Coupled UEDGE/Vorpal modeling of RF-induced ponderomotive effects on edge and SOL transport

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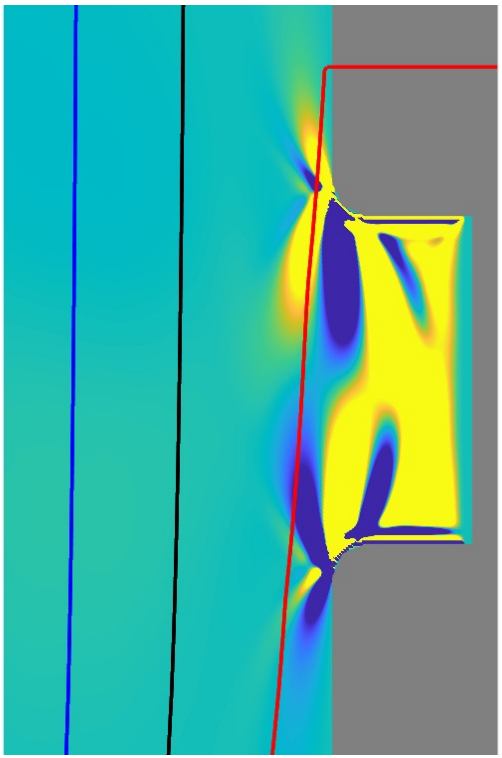

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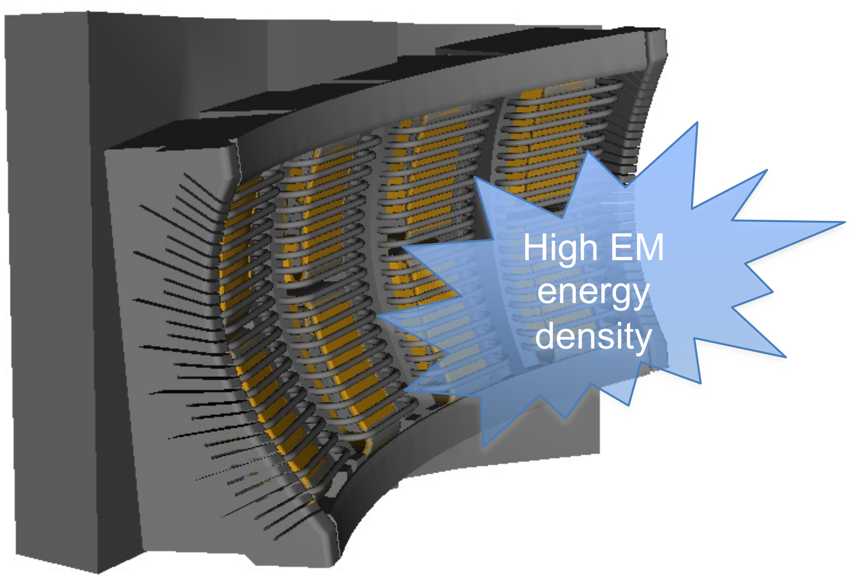
Sherwood Fusion Theory
Santa Rosa, CA
April 6, 2022

What is ponderomotive force?

- Consider electric field energy density $\frac{\epsilon_0 E^2}{2} \sim \frac{[J]}{[m^3]} \sim \frac{[N]}{[m^2]} \sim$ pressure
- Gradient of pressure or energy density = force density
- Pressure gradients drive momentum transport, e.g. in species fluid equation

$$m_\alpha n_\alpha \left(\frac{\partial \vec{V}_\alpha}{\partial t} + (\vec{V}_\alpha \cdot \nabla)\vec{V}_\alpha \right) + \nabla \cdot \overleftrightarrow{P}_\alpha = q_\alpha n_\alpha (\vec{E} + \vec{V}_\alpha \times \vec{B}) + \vec{R}_\alpha$$



- In plasma edge/SOL, energy density gradients that arise from RF antenna operation can also drive transport
- In single-particle picture,

$$\mathbf{F}_{\mathrm{p}} = -\frac{e^2}{4m\omega^2}\nabla(E^2)$$

Can pondermotive force significantly affect fusion plasmas?

Questions explored in this talk:

(1) What physics do ponderomotive forces add to edge/SOL dynamics?

(2) Can ponderomotive effects become significant enough to affect edge/SOL transport near a high-power RF antenna (~1 MW/m^2)?

RF interaction with PFCs (sheaths, impurity production, etc.) may also affect edge/SOL physics, but plasma-material interactions are not the focus here.

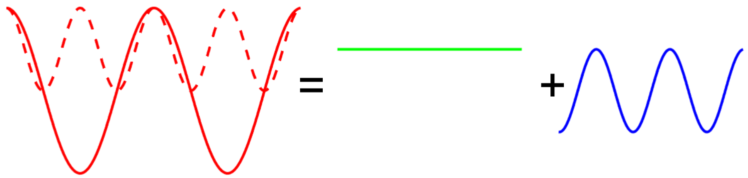
How do RF effects influence physics on transport timescales?

- When injecting RF we have both fluid (slow, 0-subscripted) and RF wave (fast, 1-subscripted) timescales

$$f_\alpha = f_{0\alpha} + f_{1\alpha} \rightarrow \quad \rho_\alpha = \rho_{0\alpha} + \rho_{1\alpha} \quad ; \quad \vec{E} = \vec{E}_0 + \vec{E}_1$$
$$\vec{J}_\alpha = \vec{J}_{0\alpha} + \vec{J}_{1\alpha} \qquad \vec{B} = \vec{B}_0 + \vec{B}_1$$
$$\ldots$$

- Terms quadratic in fast-time quantities contribute physics on slow timescales, as when DC-like terms arise in the trigonometric relation

$$[\cos x]^2 = \frac{1}{2} + \frac{1}{2}\cos 2x$$



- Slow, fast, and quadratic (mixed) components appear when fluid velocity is expressed in terms of fundamental RF physics variables. Define

flow velocity $\vec{V}_{0\alpha} = \frac{\vec{J}_{0\alpha}}{\rho_{0\alpha}}$ and jitter velocity $\vec{V}_{1\alpha} = \frac{(\vec{J}_{1\alpha} - \rho_{1\alpha}\vec{V}_{0\alpha})}{\rho_{0\alpha}}$,

then total velocity is

$$\vec{V}_\alpha = \frac{\vec{J}_\alpha}{\rho_\alpha} = \frac{\vec{J}_{0\alpha} + \vec{J}_{1\alpha}}{\rho_{0\alpha} + \rho_{s\alpha}} = \vec{V}_{0\alpha} + \vec{V}_{1\alpha} - \frac{\rho_{1\alpha}\vec{V}_{1\alpha}}{\rho_{0\alpha} + \rho_{1\alpha}}$$

- Fluid pressure also has slow, fast, and mixed components

$$\overleftrightarrow{P}_\alpha = \overleftrightarrow{P}_{0\alpha} + \overleftrightarrow{P}_{1\alpha} - \frac{m_\alpha}{q_\alpha}\frac{\rho_{0\alpha}\vec{V}_{1\alpha}\vec{V}_{1\alpha}}{\rho_{0\alpha} + \rho_{1\alpha}}$$

RF-induced ponderomotive effects contribute to slow-timescale momentum

- Momentum equation, on slow (fluid) timescale, thus has an added source term

$$m_\alpha n_{0\alpha}\left[\frac{\partial \vec{V}_{0\alpha}}{\partial t} + (\vec{V}_{0\alpha}\cdot\nabla)\vec{V}_{0\alpha}\right] + \nabla\cdot\overleftrightarrow{\mathbb{P}}_{0\alpha} = q_\alpha n_{0\alpha}(\vec{E}_0 + \vec{V}_{0\alpha}\times\vec{B}_0) + \langle\vec{F}_\alpha\rangle_0 + momentum\ sources/sinks$$

in the form of a new ponderomotive force density:

$$\vec{F}_\alpha = \rho_{1\alpha}\vec{E}_1 + \vec{J}_{1\alpha}\times\vec{B}_1 - \nabla\cdot\left[\frac{m_\alpha}{q_\alpha}\frac{\rho_{0\alpha}^2\vec{V}_{1\alpha}\vec{V}_{1\alpha}}{(\rho_{0\alpha}+\rho_{1\alpha})}\right]$$

arising from RF fields, charge densities, current densities.

- Volumetric ponderomotive source terms also arise in species energy equations, as RF waves damp and transfer power to the plasma

The underlying physics of RF-induced ponderomotive forces is rich and complex

$$\mathbf{F}_\alpha = -\frac{m_\alpha n_{0\alpha}}{4}\nabla(|\boldsymbol{V}_{1\alpha}|^2) - \frac{q_\alpha n_{0\alpha}}{2\omega}\nabla\cdot[Im(\boldsymbol{V}_{1\alpha}\boldsymbol{V}_{1\alpha}{}^*)\times\boldsymbol{B}_0]$$
$$+\frac{q_\alpha n_{0\alpha}}{4\omega}\nabla[Im(\boldsymbol{V}_{1\alpha}\times\boldsymbol{V}_{1\alpha}{}^*)\cdot\boldsymbol{B}_0] - \frac{q_\alpha}{2\omega}\boldsymbol{B}_0\times[Im(\boldsymbol{V}_{1\alpha}\boldsymbol{V}_{1\alpha}{}^*)]\cdot\nabla n_{0\alpha}$$
$$+\frac{m_\alpha n_{0\alpha}\nu_\alpha}{2\omega}\nabla\cdot[Im(\boldsymbol{V}_{1\alpha}\boldsymbol{V}_{1\alpha}{}^*)] + \frac{m_\alpha n_{0\alpha}\,\nu_\alpha}{2\omega}Im([\nabla\boldsymbol{V}_{1\alpha}]\cdot\boldsymbol{V}_{1\alpha}{}^*)$$
$$-\frac{m_\alpha}{2\omega}Im(\boldsymbol{V}_{1\alpha}\boldsymbol{V}_{1\alpha}{}^*)\cdot\nabla(n_{0\alpha}\nu_\alpha)$$

(after a Fourier transform in time, and considerable mathematical manipulation)

- Grad-V^2 term: like single-particle picture of ponderomotive force, with V_1 the "jitter velocity".
- Density gradient term: only non-zero for circular polarization, carries sign of charge. (Maybe important for RF waves launched into H-mode plasmas?)
- Green terms (like red) also carry sign of the charge; purple and black (neutral collision) terms do not.

