Efficient Calculation of and Coil Shape Optimization for Lorentz Forces

Siena Hurwitz[1], Matt Landreman[1], Thomas M. Antonsen[1]
[1]University of Maryland, College Park

Introduction

- Lorentz forces are a key coil engineering design parameter for magnetic confinement fusion reactors
- Lorentz forces are generally evaluated slowly with finite element analysis codes
- It is desirable to evaluate the forces rapidly to facilitate integration with physics design
- A simple filamentary approximation for $dF/d\ell$ with the Biot-Savart law fails as the result diverges logarithmically

Reduced Model Derivation

- Construct a quasi-cylindrical coordinate system from the coil center-line, r_c, and the Frenet-Serret unit vectors, $\{e_1, e_2, e_3\}$,

$$\boldsymbol{r}(s,\theta,\phi) = \boldsymbol{r}_c + s\cos\theta\, \boldsymbol{e}_2 + s\sin\theta\, \boldsymbol{e}_3$$

$$(\, e_1 = r_c'/|r_c'|,\ e_2 = e_1'/|e_1'|,\ e_3 = e_1 \times e_2 \,)$$

- Assume uniform current density and a circular cross-section
- Express the magnetic vector potential and Lorentz force in the system,

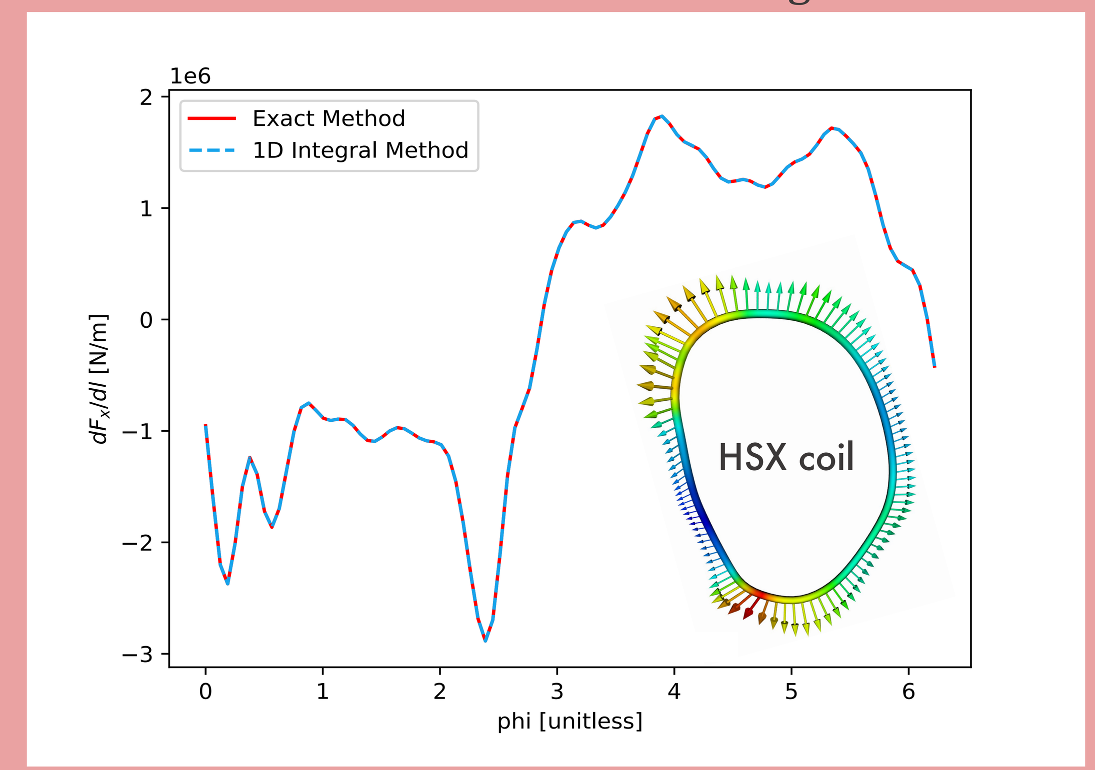
$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0 I}{4\pi a^2}\int_0^{2\pi} d\tilde{\phi} \int_0^{2\pi} d\tilde{\theta} \int_0^{a} d\tilde{s}\, \tilde{s}(1-\tilde{\kappa}\tilde{s})\frac{\tilde{\boldsymbol{r}}_c'}{|\Delta \boldsymbol{r}|}$$

$$\frac{d\boldsymbol{F}}{dl} = \frac{I}{\pi a^2}\int_0^{2\pi} d\theta \int_0^{a} ds\, s(1-\kappa s)(\boldsymbol{e}_1 \times \boldsymbol{B})$$

