Unstable whistler waves driven by runaway electron beams

Chang Liu, Eero Hirvijoki, Dylan Brennan, Amitava Bhattacharjee
Princeton Plasma Physics Laboratory

2017 Sherwood Fusion Theory Conference
P3.008
Annapolis, MD May 2, 2017

Outline

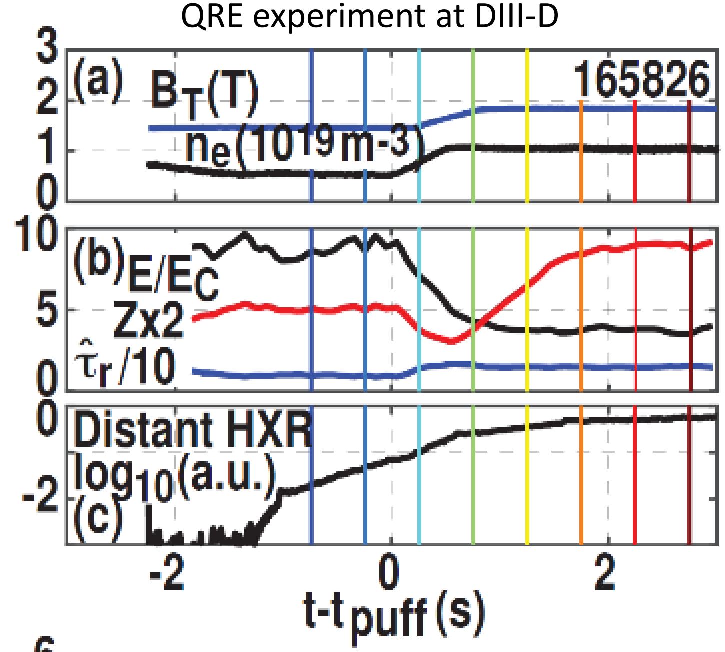
- Runaway electron generation and distribution function
  - Runaway electron beam is highly anisotropic
- Stability analysis of whistler waves in existence of runaway electron beam
  - Dispersion relation of whistler waves
  - Numerical calculation of mode growth rate
- Summary

Mitigation of runaway electrons (RE) raise big challenge for ITER

- In tokamak disruptions, runaway electrons (RE) can be generated when the plasma is cooled down abruptly.
  - Given the high energy (~25MeV) and significant number (10^{15} m^{-3}), RE can cause severe damage to the tokamak device.

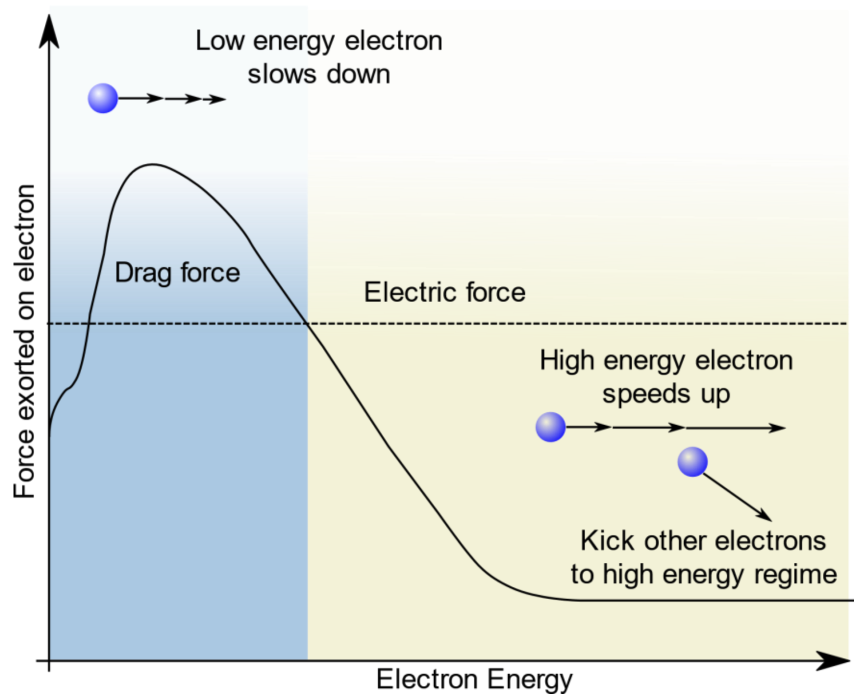
- In order to study the physics of RE, quiescent runaway electron (QRE) experiments have been conducted in several tokamak devices.
  - RE are generated in the flattop phase of discharge.
  - $T_e \sim$ several keV, insignificant MHD instabilities