RF-induced parallel momentum sources are the primary focus of this talk

- Perpendicular momentum sources may contribute interesting physics (e.g. convective cell dynamics): forces perpendicular to B induce drifts

$$\mathbf{V}_{s,\mathrm{drift}} \sim (\mathbf{F}_s \times \mathbf{b})/(q_s|\boldsymbol{B}_0|)$$

  associated with ambipolar or non-ambipolar convection.

- Restrict attention to a subset of PF terms on previous slide.

- Heuristic estimate: ponderomotive forces in plasma edge/SOL will most significantly influence parallel momentum transport, relative to other transport processes
  - Dominant effect – changes to plasma density in front of the antenna.

- In this talk: neglect ponderomotive contributions to energy equations, cross-field momentum transport, etc.

Our computations couple an edge plasma model (UEDGE) and an RF wave model (Vorpal)

- Vorpal = FDTD code, models RF antenna geometry and wave propagation
- UEDGE = implicit finite-volume code, models edge transport of plasma/neutrals

Coupling scheme:

- Run UEDGE, generate initial solution (equilibrium edge/SOL transport)
  -Map plasma profiles to Vorpal grid (uniform, larger domain)
- Run Vorpal, model RF wave propagation through specified equilibrium
  -Compute ponderomotive forces associated with normalized power flow
  -Map solution back to UEDGE grid (variable, smaller domain)
- Rerun UEDGE, adding ponderomotive force as new source term
  -PF source term modifies equilibrium transport and profiles
- Rerun Vorpal; assess how wave propagation is altered by modified profile
- Repeat cycle to convergence, if attainable/desired

  - Vorpal: ~48 hours on 32 cores, ~1M grid cells, dt = 1e-12
  - UEDGE: ~10 hours in serial (for scan), ~2k grid cells

UEDGE solves a system of fluid equations in axisymmetric tokamak geometry

- plasma density
- ion || momentum
- electron thermal energy
- ion thermal energy
- neutral density
- ad hoc radial transport
- neutral || momentum
- charge conservation
- sheath bound. cond.

$$\frac{\partial}{\partial t}(n_i) + \nabla \bullet (n_i \vec{u}_i) = -S_r + S_i$$

$$n_i u_\perp = -D_{\perp i} \nabla_\perp n_i$$

$$\frac{\partial}{\partial t}(m n_i u_{\parallel i}) + \nabla \bullet (m n_i u_{\parallel i} \vec{u}_i - \eta_i \nabla u_{\parallel i}) = -\nabla_\parallel P_i + m n_N n_i K_{cx}(u_{\parallel N} - u_{\parallel i}) + m S_r u_{\parallel N} - m S_i u_{\parallel N}$$

+ ponderomotive sources

$$\frac{\partial}{\partial t}(3/2 n_e T_e) + \nabla \bullet (\frac{5}{2} n_e T_e \vec{u}_e + \vec{q}_e) = \vec{u}_e \bullet \nabla(3/2 n_e T_e) - \Pi_e \bullet \nabla \vec{u}_e + Q_e$$

$$\frac{\partial}{\partial t}(3/2 n_i T_i) + \nabla \bullet (\frac{5}{2} n_i T_i \vec{u}_i + \vec{q}_i) = \vec{u}_i \bullet \nabla(3/2 n_i T_i) - \Pi_i \bullet \nabla \vec{u}_i + Q_i$$

$$q_\perp = -n \chi_\perp \nabla_\perp T$$

$$\frac{\partial}{\partial t}(n_N) + \nabla \bullet (n_N \vec{u}_N) = S_r - S_i$$

$$n_N u_{\perp N} = -D_{\perp N} \nabla_\perp n_N$$

$$\frac{\partial}{\partial t}(m n_N u_{\parallel N}) + \nabla \bullet (m n_N u_{\parallel N} \vec{u}_N - \eta_N \nabla u_{\parallel N}) = -\nabla_\parallel P_N - m n_N n_i K_{cx}(u_{\parallel N} - u_{\parallel i}) - m S_r u_{\parallel N} + m S_i u_{\parallel N}$$

$$\nabla \bullet J(\phi) = 0$$

$$J_\parallel = \frac{en}{0.51 m \nu} \frac{B_x}{B} \left(\frac{1}{n} \frac{\partial P_e}{\partial x} - e \frac{\partial \phi}{\partial x} + 0.71 \frac{\partial T_e}{\partial x} \right)$$

+ ponderomotive sources

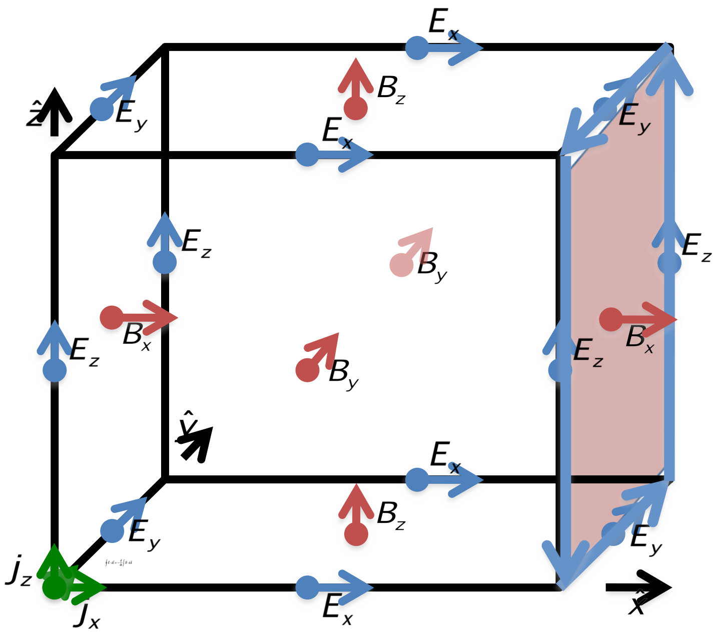
$$J_r = \sigma_\perp E_r$$

+ all cross-field drifts

$$\phi = \frac{-Te}{e} \ln\left[2\sqrt{\pi} \left(\frac{J_\parallel - e n u_{\parallel i}}{e n v_{te}} \right) \right]$$

*T. D. Rognlien et al., J. Nucl. Mater. 196–198, 347 (1992)

Vorpal models RF and plasma waves using finite-difference time-domain methods



K. S. Yee, *IEEE Trans. Antennas Propag.* **14**, 302 (1966).

- FDTD approach: preserve vector operations ($\nabla\times, \nabla\cdot, \nabla$); center fields in space & time, on discrete cells (***E*** @ cell edges, ***B*** @ cell faces)

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

- Time-centered - leap-frog time staggering puts ***E*** at full timesteps, ***B*** at half timesteps

- Adding current sources enables cold plasma waves to be modeled (J @ cell nodes). Semi-implicit method avoids $\omega_p \Delta t \le 2$ constraint.

*Explicit Faraday step

$$\frac{\partial \vec{B}_1}{\partial t} = -\vec{\nabla}\times\vec{E}_1$$

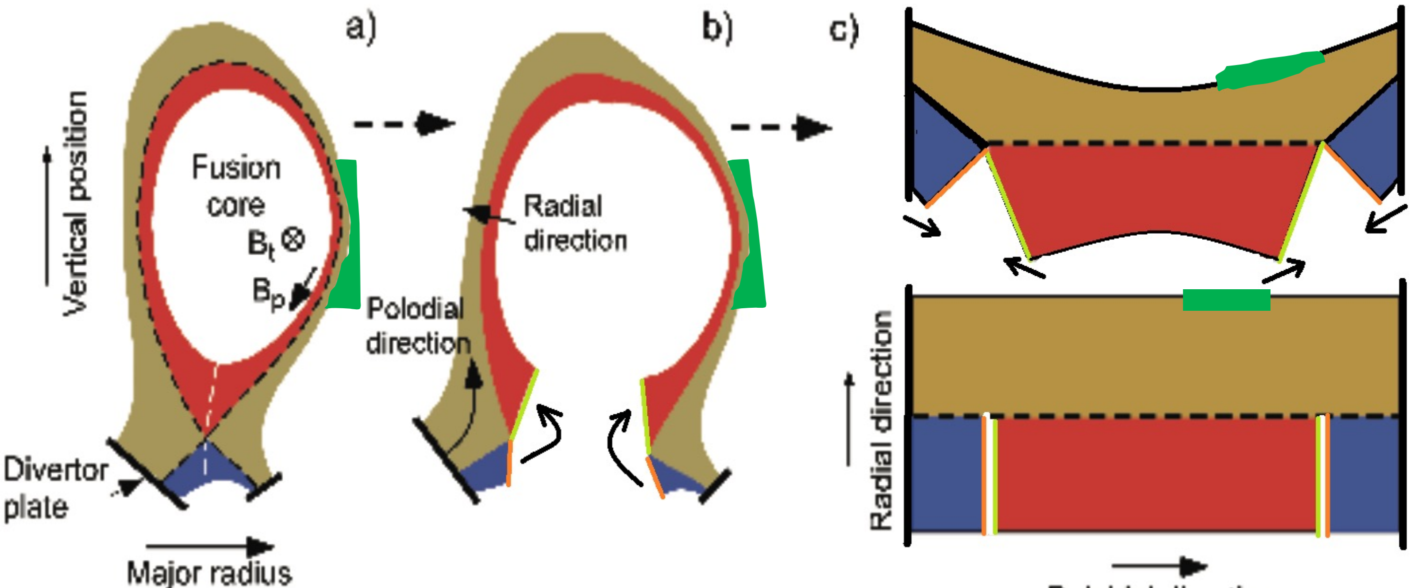
*Implicit step advances Ampere and cold current density equations in time [D. N. Smithe, Phys. Plasmas **14**, 056104 (2007)].