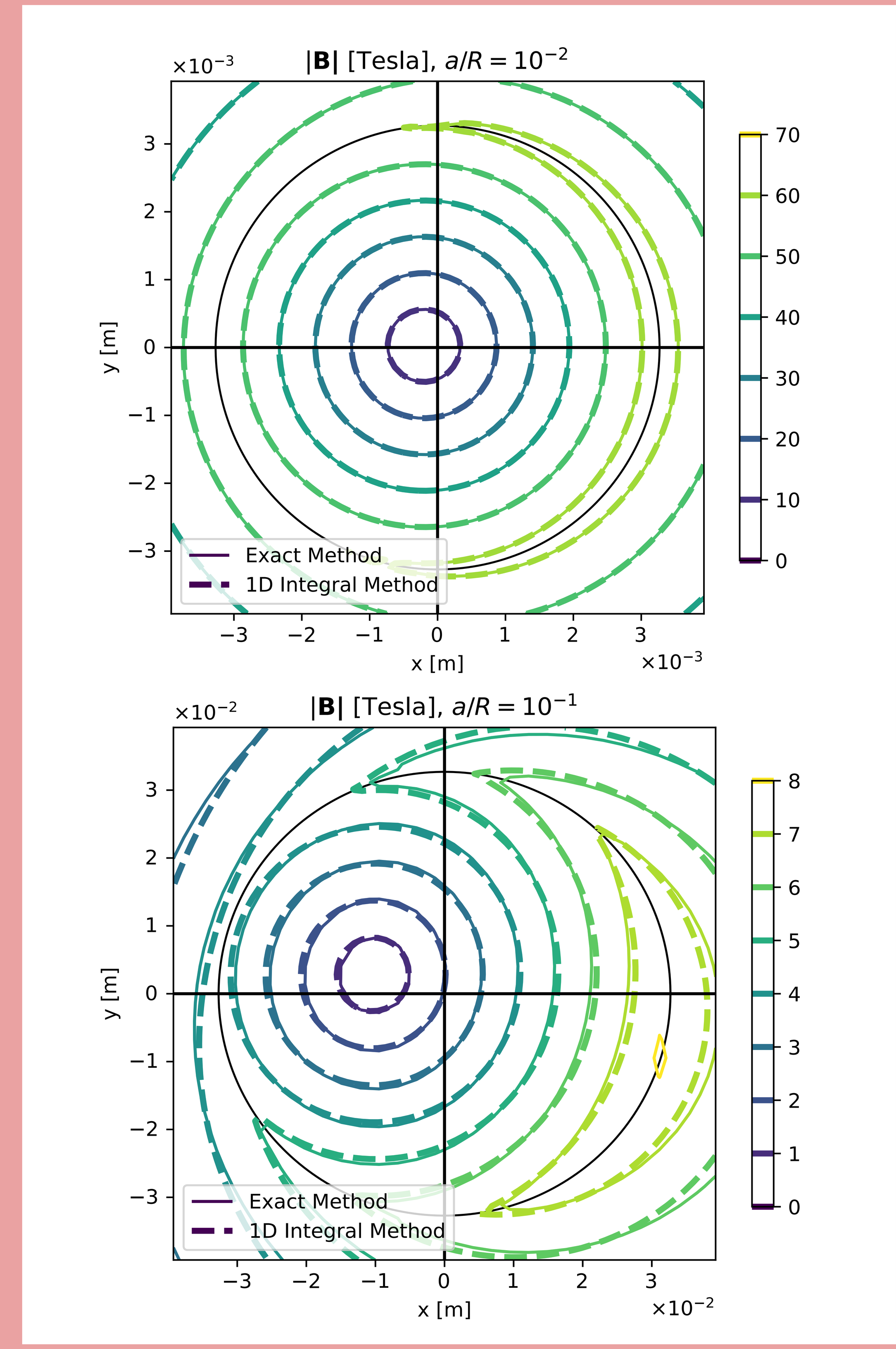
- Assume $a/\mathcal{L} \ll 1$, where $\mathcal{L}$ is a length scale of r_c (e.g., $|r_c'|, \kappa^{-1}$)
- Partition the integral over $\tilde{\phi}$ in $\boldsymbol{A}(\boldsymbol{r})$ into two sections about ϕ_0, where $a/\mathcal{L} \ll \phi_0 \ll 1$
  - The near region ($|\phi - \tilde{\phi}| < \phi_0$) satisfies $|\Delta\phi| \ll 1$
  - The far region ($|\phi - \tilde{\phi}| > \phi_0$) satisfies $a/|\Delta \boldsymbol{r}| \ll 1$
- Solve for $\boldsymbol{A}(\boldsymbol{r})$ and simplify under these assumptions
- Posit a regularized form of the magnetic field, $\boldsymbol{B}_{\text{reg}}$, then calculate $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ and $d\boldsymbol{F}/d\ell$ explicitly and compare to $\boldsymbol{B}_{\text{reg}}$

Reduced Model Results

$$d\boldsymbol{F}/d\ell = I\boldsymbol{e}_1 \times \boldsymbol{B}_{\text{reg}}$$



$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{B}_{\text{reg}} + \boldsymbol{B}_{\text{cylinder}} + \boldsymbol{\mathcal{O}}(\kappa s)$$



