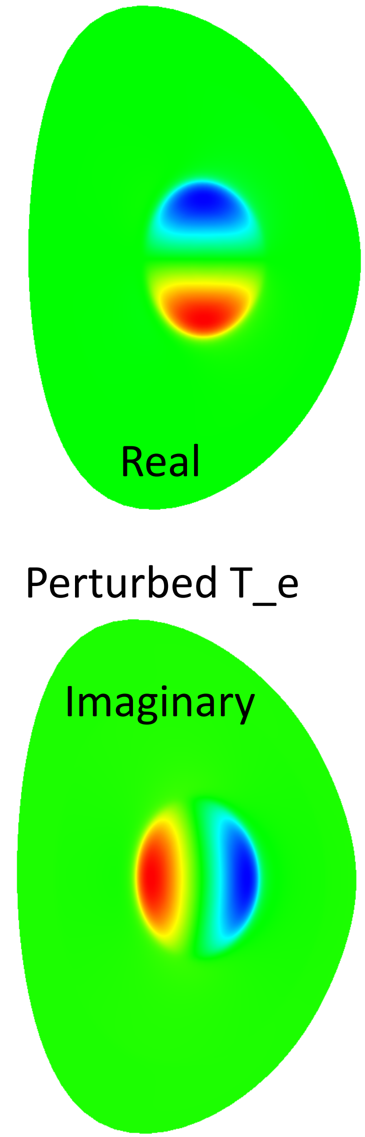


(plots from Eric Held)

NIMROD's new HDF5 interface improves our visualization capabilities

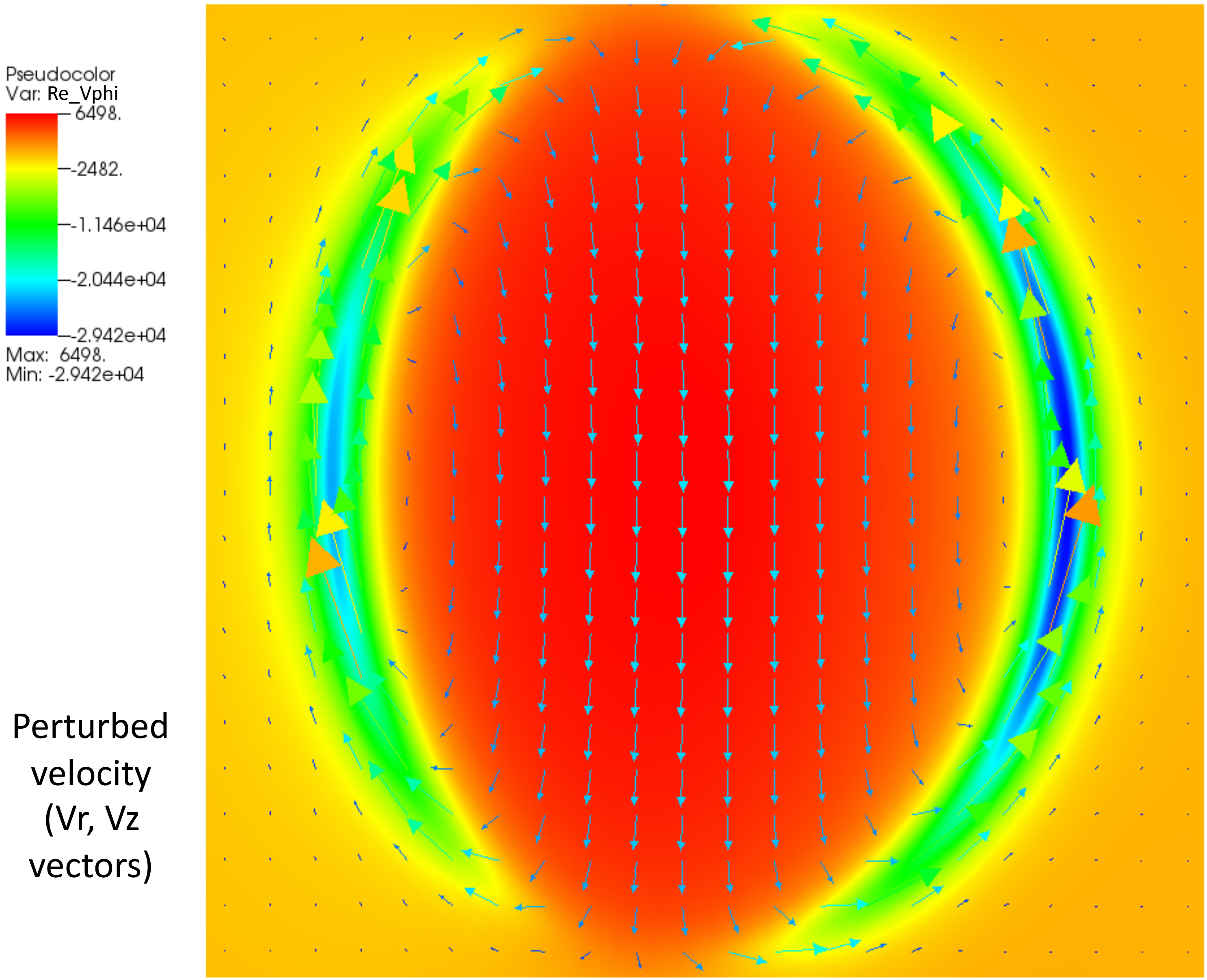


Perturbed magnetic field (Br, Bz vectors)



Perturbed T_e

NIMROD's new HDF5 interface improves our visualization capabilities



Perturbed velocity (Vr, Vz vectors)

Project milestones (CEMM)

	Year 4	Year 5
Sawtooth	•Apply continuum closure models for energetic and thermal ions to the Giant Sawtooth problem (Tech-X).	•Continue linear modeling of sawtooth stabilization in DIII-D shot 96043 (Tech-X). •Demonstrate nonlinear evolution of sawtooth with continuum kinetic closures and extended MHD Ohm's law (Tech-X/USU).
Model development - continuum kinetic (with Eric Held)	•Improve parallel scaling of kinetic closures (USU).	•Demonstrate applicability by applying to a 3D coupled problem (USU/Tech-X)
Model development - kinetic PIC	•Begin new particle parallelization development for NIMROD (Tech-X).	•Complete, test, and apply the new particle parallelization in NIMROD (Tech-X).

Plan of action going forward

- Continue exploring the extent to which present model can accurately characterize the MHD and 2-fluid(?) behavior of linear sawtooth onset – improve model as needed
- Exercise different combinations of physics components – MHD, 2-fluid, parallel closure, particles (all of them important for this work at some level)
- Near-term goal – ensure self-consistency between PIC and continuum approaches, in collaboration with Eric Held. Leverage code improvements to NIMROD since this project was initiated (cleaner, more stable equilibria).
- Longer-term goals – gaining physics/computational insights with NIMROD
  - -get experience using particle capabilities and continuum kinetic capabilities
  - -code performance improvements for development milestone
  - -examine the effect of more general hot-particle distribution functions
- Eventual milestone – DIII-D shot 96043 modeling of hot-particle induced giant sawtooth stabilization.