$$\frac{\partial \vec{E}_1}{\partial t} = c^2\vec{\nabla}\times\vec{B}_1 - \sum_\alpha \frac{\vec{J}_{1\alpha}}{\epsilon_0}$$

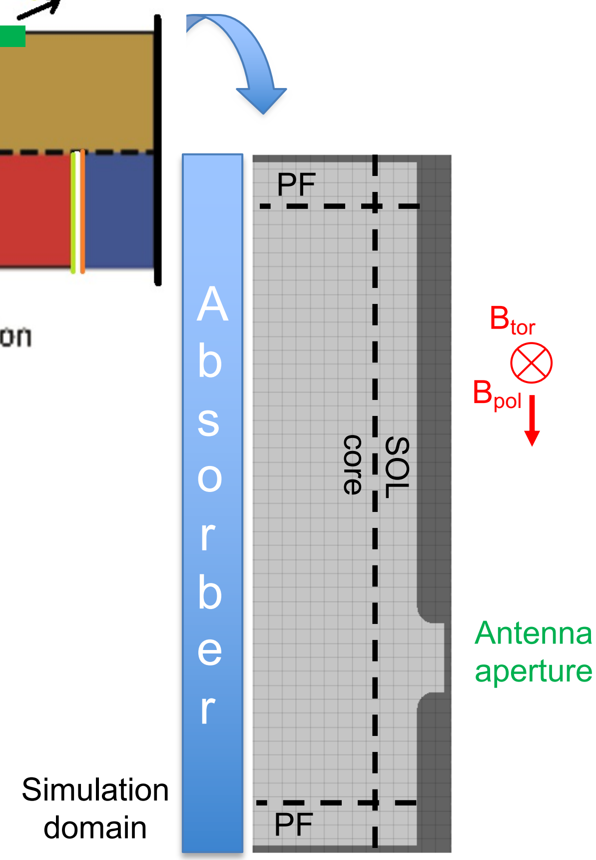
$$\frac{\partial \vec{J}_{1\alpha}}{\partial t} + \frac{q_\alpha \vec{B}_0}{m_\alpha}\times\vec{J}_{1\alpha} = \frac{q_\alpha \rho_{0\alpha}}{m_\alpha}\vec{E}_1$$

- Ponderomotive forces are computed directly from wave fields, then time-averaged in Vorpal post-processing

Vorpal/UEDGE cases are run in a 2D edge/SOL slab model, generalized to include an RF antenna (green)

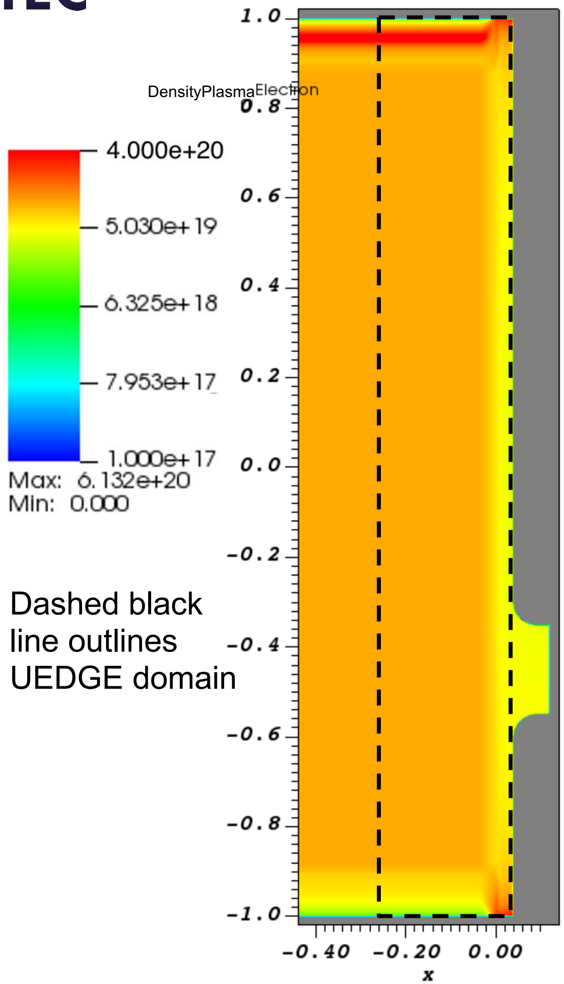


Modified from McCourt et al., Comp. Sci. Discov. **5**, 014012 (2012).



- Vorpal domain is broadened on the core side relative to UEDGE domain, with constant density and absorbing boundary conditions there
- Unlike a physical antenna, model is independent of toroidal angle

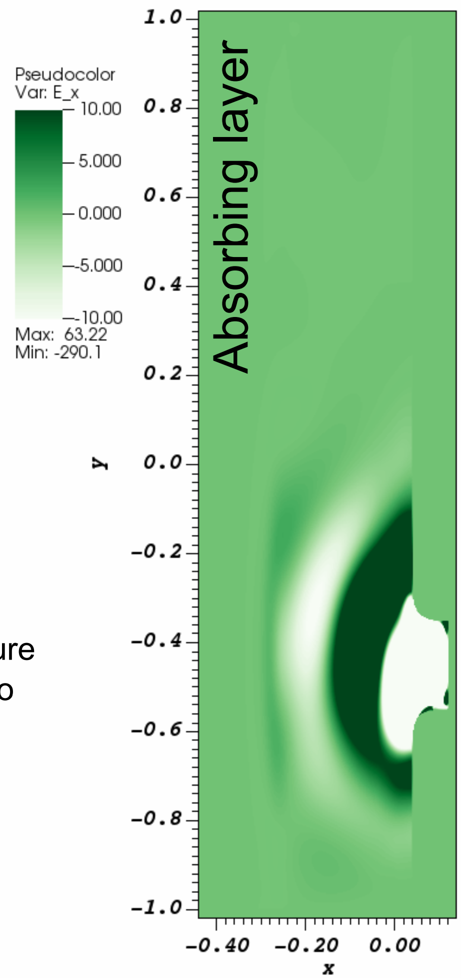
Fast wave propagates toward the core plasma after tunneling through the low-density evanescent region



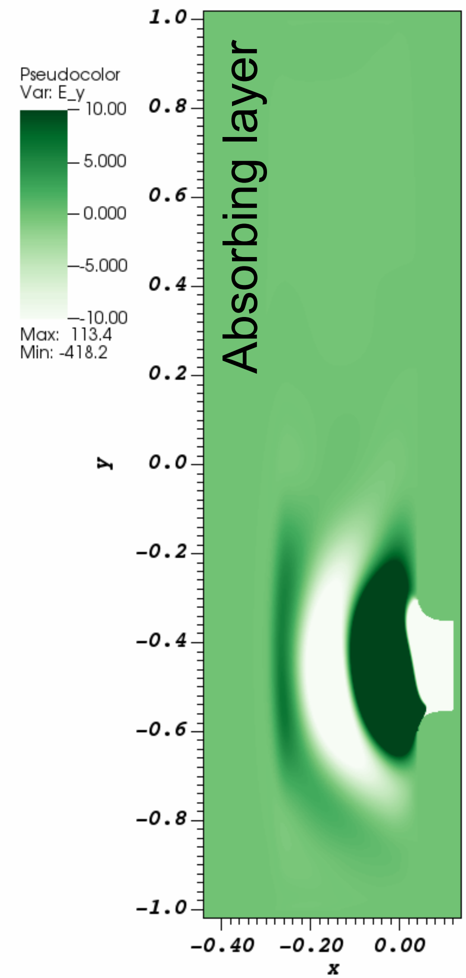
Plasma density

Dashed black line outlines UEDGE domain

Normalize RF power from antenna aperture to ~1 MW/m^2 to compute PF



E_x (radial field)

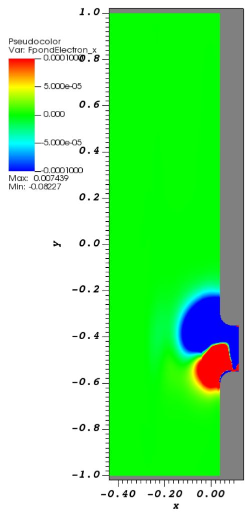


E_y (~poloidal field)

Parameters shown are for an Alcator C-Mod-like plasma:

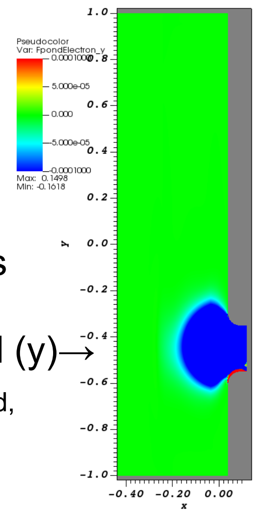
B_{tor} = 5.0 T B_{pol} = 0.5 T n_{core} ~ 1.0 x 10^{20} m^{-3} f = 40 MHz

Ion/electron PFs are oppositely directed in this fast wave scenario, yielding net current flow (small)

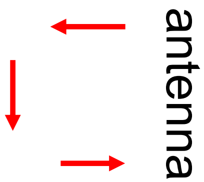


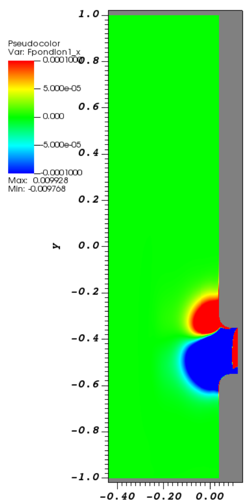
Electron forces
←radial (x)
poloidal (y)→

Blue = downward,
red = upward



Net forces (electron current in opposite direction)