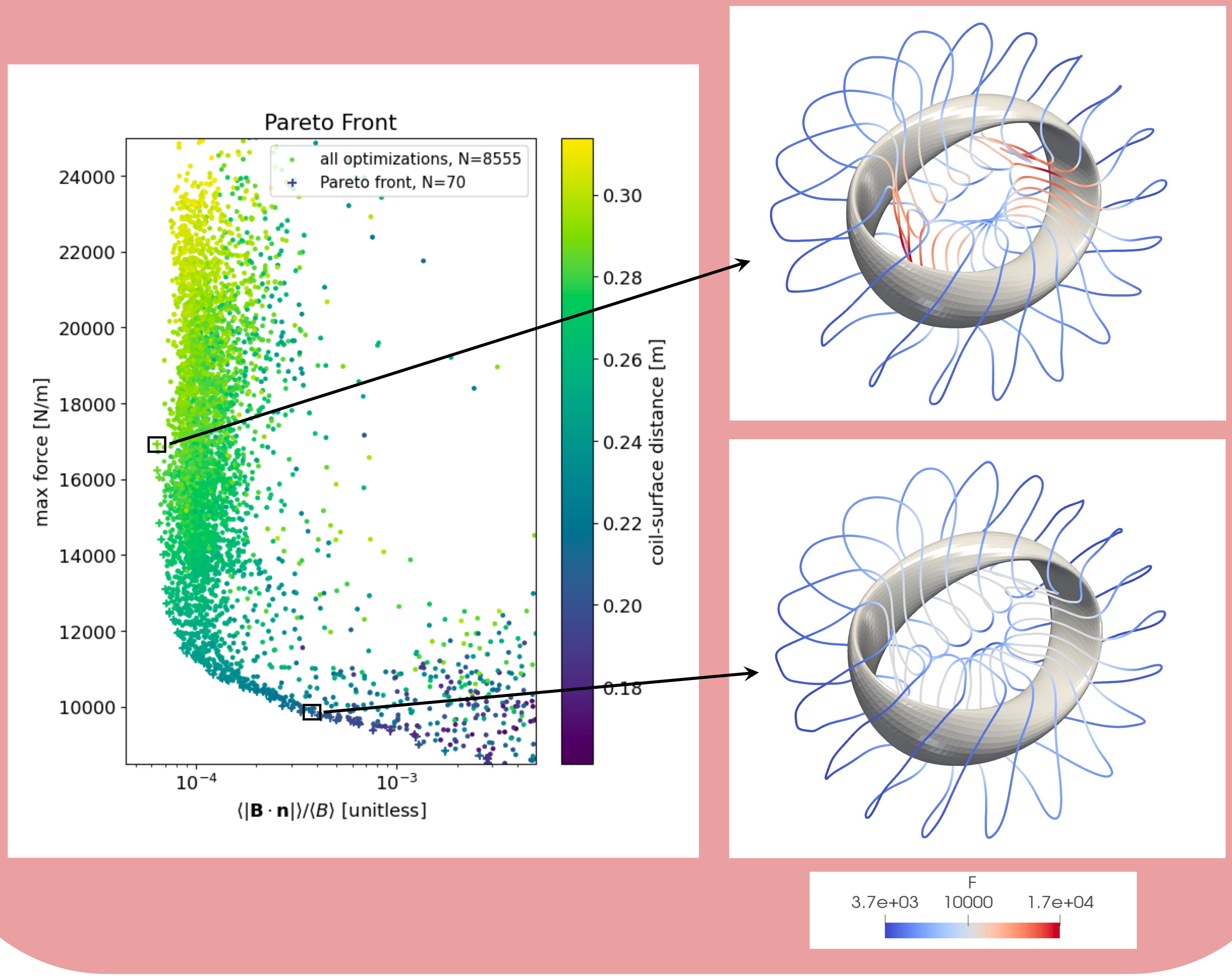
~10,000x faster than exact computations!

$$\boldsymbol{B}_{\text{reg}} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} d\tilde{\phi}\, \frac{\tilde{\boldsymbol{r}}_c' \times \Delta \boldsymbol{r}_c}{(|\Delta \boldsymbol{r}_c|^2 + a^2/\sqrt{e})^{3/2}}$$

Force Optimization Objective

- Optimized coils for the Landreman-Paul quasi-axisymmetric configuration, $N_{fp} = 2$
- Objective function penalizes
  - quadratic flux
  - Lorentz force
  - coil lengths
  - arc length variance
  - curvature
  - mean-squared curvature
  - coil-coil distance
  - coil-surface distance

Force Optimization Results



References

Hurwitz, S., Landreman, M., & Antonsen Jr, T. M. (2023). Efficient calculation of self magnetic field, self-force, and self-inductance for electromagnetic coils. *arXiv:2310.12087*.

Landreman, M., Hurwitz, S., & Antonsen Jr, T. M. (2023). Efficient calculation of self magnetic field, self-force, and self-inductance for electromagnetic coils. II. Rectangular cross-section. *arXiv:2310.12087*.