QRE experiment at DIII-D



Basic picture of electron runaway

- In plasma, drag force on electrons due to Coulomb collision is a non-monotonic function of p. For $v \gg v_{th}$, collision frequency$\sim v^{-3}$, collisional drag force decrease with p.
- Under $E>E_{CH}$ (Connor-Hastie field), electrons with momentum larger than p_{crit} can run away in momentum space.
- Knock-on collision of high energy electron with thermal electron can lead to avalanche growth of RE.
- Synchrotron radiation force can dissipate electrons' energy and limit runaway process.



Kinetic equation of RE in momentum space

$$\frac{\partial f}{\partial t}+E\left(\xi\frac{\partial f}{\partial p}+\frac{1-\xi^2}{p}\frac{\partial f}{\partial \xi}\right)$$

Electric force

$$-\frac{1}{\tau}\left\{\frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[\frac{\Gamma\psi}{v}\frac{\partial f}{\partial p}+\frac{\Gamma\psi}{T}f\right]+\frac{1}{2vp^2}\left[Z+\phi-\psi+\frac{v^2T}{2c^4}\right]\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial f}{\partial \xi}\right\}$$

Collision operator

$$+\frac{1}{\tau_r}\left\{-\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^3\gamma(1-\xi^2)f\right]+\frac{\partial}{\partial\xi}\left[\frac{1}{\gamma}\xi(1-\xi^2)f\right]\right\}$$

Synchrotron radiation damping

$$=S_{RP}$$

$$S_{RP}=\frac{n_r}{\tau\ln\Lambda}\delta(\xi-\frac{p}{1+\gamma})\frac{1}{p^2}\frac{\partial}{\partial p}\left(\frac{1}{1+\sqrt{1+p^2}}\right)$$

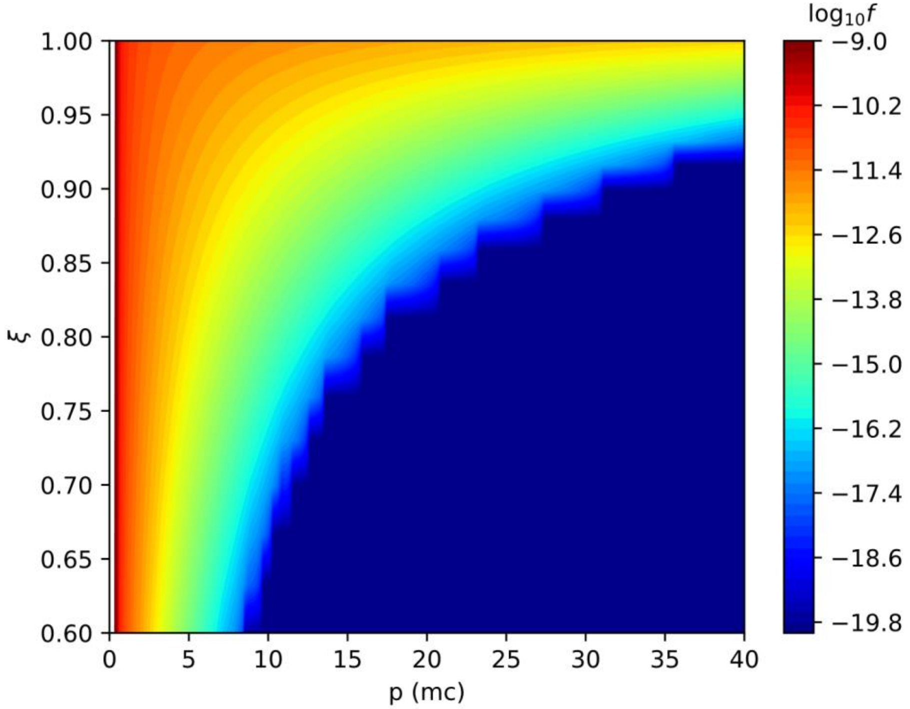
Rosenbluth-Putvinksi secondary RE generation