Ion forces
←radial (x)
poloidal (y)→

Blue = downward,
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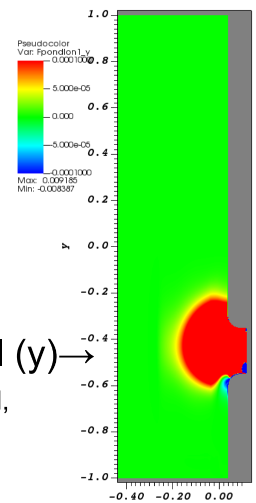
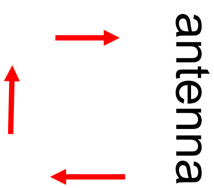


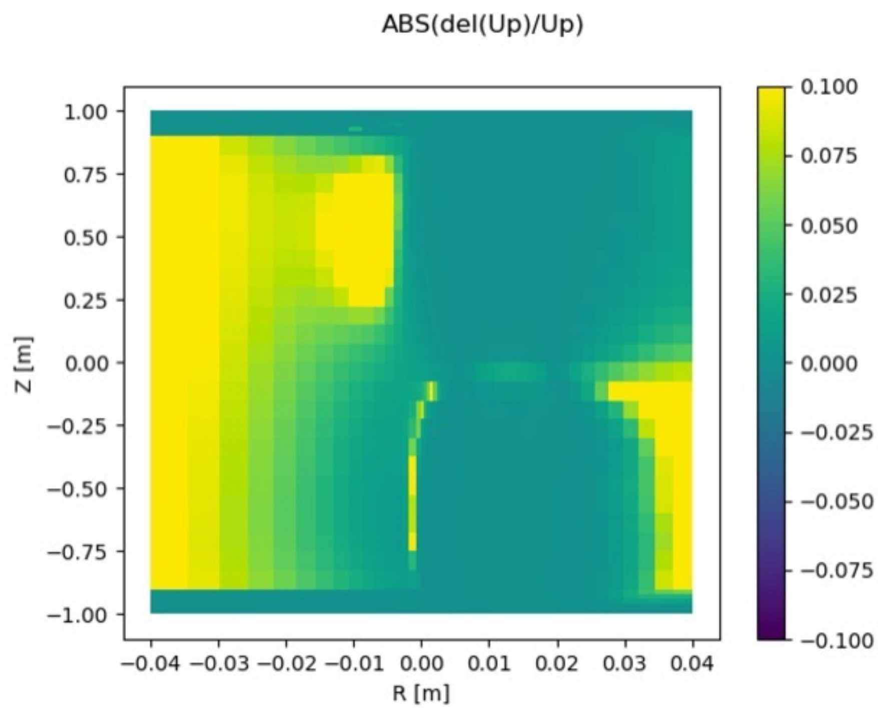
Net forces (ion current in same direction)



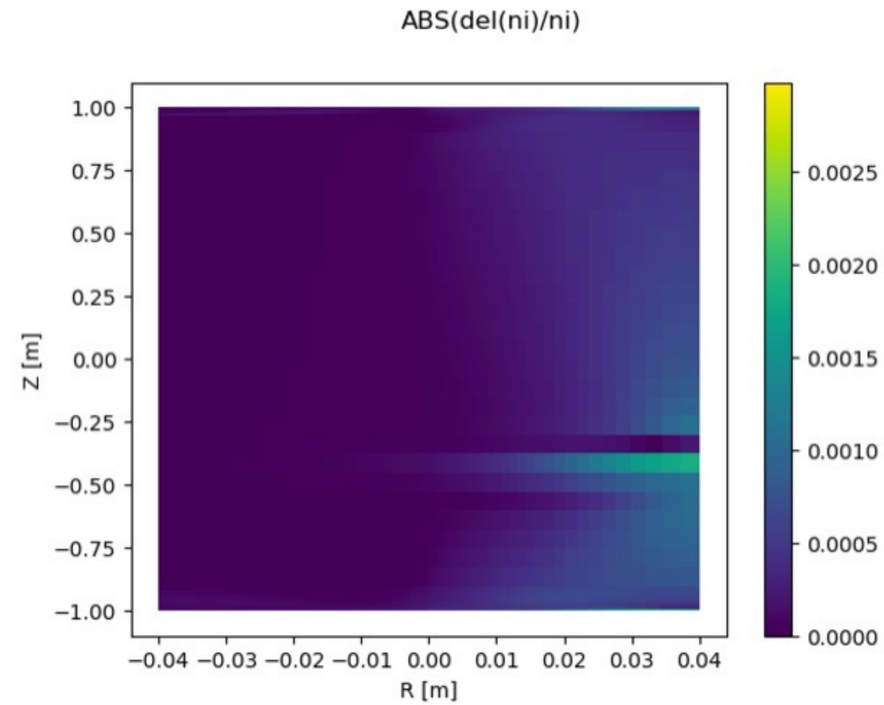
Ponderomotive effects primarily move density around in front of the antenna (non-ambipolar convective behavior with net current).

More complex force patterns might ensue in other plasma/RF regimes, but effect in this scenario is relatively inconsequential.

UEDGE results: forces, though detectable, have little effect on edge/SOL transport in this FW scenario



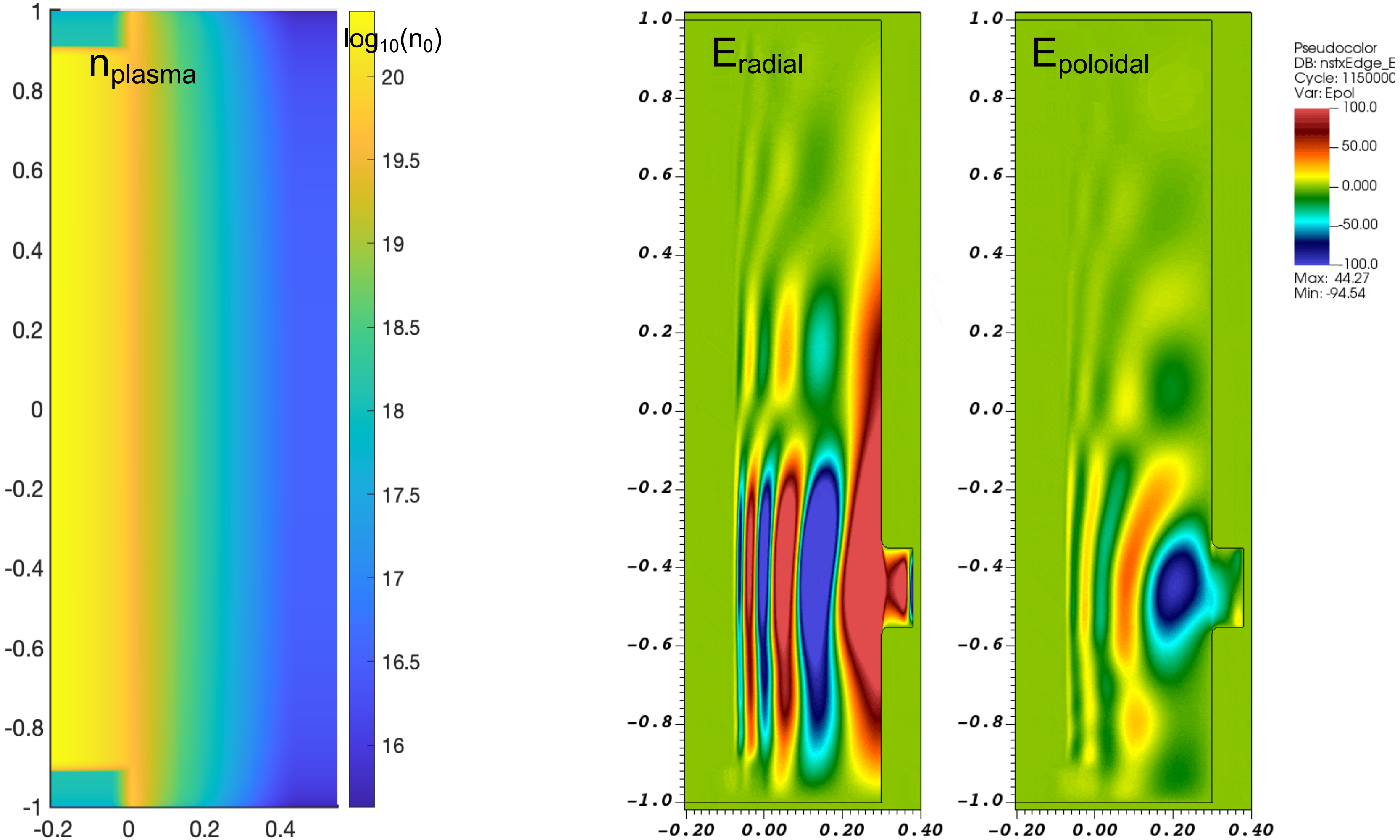
Ponderomotive effects induce > 10% changes in relative magnitude of parallel flow velocity



Overall effect on bulk plasma density is minimal (<1% change in relative magnitude)

- Generally, PF effects are smaller in cases with high plasma density and low density gradients near the antenna.