M. Landreman, A. Stahl, and T. Fülöp, Comp. Phys. Comm. 185, 847 (2014).

Runaway electron beam is highly anisotropic

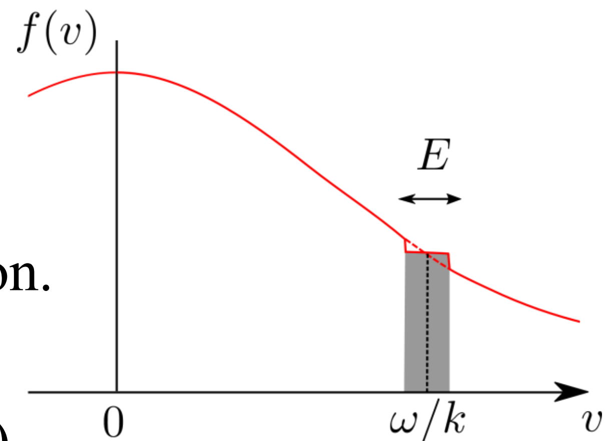
- Kinetic equation is solved numerically using finite element method (FEM) in p-ξ space.
- Runaway electron tail is generated from Maxwellian.
- E=9E_{CH}, Z_{eff}=2, T_{e0}=1.8keV, B=1.5T

Runaway electron distribution at t=2s

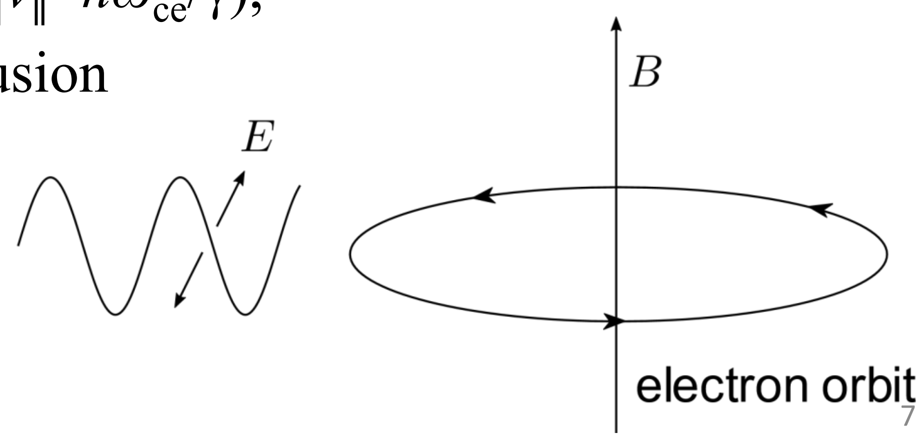


Electron diffusion in momentum space through wave particle interaction

- The electrons can be diffused in momentum space by the electromagnetic waves in the plasma, if satisfying resonance condition.



- For Cherenkov resonance (ω-$k_{\parallel}v_{\parallel}$=0), diffusion happens in the parallel direction.
- For cyclotron resonance (ω-$k_{\parallel}v_{\parallel}$=$n\omega_{ce}/\gamma$), wave causes both energy diffusion and pitch angle scattering.



Growth of unstable modes due to momentum space diffusion

$$\Gamma = \frac{\omega_{pe}^2}{D}\int d^3p \sum_{n=-\infty}^{n=\infty} Q_n \pi\delta(\omega - k_\parallel v\xi - n\omega_{ce}/\gamma)\left[v\frac{\partial f}{\partial p} - \frac{v}{p}\frac{n\omega_{ce}/\gamma - \omega(1-\xi^2)}{\omega\xi}\frac{\partial f}{\partial \xi}\right]$$

$$Q_n = \left[E_x \frac{n\omega_{ce}}{\gamma k_\perp v} J_n(k_\perp\rho) + E_z \xi J_n(k_\perp\rho) + iE_y\sqrt{1-\xi^2}\,J'_n(k_\perp\rho)\right]^2$$

$$D = \frac{1}{\omega}\mathbf{E}^* \cdot \frac{\partial}{\partial\omega}(\omega^2\epsilon)\cdot\mathbf{E}$$

For RE beam distribution

- $\partial f/\partial p<0$: parallel diffusion cause Landau damping.
  - Bump-on-tail distribution can give positive growth rate
- $\partial f/\partial \xi>0$: for anomalous Doppler resonance ($n\omega_{\mathrm{ce}}<0$), beam distribution gives a positive growth rate.