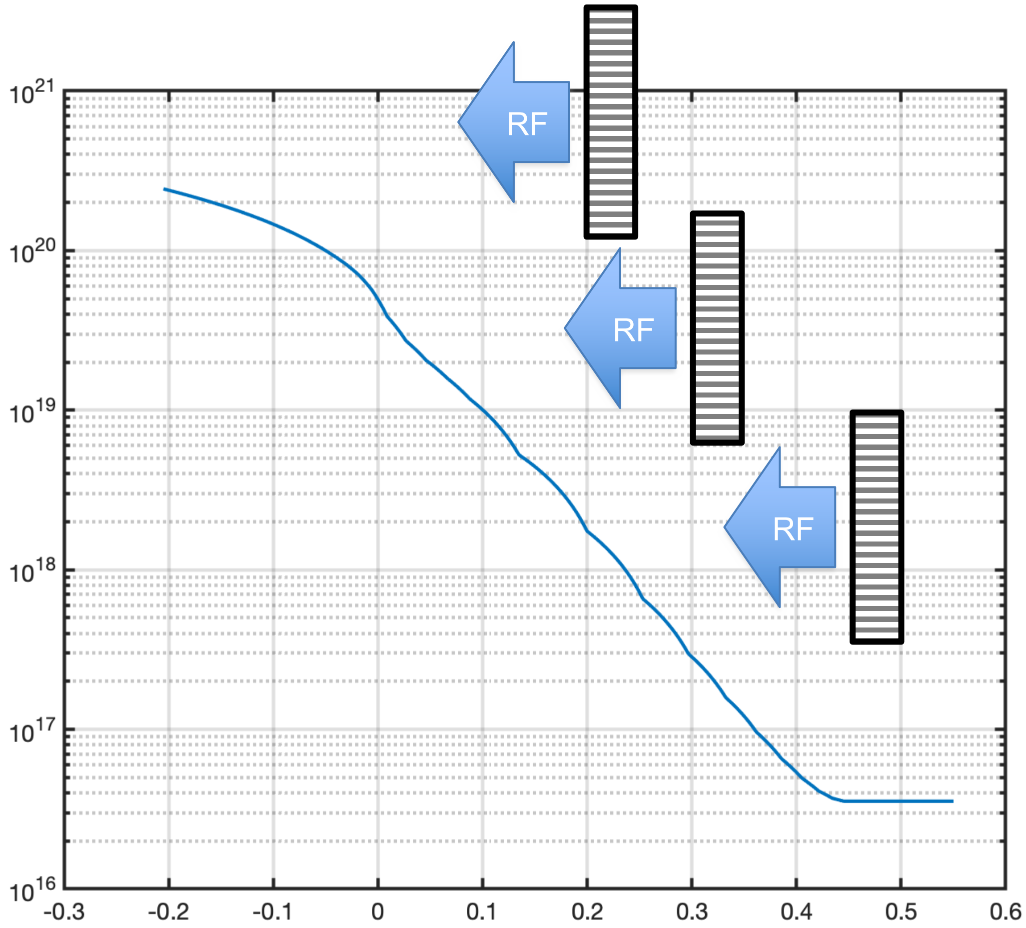
Ponderomotive forces influence edge/SOL dynamics more strongly in NSTX-like scenarios

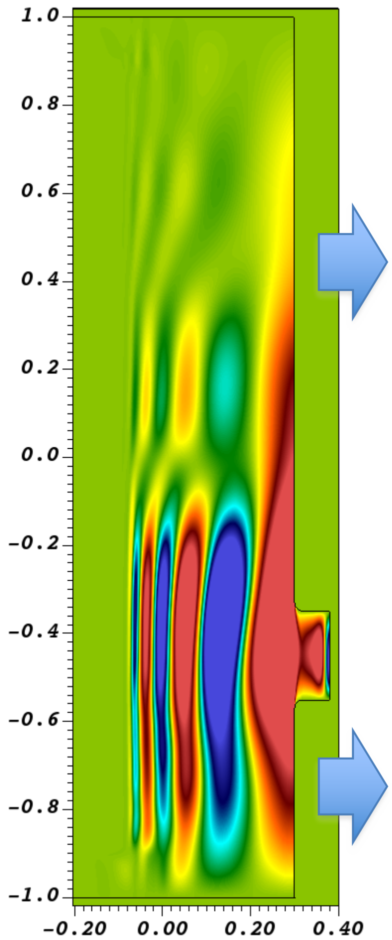


Parameters shown are for an NSTX-like plasma:

B_{tor} = 1.0 T B_{pol} = 0.5 T n_{core} ~ a few x 10^{20} m^{-3} f = 30 MHz

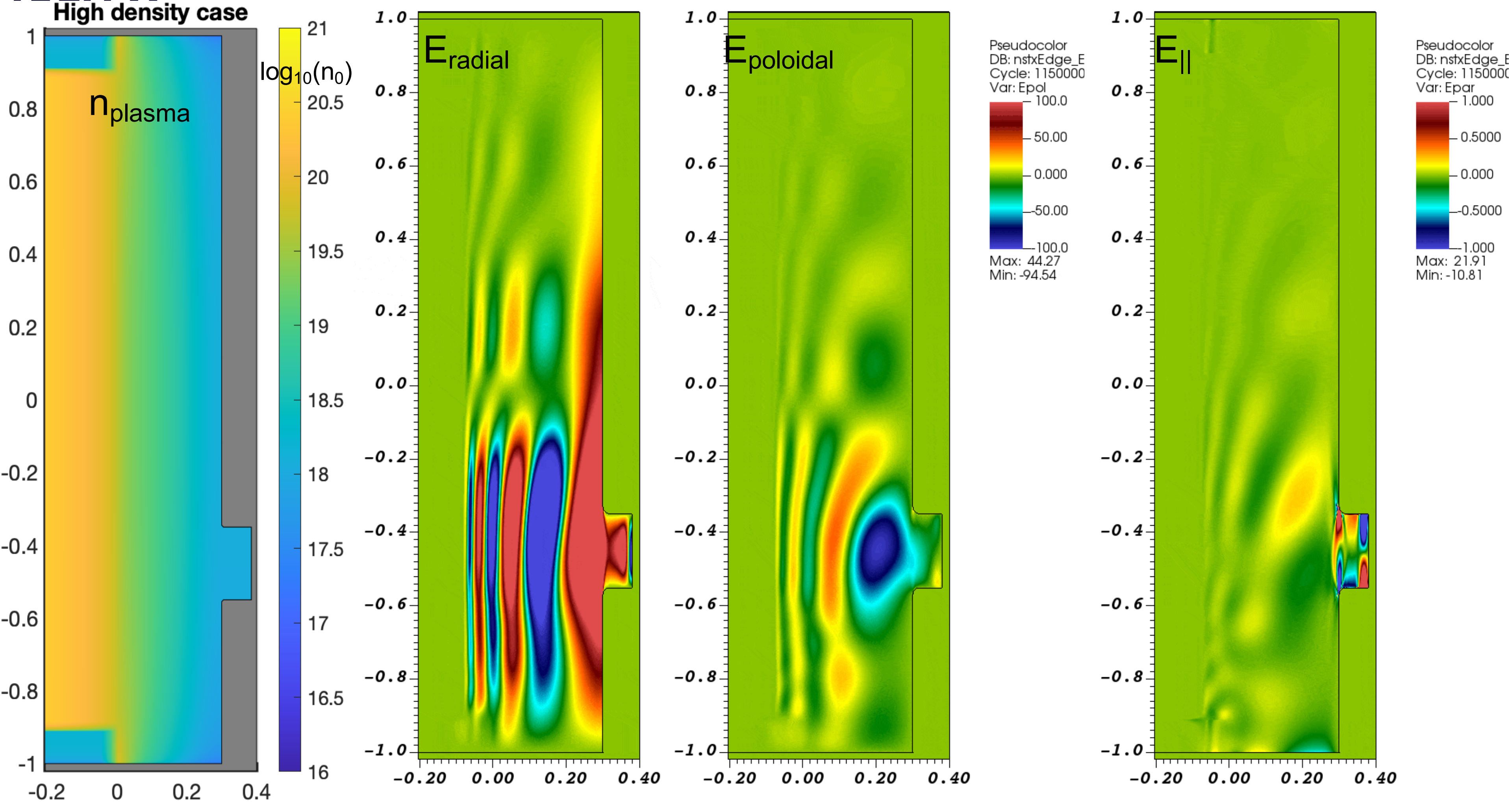
We situate the antenna at various points on the density profile and assess the ensuing PF effects





- By moving antenna and wall outward, can examine how ponderomotive effects influence edge/SOL transport as we move to lower-density regimes

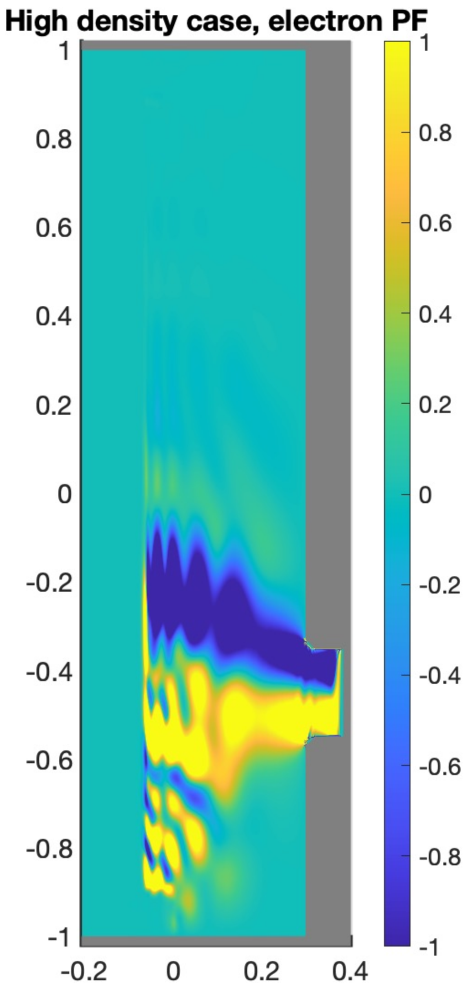
Wave propagation, high density (~10^{18} m^{-3} @ wall) case: predominantly a fast wave