P. Aleynikov and B. Breizman, Nucl. Fusion 55, 043014 (2015).

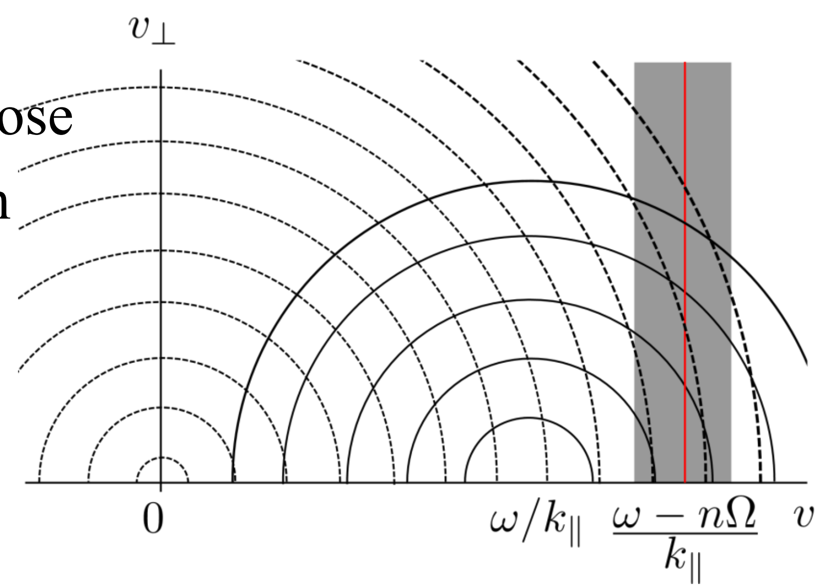
Particle diffusion in cyclotron resonance

- For a transverse wave propagating along B field direction, particle interacting with the wave will conserve its energy in the wave phase velocity

$$\left(v_{\parallel} - \omega / k\right)^2 + v_{\perp}^2 = \text{const}$$

- For anomalous Doppler resonance (ω-$k_{\parallel}v_{\parallel}$=$n\omega_{ce}/\gamma$, $n\omega_{ce}<0$), particles loose energy when diffuse from small pitch angle to large pitch angle.
  - The mode is unstable for beam-like anisotropic electron distribution.

Particle diffusion in anomalous Doppler resonance

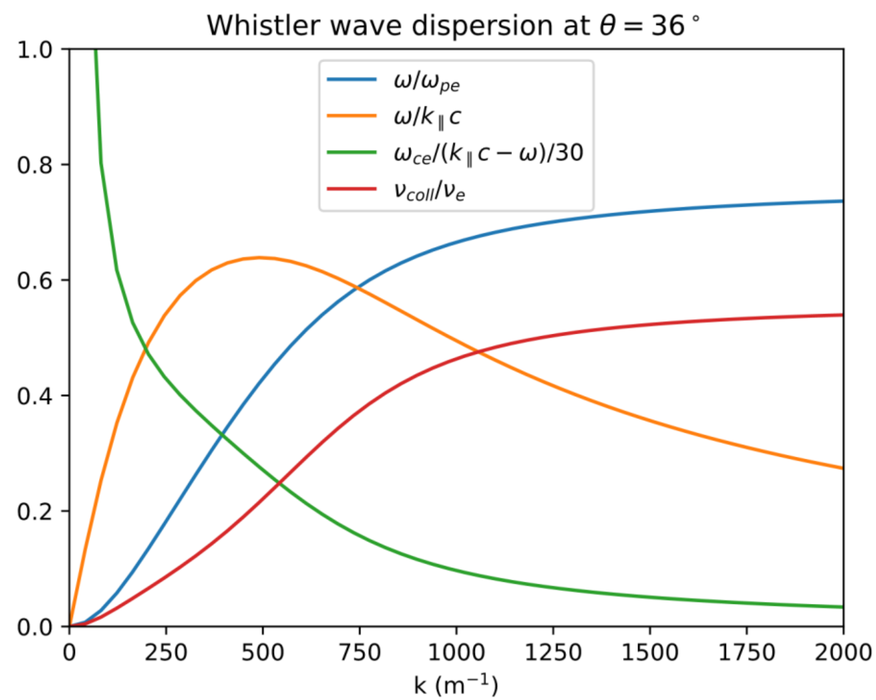


Whistler waves can satisfy anomalous Doppler resonance with RE beam

- With $\omega < \omega_{ce}$, whistler waves are likely to satisfy anomalous Doppler resonance for RE beam.
  - Here we only consider n=0, ±1

- Whistler wave with very small k and very large k are prone to Landau damping of thermal electrons.