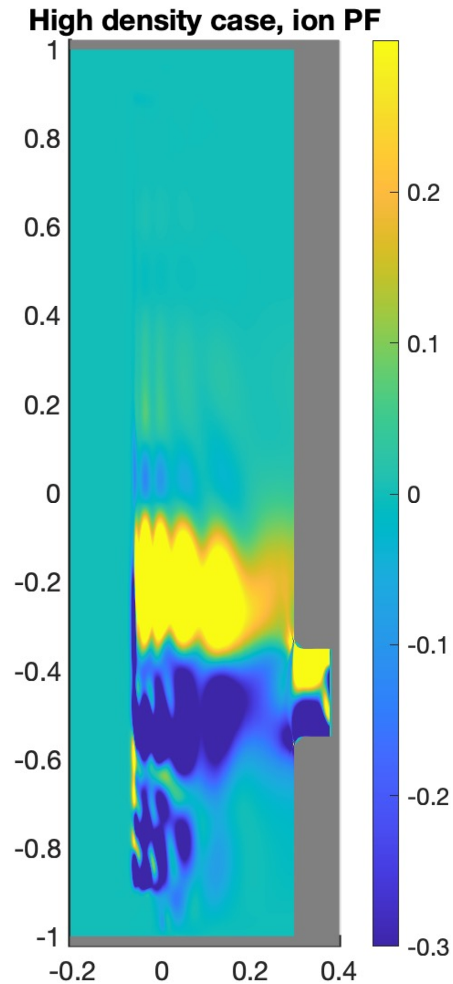


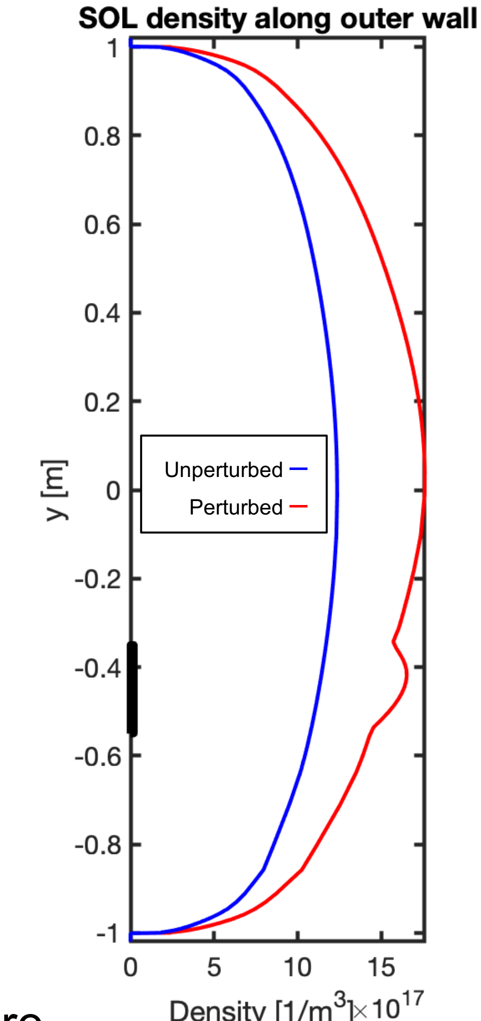
- Parallel E 100x smaller than radial, poloidal E (generally true for subsequent plots)
- Fast wave propagates at high densities

High-density (~10^{18} m^{-3}) scenario: ponderomotive forces increase density in front of the antenna

Yellow = upward force, blue = downward force

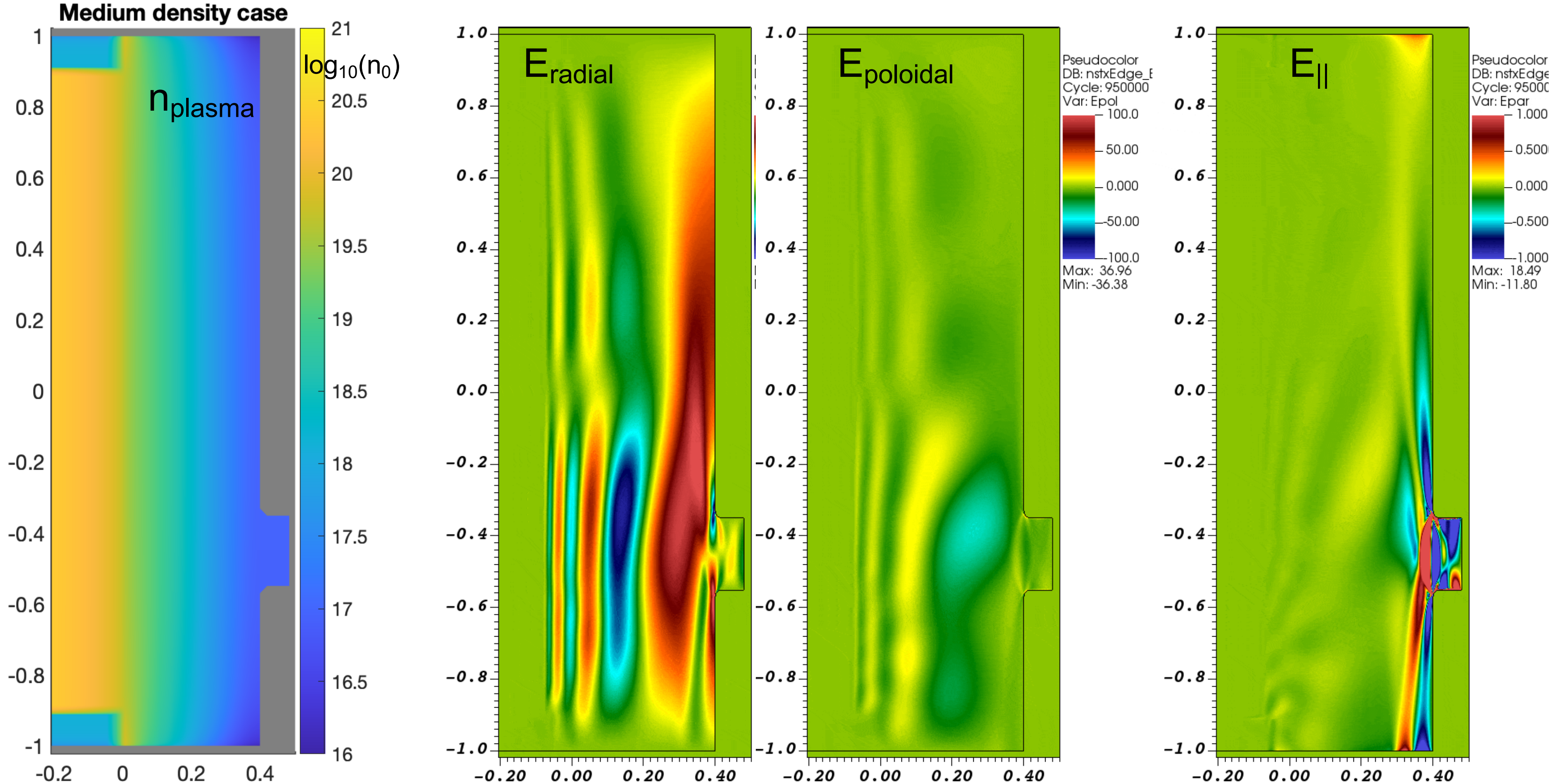






A broad region of PF density fans out from the antenna aperture, predominantly pulling electrons toward antenna and inducing locally increased density there. Broader effects on edge transport also push up the density profile generally, as both ion and electron forces impart momentum to the plasma.

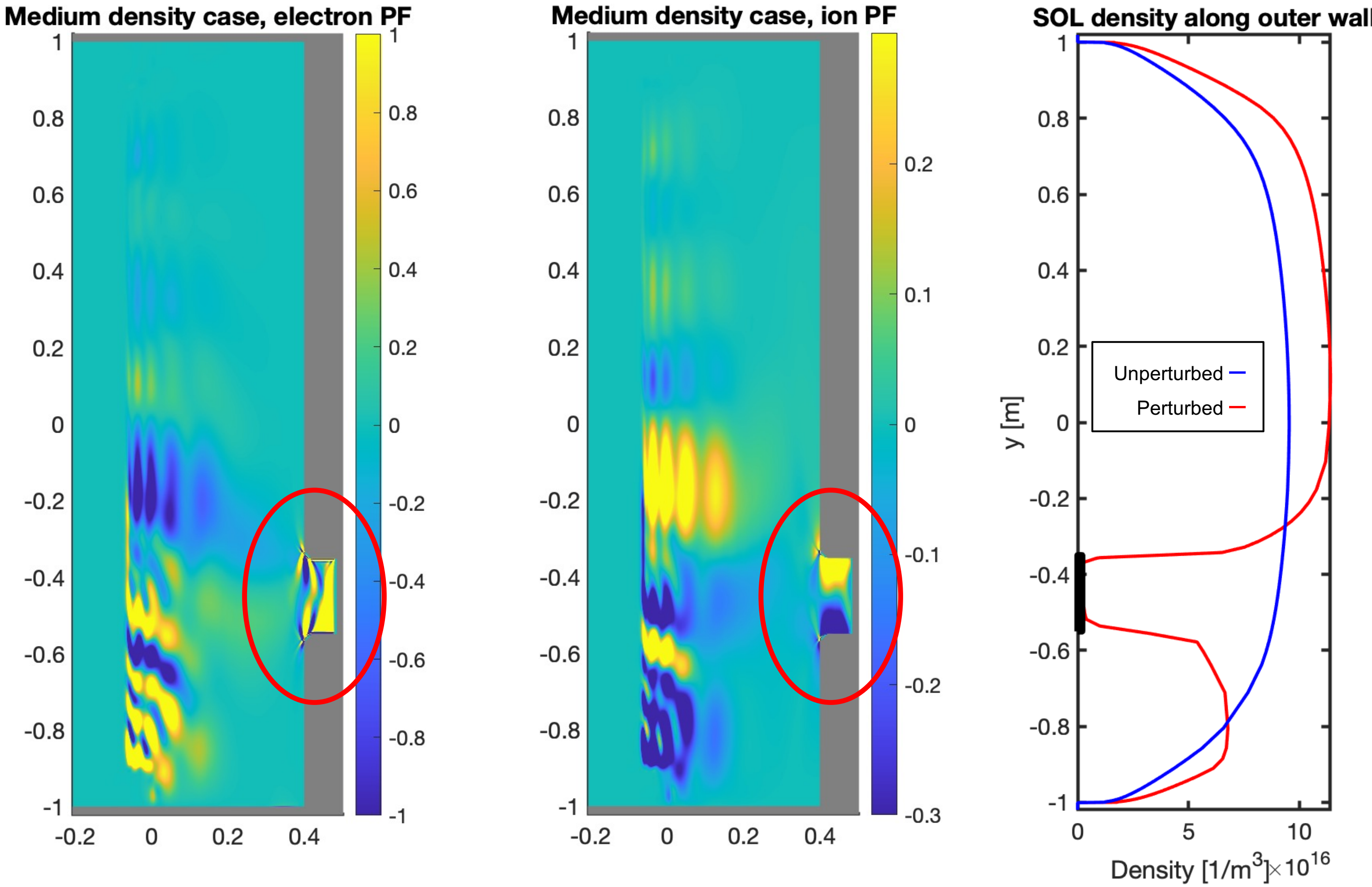
Wave propagation, medium density (~10^{17} m^{-3} @ wall) case: long-wavelength fast wave is joined by an evanescent slow wave near antenna