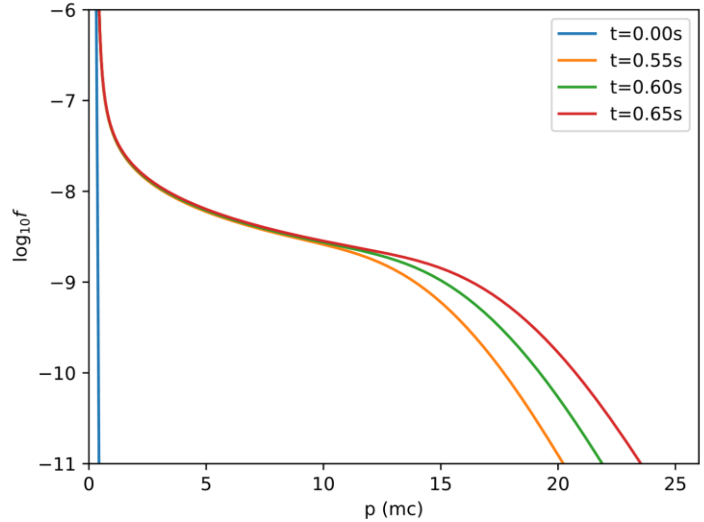
- Collisional damping rate (ν_{coll}) is stronger for high k mode

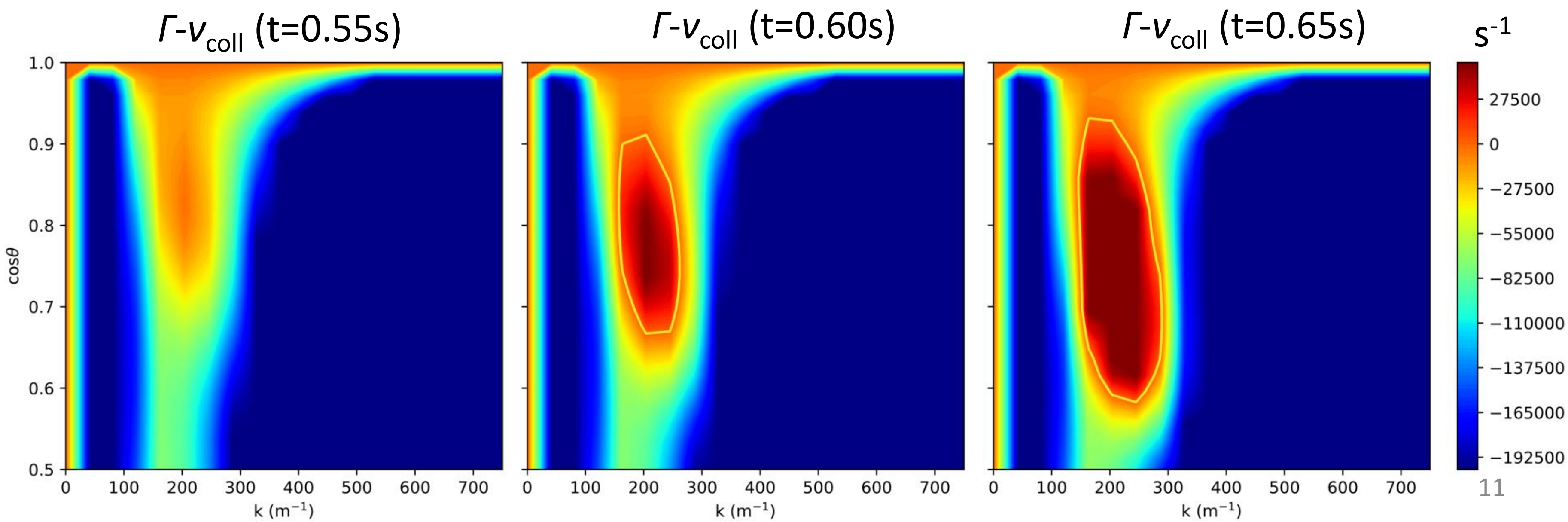


Whistler waves can be driven unstable by growing RE beam

- Numerical calculation of γ shows that, as RE beam extends to higher energy, whistler waves become unstable.

- E=9E_{CH}, Z_{eff}=2, T_{e0}=1.8keV, B=1.5T





Next step: Adding quasilinear diffusion operator to study wave-particle interaction

- Quasi-linear diffusion operator (including both electric and Lorentz force)

$$\frac{\partial f_0}{\partial t} = \frac{1}{2} e^2 \sum_{n=-\infty}^{\infty} \int d^3\mathbf{k}\, \hat{L} p_\perp \delta(\omega - k_\parallel v_\parallel - n\Omega) \left|\psi(n,\mathbf{k},\omega)\right|^2 p_\perp \hat{L}\, f_0$$

$$\hat{L} = \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} + \frac{k_\parallel v_\perp}{\omega} \frac{1}{p_\perp} \frac{\partial}{\partial p_\parallel}$$

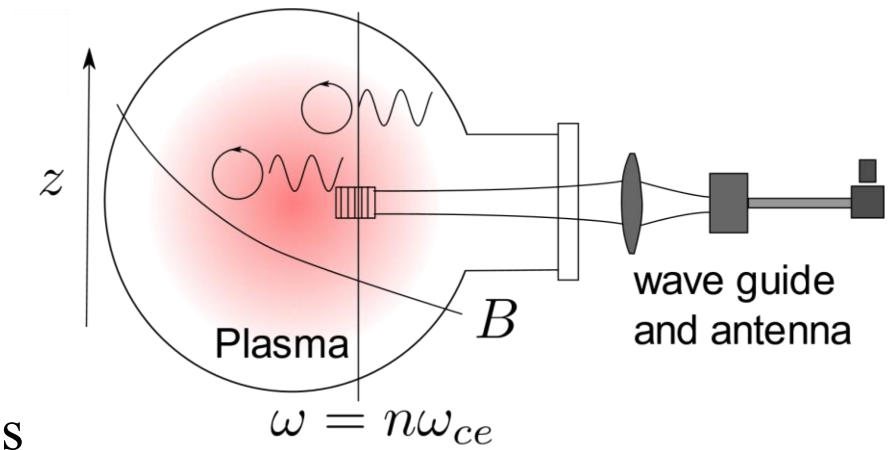
$$\psi(n,\mathbf{k},\omega) = \frac{1}{2}(E_x + iE_y) J_{n-1}(k_\perp \rho_L) + \frac{1}{2}(E_x - iE_y) J_{n+1}(k_\perp \rho_L) + \frac{p_\parallel}{p_\perp} E_z J_n(k_\perp \rho_L)$$

T. H. Stix, Waves in Plasmas

A.N. Kaufman, Physics of Fluids 15, 1063 (1972).

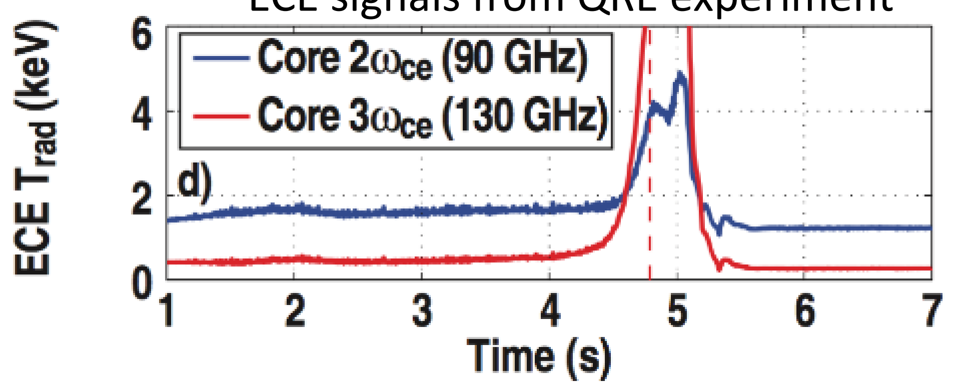
Next step: Study ECE radiation of RE using synthetic diagnostic

- Electron cyclotron emission (ECE) is a widely used diagnostic tool for tokamak plasma.



- Prompt ECE signal growth observed in RE experiments
  - Previous studies shows that this is linked to scattering of RE beam from the excited whistler waves

ECE signals from QRE experiment



Chang Liu, PhD thesis, Princeton University
Lei Shi, PhD thesis, Princeton University
C. Paz-Soldan et al., Nucl. Fusion 56, 56010 (2016).

Summary

- Given its anisotropy, runaway electron beam generated in tokamaks provides an energy source to excite unstable normal modes through anomalous Doppler resonance.
- Whistler waves can become unstable due to the RE beam in QRE experiments.
- The excited whistler waves can cause significant scattering of RE beam.
- Future topics to study:
  - Spatial propagation of excited whistler waves
  - Trapping effect of runaway electrons
  - Include the collision effect using the line broadened quasilinear model