- Power coupling to plasma core (at left) is diminished due to losses associated with the slow wave (bottom right of E_{radial} plot)

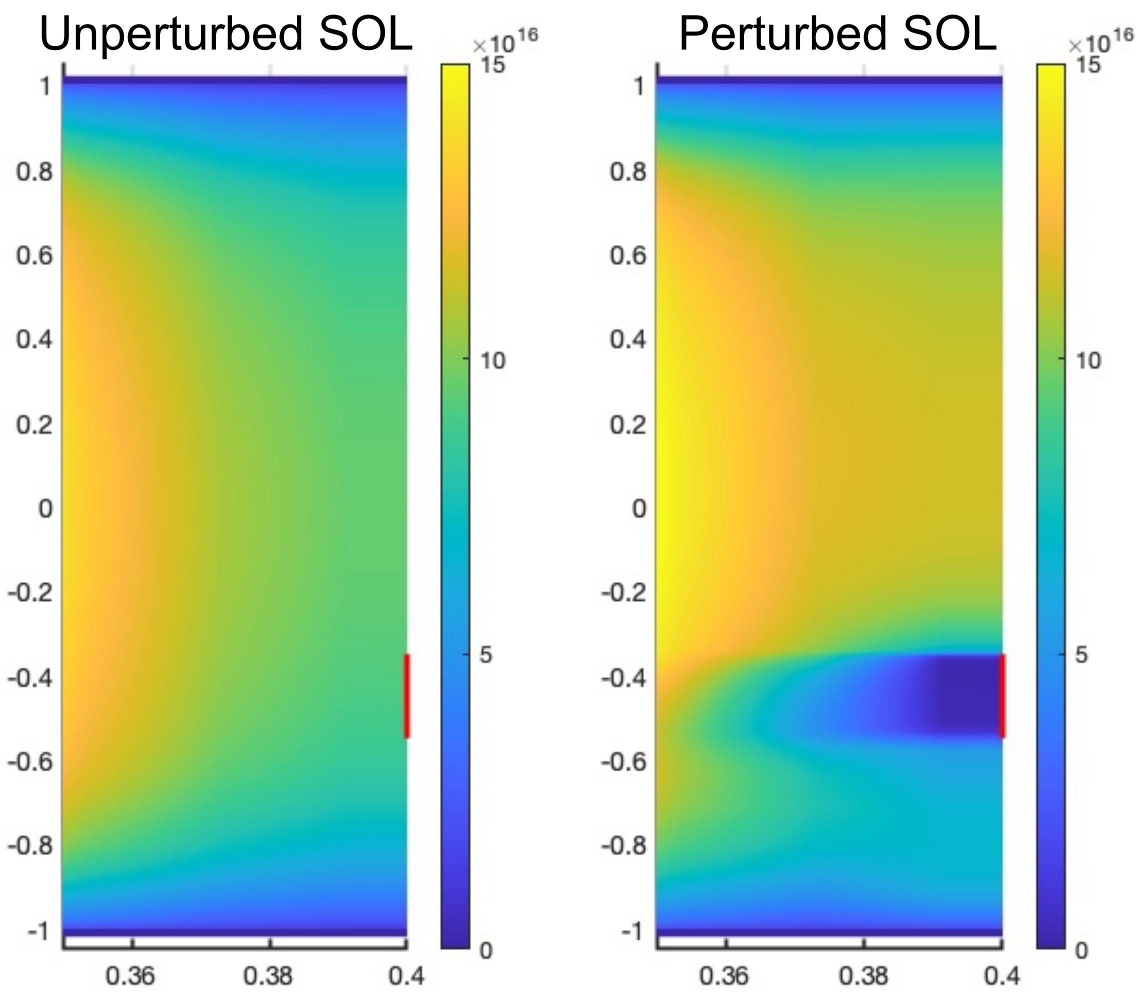
Medium-density (~10^{17} m^{-3}) scenario: ponderomotive forces begin to localize near the antenna and significantly depress the density there

Input power flux 1.0 MW/m^2



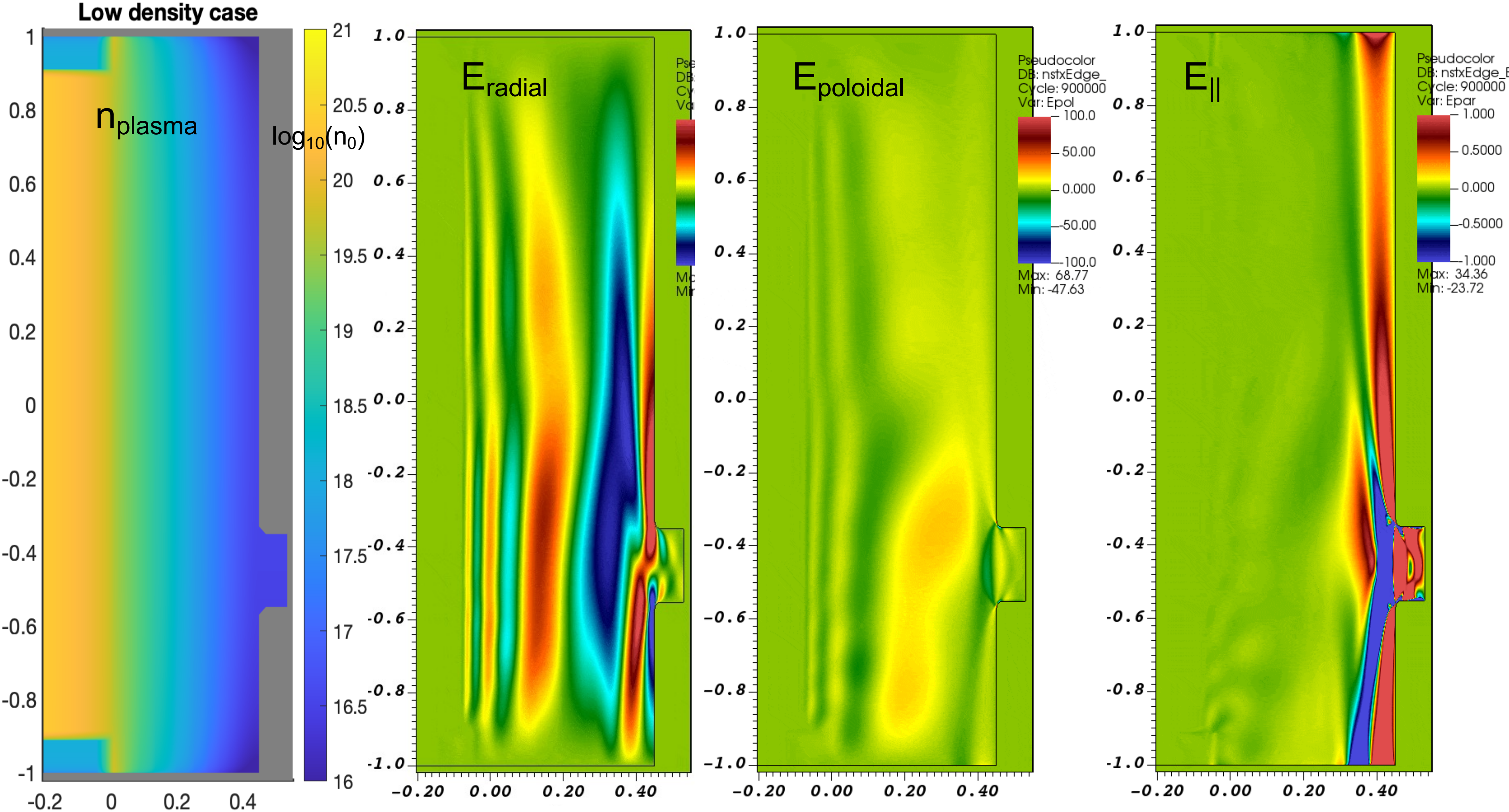
Although background density is again slightly enhanced away from the antenna aperture, the localized PF expels density from the region immediately in front of the aperture (opposite to the effect observed at higher density).

The density depression persists a few centimeters into the plasma, and will nonlinearly influence subsequent wave propagation



- The fast wave cannot propagate when the plasma density is too low
- Coupling of antenna power to plasma will potentially be strongly affected by the induced density reduction, especially if it is self-perpetuating
- Simulations of this effect are ongoing.

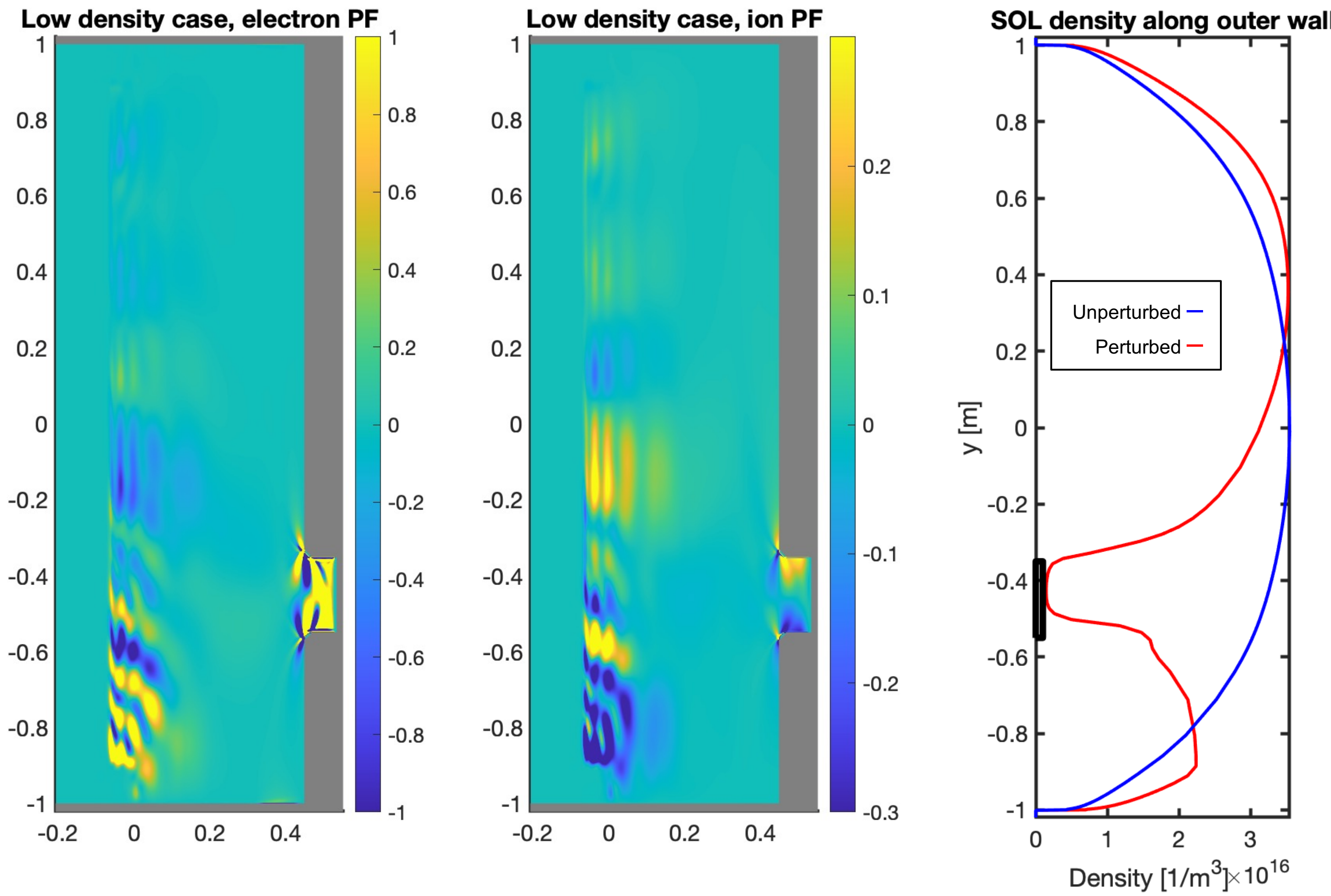
Wave propagation, low density (~10^{16} m^{-3} @ wall) case: both fast and slow waves are present at various profile locations



- Slow wave is a backward wave; both right- and left-propagating wavefronts can be seen in E_{radial} fields

Low-density (~10^{16} m^{-3}) scenario: PFs again reduce local density near antenna; provide little global density enhancement

Input power flux 0.25 MW/m^2

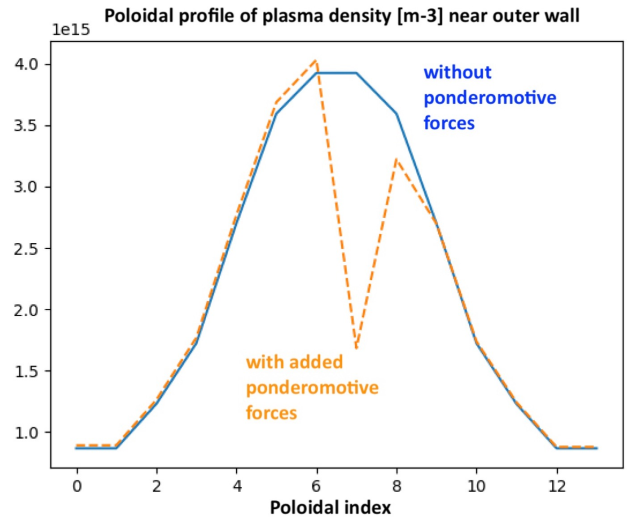


- Diminished density in front of the antenna persists; will influence subsequent coupling of antenna power to plasma

Results are consistent with heuristic estimates and UEDGE predictions: PF effects likely dominate edge transport at low densities

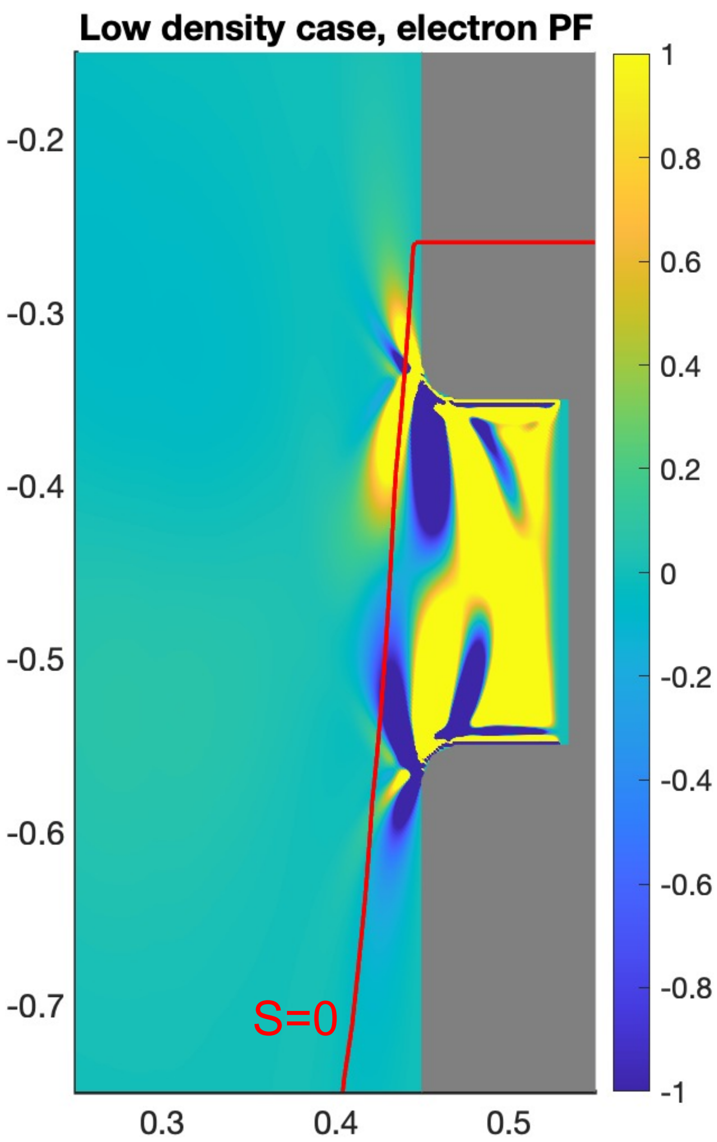
- Electron $PF_{||}$ – localized near antenna, of magnitude O(1) [N/m^3].
- Compare with thermal pressure gradient $dP/dx_{||} \sim nT/L$: for representative parameters [$n = 10^{16}$ m^{-3}, T = 10 eV, L ~ 1 m], we find $dP/dx_{||} \sim O(10^{-2})$ [N/m^3].
- Conclusion: PF terms >> other important edge transport effects.

- UEDGE computations confirm this hypothesis; forces of this magnitude significantly modify edge densities.



TECH-X

Near lower hybrid resonance, fast and slow waves contribute very differently to PF effects



- Structure of localized PF is well-resolved on Vorpal grid (dx ~ 1.25e-3) – not a numerical effect
- Lower hybrid resonance at S=0 significantly modifies launched RF waves (evanescent at low density, propagating at high density: i.e. FW above, SW below)
- Working hypothesis – PF induced below LH resonance reduces density, PF induced above LH resonance enhances it
- Potentially yields volatile RF coupling behavior due to bifurcation:

high n: PF further increases n ↑ — $S < 0$

$S = 0$

low n: PF further decreases n ↓ — $S > 0$

- Exploration of these ideas under the RF-SciDAC effort will continue throughout FY22.

Implications for ITER: ponderomotive effects are relevant and important

- ITER ICRF antennas will transmit 20 MW of power through a low-density evanescent region
  - Ponderomotive forces will be substantial due to the high power flux
  - To achieve optimal power coupling, their effects need to be well understood
- Vorpal can simulate the ITER antenna with full geometric fidelity; associated PF effects on edge/SOL transport can also be modeled using the toolset we've built
  - Upcoming RF-SciDAC efforts will focus on this modeling
  - Extension of present work to 3D is of interest
- Work thus far strongly validates the concept of gas puffing near the ITER ICRF antennas, to prop up density locally and promote good RF power coupling
  - See, e.g., W. Zhang et al., "Scrape-off layer density tailoring with local gas puffing to maximize ICRF power coupling in ITER", Nucl. Mater. Energy **19**, 364 (2019).

TECH-X

Summary/Conclusions

- The physics of ponderomotive forces is rich and complex, and will become increasingly consequential at the RF power fluxes needed for burning plasma experiments (ITER, SPARC, etc.).
- We have successfully coupled Vorpal and UEDGE to model ponderomotive effects on edge/SOL transport.
- RF waves transitioning from evanescence to propagation can alter edge/SOL transport in quite different ways on opposite sides of the transition point, possibly leading to bifurcation.
- Much interesting work remains to be done.

This talk will be available on my website: https://nucleus.txcorp.com/~tgjenkins